Cosmology

No static Solution.

Assumption Universe is Isotropic & Homogeneous in space.

$$\overbrace{T^2}^{T,} - \overbrace{W^2 - X^2 - Y^2 - Z^2}^{w,\, x,\, y\; z\; \rho^2} = -\underbrace{R^2}_{\text{const.}}$$

Same Sym. gp$^{\#}$ as Flat ST.

$$dt^2 - dw^2 - dx^2 - dy^2 - dz^2 \Longrightarrow dt - \sinh^2(t)\,(dr^2 + r^2 d\Omega^2)$$

cosm. <u>const</u>

$$G_{\underline{\mu\nu}} = \underline{\Lambda}\ \underline{g_{\mu\nu}}$$

In order to ge static universe he had to have both Λ, and matter.

Constraint eqns. of GR.

Set up metric.

Spatial metric which is Hom & isotropic. $\rightarrow$ $\underset{\sim}{6}$ Killing vectors.

$$w^2 + x^2 + y^2 + z^2 = \tilde{R}.$$

$$-w^2 + x^2 + y^2 + z^2 = +R^2$$

$$\boxed{\frac{dr^2}{1-kr^2}} + r^2(d\theta^2 + \sin^2\theta \, d\tilde{\phi})$$

$k = -1, 0, 1$ $\quad$ $k=-1$ $\quad r = \sinh \rho.$

$$d\rho^2 + \sinh^2(\rho)(d\theta^2 + \sin^2\theta \, d\phi^2)$$

flat $k=0$

$$dr^2+r^2(d\theta^2+\sin^2\theta\, d\phi^2)$$

$k=1$

sphere

$$d\rho^2+\sin^2\rho\,(d\theta^2+\sin^2\theta\, d\phi^2)$$

$$a^2(t)\left(\frac{dr^2}{1-kr^2}+r^2 d\Omega^2\right)$$

time space $dt\,dr,\ dt\,d\theta,\ dt\,d\phi$

$$\cancel{dt\left((\ )\frac{dr}{ds}+(\ )\frac{d\theta}{ds}+(\ )\frac{d\phi}{ds}\right)} \quad 0$$

by isotropy

$$-a(t)^2 dt$$

$$\tau=\int a(t)\,dt$$

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$G^{00} \Longrightarrow \quad H = \frac{\dot a}{a} = \partial_t \ln(a(t))$$

$$H^2 = -\frac{k}{a^2} + \frac{8\pi}{3}\rho$$

$\partial\rho \quad \rho$ (coord volume)

$r < r_0 \simeq 0$

ρ (physical volume) $= \rho\,(a^3$ coord vol$)$

$\partial_t\,(\rho$ phys vol$) = -p\,\partial_t\,($phys vol$)$

$$\partial_t(\rho a^3) = -p\,\partial_t(a^3)$$

If $p = v^2 \rho$

$$\frac{p}{\rho} = (\text{vel of sound})^2$$

$$\partial_t\left(\rho\, a^{3(1+v^2)}\right) = 0$$

$$\rho = \frac{\rho_0}{a^{3(1+v^2)}}$$

Diff. of matter. (non interacting)

$$H^2 = -\frac{k}{a^2} + \frac{8\pi}{3}\sum \frac{\rho_{i0}}{a^{3(1+v_i^2)}}$$

Static universe.

If $\rho_i > 0$ then $k = 1$

But also need $\ddot{a}$ to be 0.

$$\dot{a}^2 = -k + \frac{8\pi}{3}\sum_i \frac{\rho_{i0}}{a^{(1+3v_i^2)}}$$

$$2\ddot{a}\cancel{a} = \frac{8\pi}{3}\sum_i \frac{\rho_{0i}\left(-(1+3v^2)\right)}{a^{(2+3v^2)}}\cancel{a}$$

$v^2 = -1$

Need Cosm const ($v^2 = -1$) and ordinary matter.

unstable ($\ddot{a}$ positive if
a increases away from
equil. ($\dot{a}=0$)

1929 Hubble.
Hyades. — brightness
Varies (oscillates)
oscil. freq depends on
brightness.
– Find Cepheid variable stars. in fuzzy
blobs of stars. ⟶ find distance.

Spectroscopically measure the H line. $\longrightarrow$ displaced to red.

Further away the greater the red shift.

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$$

physical dist changes as $\frac{da}{dt}$ comov dist $= \dfrac{da/dt}{a}$ phys dist

H

Densities large in past.

$p \simeq \frac{1}{3}\rho$ $\qquad \rho \sim a^{-4}$

#relpart $\sim a^{-3}$

energy/part $\sim \frac{1}{a}$

Temp drops.

Nonrelmatter

$\rho \sim \frac{1}{a^3}$

electrons captured by protons

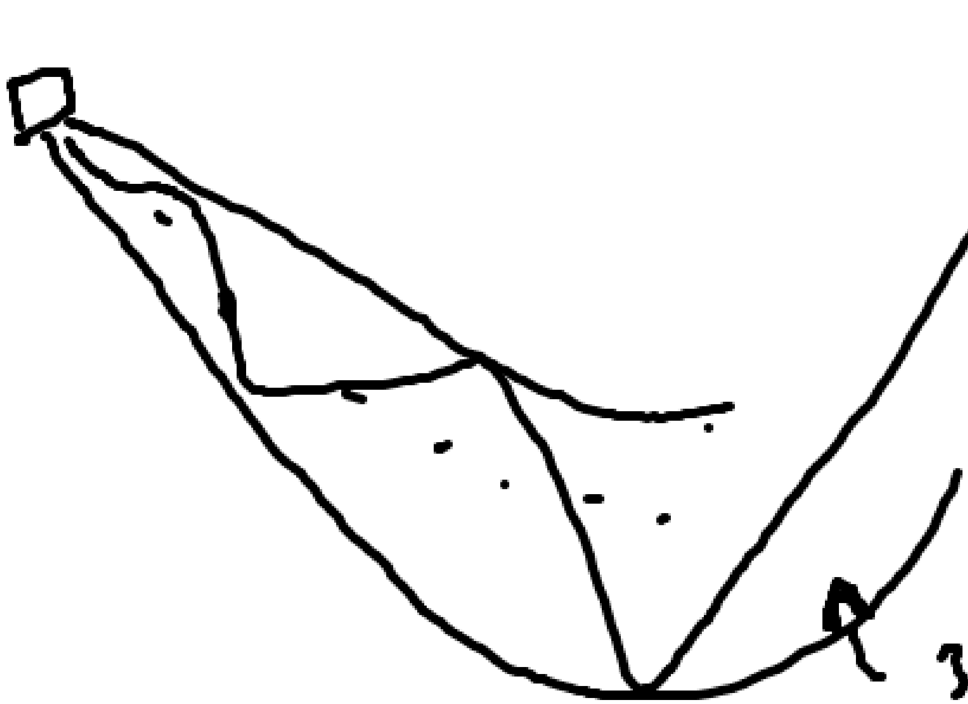
etc. – matter becomes transparent

- Gamov ~ 10's °K.

Peebles $\sim 5-10$°K.

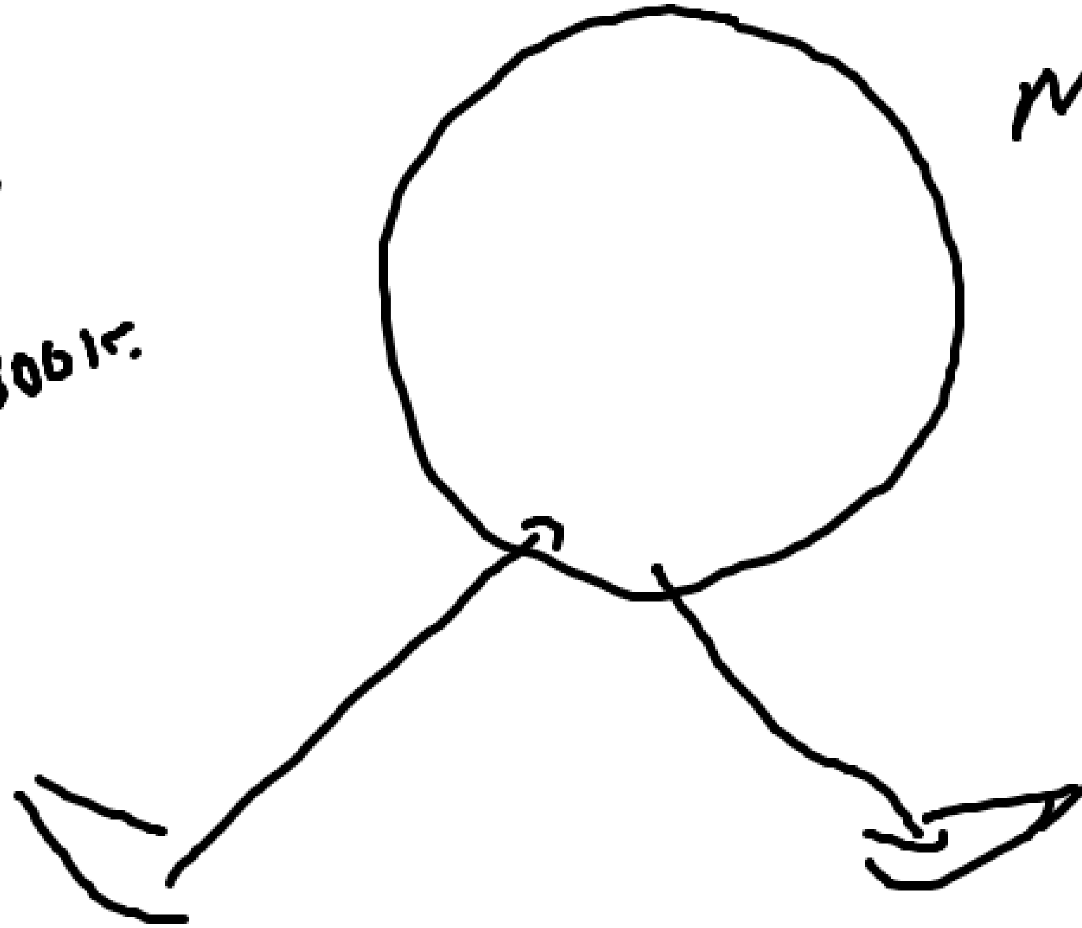
Dicke. Experiment Roll, Wilkinson

Penzias. ATT Bell Labs.



Echo satt.

Comm. sat.



Mylar sphere.