Physics 530-11
Penrose diagram of Schwartzscild

## 0.1 Kruscal Schwartzschild

The Schwartzschild metric is

$$ds^2 = (1 - \frac{2M}{r})dvdu - r^2 d\Omega^2 \quad (1)$$

wehre $d\Omega^2 = d\theta^2 + sin(\theta)^2 d\phi^2$ and

$$u = t - r^* \quad (2)$$

$$v - t + r^* \quad (3)$$

$$r^* = r + 2 * M \ln(\frac{r}{2M} - 1) \quad (4)$$

Thus$v - u = 2r^* + 4\ln(\frac{r-2M}{2M})$ and $e^{(v-u)/4M} = (\frac{r-2M)}{2M} e^{r/2M}$ and we can write

$$ds^2 = \frac{2M}{r} e^{-r/2M} e^{(v-u)/4M} dudv - ((v-u)/2 - 2Mln(\frac{r}{2M} - 1)) \quad (5)$$

We now define

$$\begin{array}{ll} \hat{U} = -4Me^{-u/4M} & u > 0 \\ \hat{U} = u - 4M & u < 0 \\ \hat{V} = 4Me^{v/4M} & v < 0 \\ \hat{V} = v + 4M & v > 0 \end{array} \quad (6)$$

Now define

$$V = tan(\hat{V}/4M) \quad (7)$$

$$U = tan(\hat{U}/4M) \quad (8)$$

and continue to $\hat{U} > 0$ and $\hat{V} < 0$ by $\hat{U} \leftrightarrow \hat{V}$

for large r, $v - u = 2r^* = 2r(1 + 2M\frac{\ln(r/2M-1)}{r})$ This approaches r for large r, but the $2M\ln(r/2M - 1)$ is a singular function as $r = sin(V - U)/(cos(V)cos(U))$ goes to infinity. Thus the $U = -\pi/2$ and $V = \pi/2$ surfaces are well behaved but the metric as one approaches them is not very well behaved. and is especially badly behaved as one approaches both of those limits. $I_0$, spacelike infinity, is badly behaved not only in the conformal factor going to infinity there, but also that the conformal metric is badly behaved there. Similarly, the metric at $I^{\pm}$ is badly behaved unlike the case for flat spacetime.

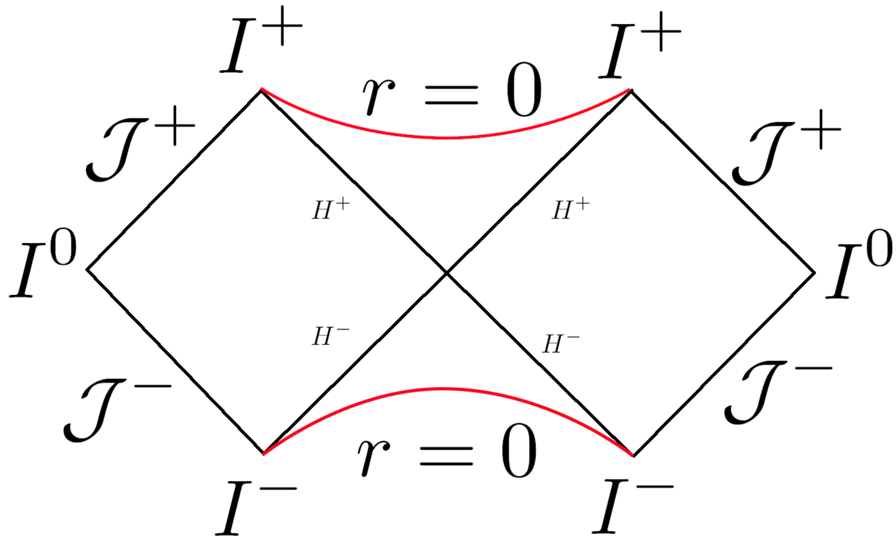


Figure 1: The Penrose conformal diagram for the eternal (Kruscal) black hole.