Cosm.

$$ds^2 = -dt^2 + \frac{dr^2}{1 - k\left(\frac{r}{r_0}\right)^2} + r^2 d\Omega^2$$

$$H^2 = -\frac{k}{r_0^2 a^2} + \sum_i \frac{8\pi G}{3} \frac{\rho_{i,1}}{\underline{a^{3(1+\alpha)}}}$$

$$\alpha_i = \frac{p_i}{\rho_i}$$

$\alpha = 0$ Dust $\quad a = \left(\frac{t}{t_0}\right)^{2/3} a_0$

$\alpha = 1/3$ Relativ. $\quad a = \left(\frac{t}{t_0}\right)^{1/2} a_0$

$\alpha = -1$ "Cosm. Const." $\quad a = e^{+\lambda t} a_0$

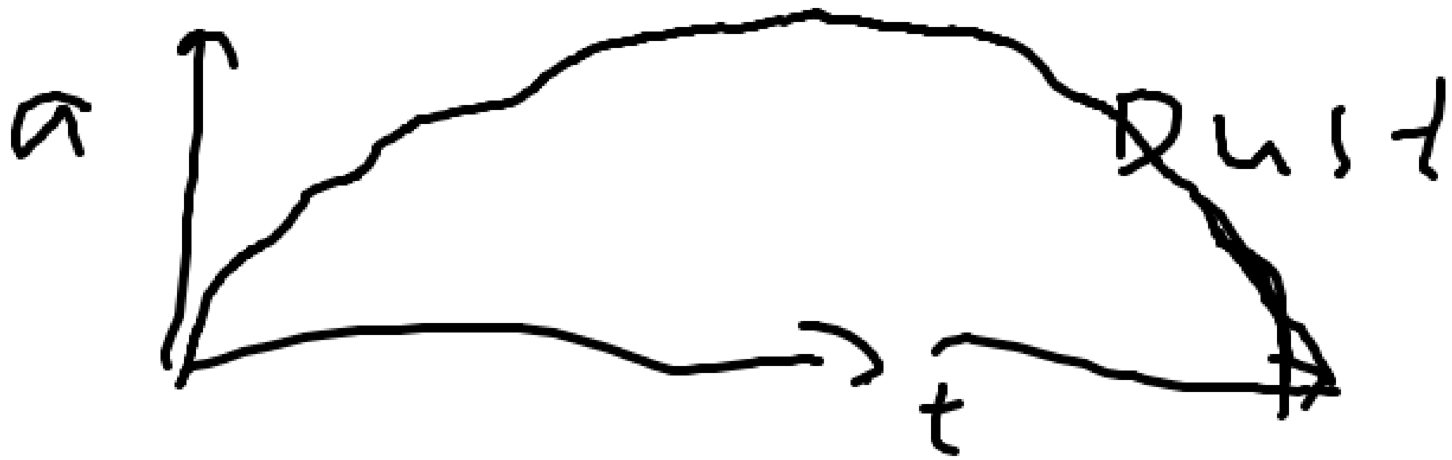
$\alpha < 0$, $p < 0$ tension

all $k = 0$ flat slices.

If $k = +1$ and $\alpha > -1/3$

$\frac{k}{a^2 r_1^2}$ term eventually dominates.

- There is a max to radius of universe. (a has a max)



If $\alpha < -1/3$

Then $\frac{k}{r_0^2 a^2}$ dominates for

small a. $\rightarrow$ minimum

size of universe.

($\alpha = -1$ then

get minimum

for $k = 1$

Dust with holes

Swiss Cheese model

schw. metric.

mass.
B.H. M

Dust follows Geodesic.

(r, θ, ϕ const) choose mass so boundary dust is geod for both cosm. + Schw.

If the universe is flat $(k=0)$ $a = t^{2/3}$ $\Rightarrow$

$$\underbrace{a^2 r^2}_{t^{2/3} r_0} (d\Omega^2)$$

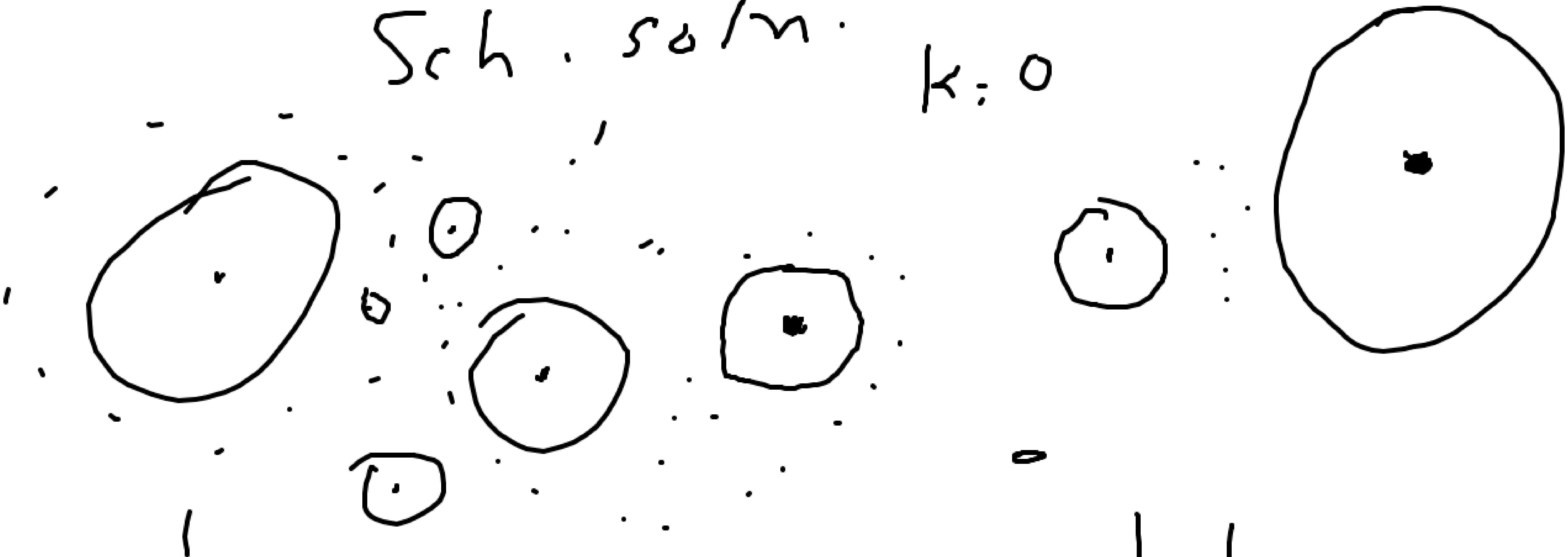
$\Rightarrow$ radial geod which have 0 rad. vel. at ∞

If $k=+1$ $\longrightarrow$ geodesic is bound geodesic.

but $a(t)$ has a max.

$k=-1$, corresponds to unbound vel of particle in Sch. soln.

$k=0$



Inhomogeneous model.

Limitations.

1) Masses in center of empty regions → tied to dust density outside and size of empty region.

\+ all "holes" have dust bound in same way (bound, marginal, unbound in same way.)

k = 0 universe

$$ds^2 = -dt^2 + a^2(t)\,(\underbrace{d\Sigma^2}_{\text{spatial metric}})$$

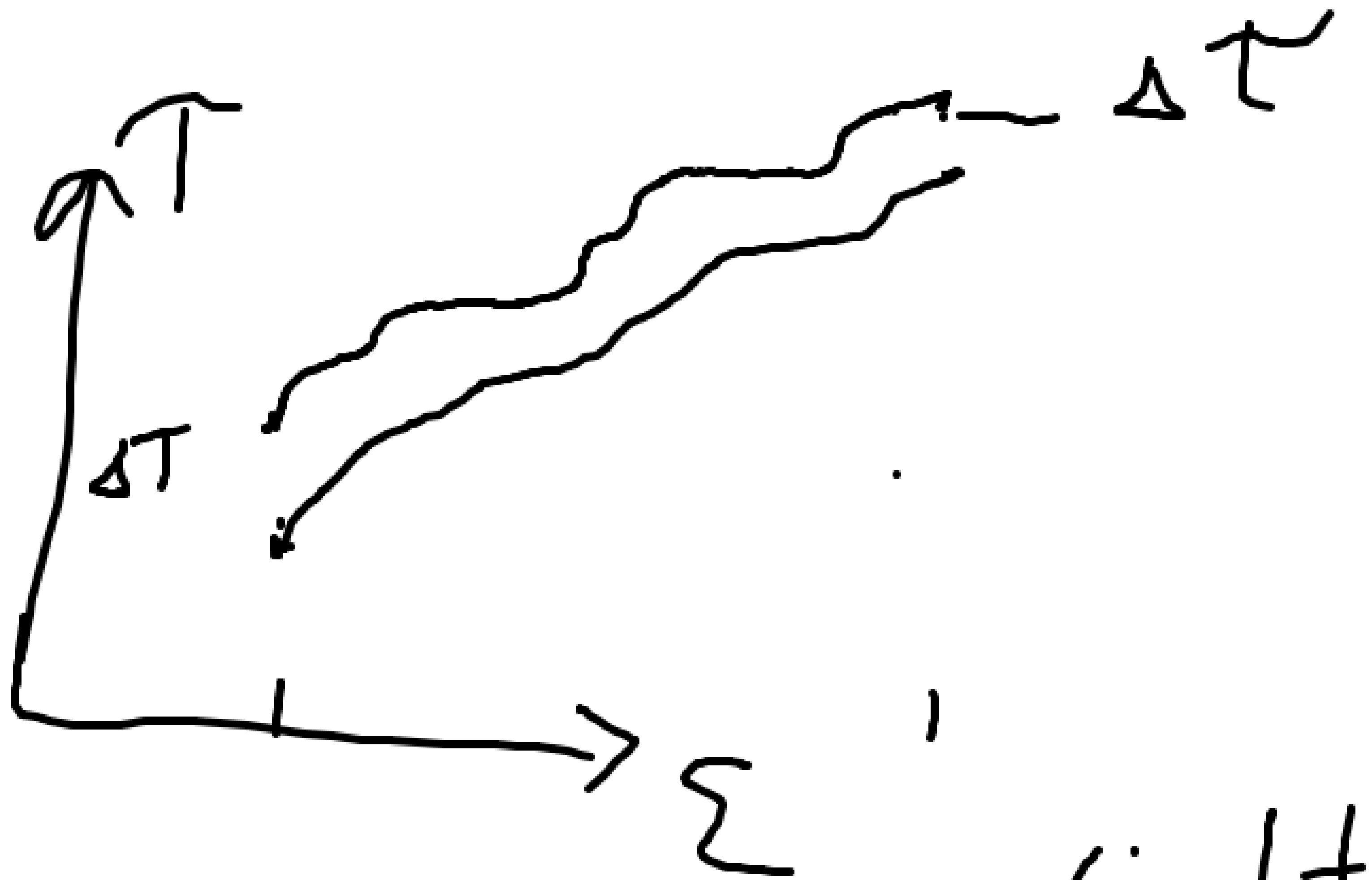
$$\frac{dr^2}{1-k\left(\frac{r}{r_0}\right)^2} + r^2 d\Omega^2$$

$$= \underbrace{a^2(t)}_{\text{conf. factor}} \left(-\underbrace{\left(\frac{dt}{a(t)}\right)^2}_{dT^2} + d\Sigma^2 \right)$$

$$T = \int \frac{dt}{a(t)}$$

Null Geodesics are same.

$$-d\tau^2 + d\Sigma^2$$



$\Delta\tau$ is same at each spatial point.

$$\Delta\tau = \Delta\int\frac{dt}{a(t)} \simeq \frac{\Delta t}{a(t)}$$

$$\Delta t_f = \Delta t_i \frac{a_f}{a_i}$$ → grav. cosm redshift

$$V_f = V_i \frac{a_i}{a_f} \rightarrow \text{Cosm red shift.}$$

$$\frac{a_i}{a_f} = \frac{a_f - \dot{a}T}{a_f} = 1 - H\underline{T}$$

$$= 1 - HL$$

Hubbles Law.

T is proper time sep. betw. ommission + abs.

and is distance light travel.

$k=0$ radial motion

$$\frac{\delta}{\delta r}\int -\left(\frac{dt}{ds}\right)^2 + a^2\,(dr/ds)^2 + r^2\left(\frac{d\theta}{ds}^2 + \sin^2\theta\,\frac{d\phi}{ds}^2\right)$$

$$= -\frac{d\,a^2\,dr/ds}{ds} + 2r\left(\frac{d\Omega^2}{ds}\right) = 0$$

If $\frac{d\theta}{ds} = \frac{d\phi}{ds} = 0$

$$a^2\,dr/ds = \text{const.}$$

$$a\,dr/ds = \frac{\text{const}}{a}$$

peculiar motions of
geodesics die out.

De Sitter $\quad \alpha = -1$

$k = 0$

$$\left(\frac{\dot{a}}{a}\right)^2 = G\frac{8\pi}{3}\rho$$

$$a = e^{\pm\sqrt{G\frac{8\pi}{3}\rho}\; t}$$

$k = 1$

$$\dot{a}^2 = -\frac{1}{r_0^2} + \overbrace{\frac{8\pi G}{3}\rho}^{\lambda^2} a^2$$

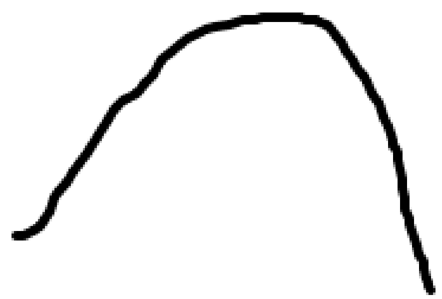
$$\dot{a}^2 - \lambda^2 a^2 = -\frac{1}{r_0^2}$$

$$a = \frac{1}{\lambda r_0}\cosh \lambda t$$

If $k=-1$

then $a = \dfrac{1}{\lambda r_0} \sinh \lambda t$

$\longrightarrow$ singularity ??



All these solutions are the same S.T. in diff coords.

$k=0 \to t \to -\infty$

$k=-1 \to t \to 0$

only complete soln. is $k=+1$

only complete solution all geodesics have parameter from $-\infty$ to ∞.

The $k=0$ $t=$const slices are "null" slices parallel to one assymptotes.

Static coord

$$-dt^2 + e^{2\lambda t}\left(dr^2 + r^2 d\Omega^2\right)$$

$$\rho = e^{\lambda t} r$$

$$dr = e^{-\lambda t}\left(d\rho - \lambda\rho\, dt\right)$$

$$-(1-\lambda^2\rho^2)\,dt^2 - 2\lambda\rho\, d\rho\, dt + d\rho^2 + \rho^2 d\Omega^2$$

$$= -(1-\lambda^2\rho^2)\underbrace{\left(dt + \frac{\lambda\rho}{1-\lambda^2\rho^2}\,d\rho\right)}_{d\tau}{}^2 + \frac{d\rho^2}{1-\lambda^2\rho^2} + \rho^2 d\Omega^2$$

$$ds^2 = -(1-\lambda^2\rho^2)\,d\tau^2 + \frac{d\rho^2}{1-\lambda^2\rho^2} + \rho^2(d\Omega^2)$$

Static! $\rho = \frac{1}{\lambda}$ **sing**

Looks a lot like Schw.

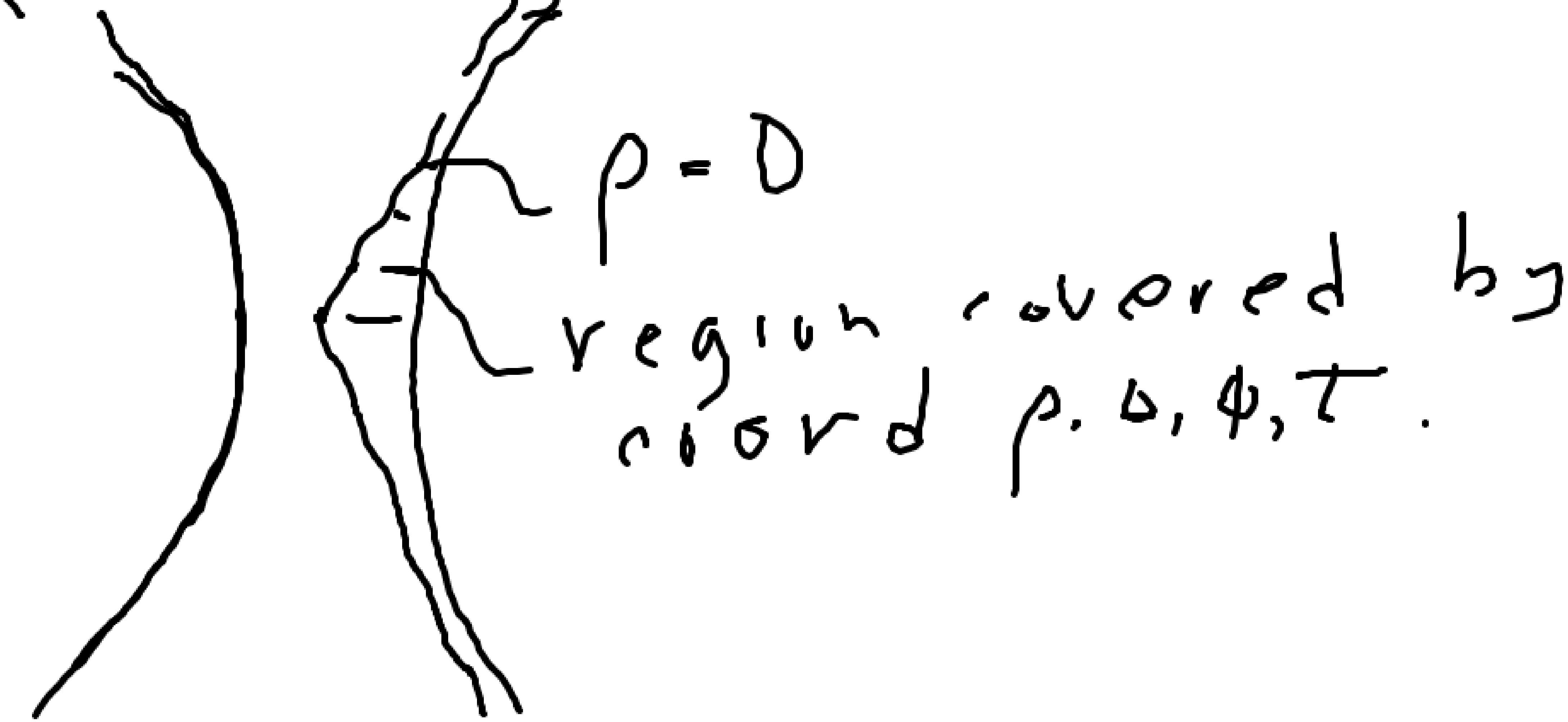
"singularity") at $\rho = \frac{1}{\lambda}$

Horizon. for $\rho=0$.

Anything at $\rho > \frac{1}{\lambda}$ cannot comm. with $\rho=0$.

$\rho = \frac{1}{\lambda}$ is genuine horizon.

$\rho = \frac{1}{\lambda}$ is a light like surface.



(only case where term horizon is properly used.

common useage. $\boxed{Hr = 1}$

radius. where (rate of change of dist is light speed) (commonly called horizon. WRONG

Hubble RADIUS.