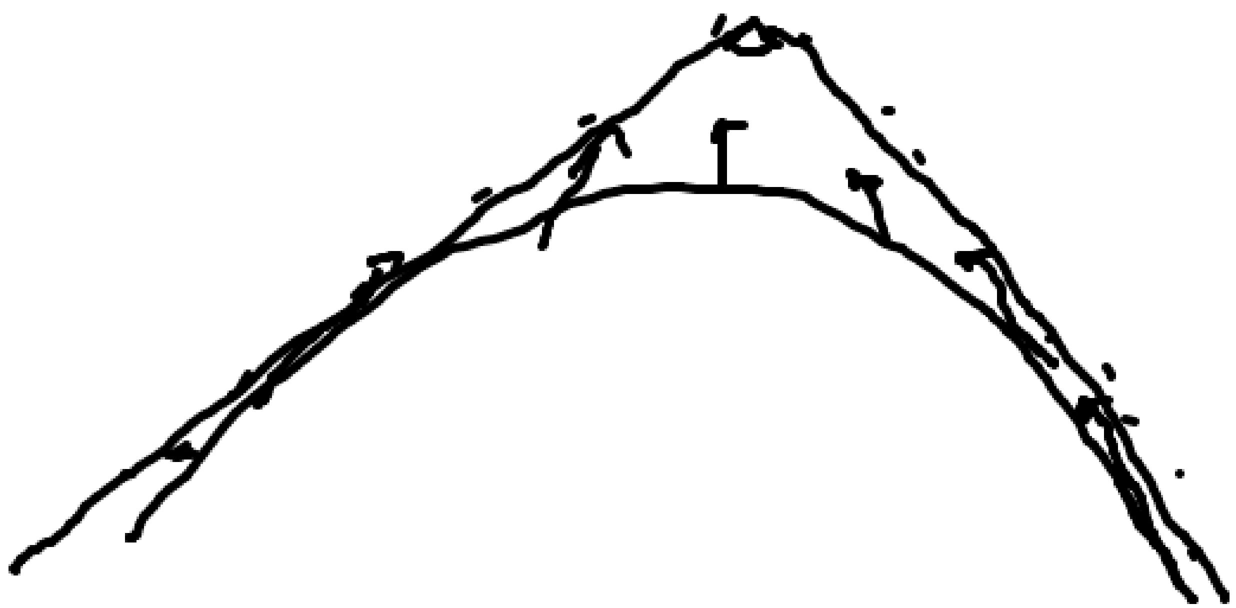
Coord Conditions

$\alpha = 1$, $\beta^i = 0$



$\underline{K = 0}$, $\dfrac{\partial K}{\partial t} = 0$

Volume of coord sphere does not change in time

Harmonic coods.

Linearized gravity:

$$\partial_i \bar{h}^{ij} = \partial_i \left(h^{ij} - \frac{1}{2} h \eta^{ij} \right) = 0$$

$$\Box \bar{h}^{ij} = \ldots$$

Demand coods - obey wave eqn.

$$\nabla_\alpha g^{\alpha\beta} \nabla_\beta \boxed{X^\gamma} = 0$$

$$\nabla_\alpha g^{\alpha\beta} \partial_\beta X^\gamma = 0 = \frac{1}{\sqrt{g}} \overbrace{\partial_\alpha \left(\sqrt{g}\, g^{\alpha\gamma} \right) = 0}$$

$$\partial_\alpha\left(\sqrt{g}\, g^{\alpha\beta}\right) = 0$$

$$\underbrace{\partial_\alpha\left(\sqrt{g}\, g^{\alpha\beta}\right)}_{\underbrace{\partial_0\left(\alpha\sqrt{\gamma}\, \overset{-\frac{1}{\alpha^2}}{g^{00}}\right)}_{\partial_0\left(\frac{1}{\alpha}\right) + \cdots}} = \underbrace{F(K_{ij}, \gamma_{ij})}_{\partial_i\left(\alpha\sqrt{\gamma}\,\beta^i\right) + \cdots}$$

$$\partial_0\left(\frac{1}{\alpha}\right) + \cdots = F$$

Eqn of motion for α.

Pretorius used in order to produce stable evolution of numerical B.H. orbits.

Problem

"Real" singularities.

Matter collapse to B.H.
real singularity in center
of B.H.

B.H. Horizon.

Horizon is surface out of which
nothing can get to infinity
(far away from the B.H.)

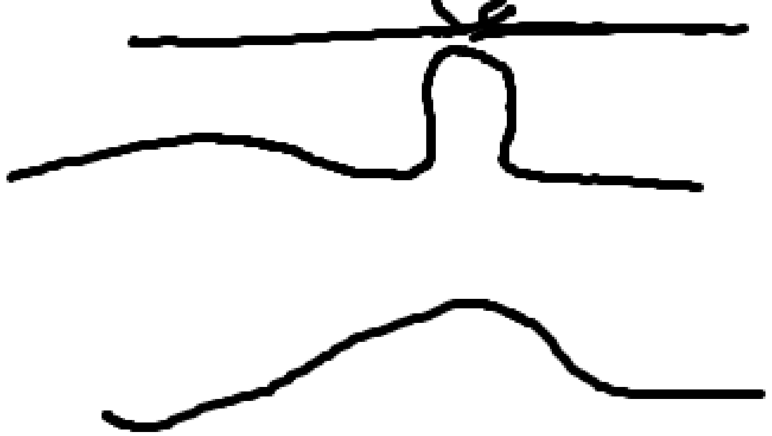
Black hole excision.
(Don't calculate anything inside the Horizon)

Hawking proved Horizon has to be a null surface. – 3-D surface which contains null geodesics.

Horizon is ruled by null geod.

(assumptions. null energy cond.

$T_{\mu\nu} l^\mu l^\nu > 0$.



How do we find the horizon?

can't, but can come close.

Apparent Horizon

local condition, depends on the choice of time

$n_\beta = \alpha \, \underline{\partial_\beta t}$

$t = \text{const}.$

2 surface.

perp to 2 surface (unit length) inside $t=$ const surface)

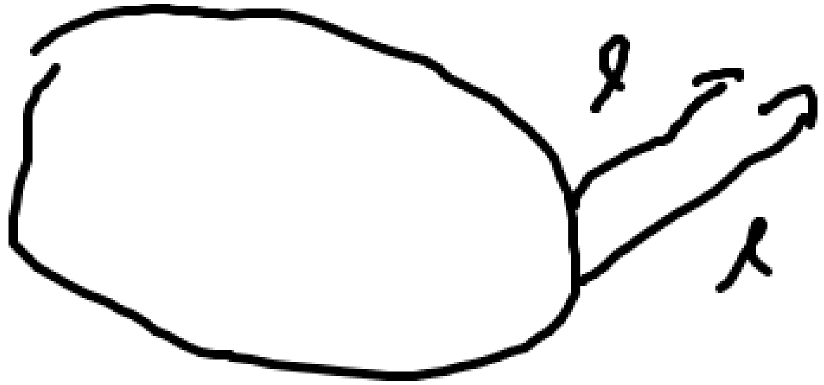
$l^\alpha = n^\alpha \pm s^\alpha$ - Null vector.

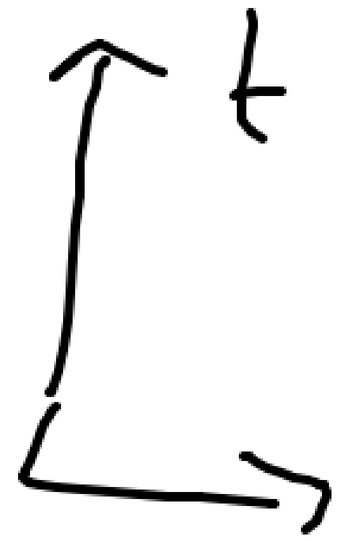
$$g_{\alpha\beta} l^\alpha l^\beta = \underbrace{g_{\alpha\beta} n^\alpha n^\beta}_{-1} \pm \overbrace{2 g_{\alpha\beta} n^\alpha s^\beta}^{0} + \underbrace{g_{\alpha\beta} s^\alpha s^\beta}_{+1}$$

$$= 0$$

At every point we have 2 null vectors. perp to 2 surface.
Lets. choose the outward pointing s^α. Ie we assume the 2 surface is closed & has an inside & outside.

Parallel transport l^a along the surface.





Demand that on average over the 2-d piece of surface the Diverg is 0.

surface not change area.

$$\Theta = \nabla_\alpha l^\alpha + n^\alpha n^\beta \nabla_\alpha l_\beta - s^\alpha s^\beta \nabla_\alpha l_\beta = 0.$$

- Along l^α the area does not change.

True everywhere along the closed 2 surface, then this is called an apparent horizon.

If it is neg or 0
every where, then trapped
surface.
Closed. nature of surface is
important.
flat spacetime everywhere
ha surfaces which have
0 expansion

$t - x^2 - y^2 - z^2 = 0$

$t = z + z$.

expansion = 0.
plane on cone are orthog to intersection
parabola.
Apparent Horizon except
not closed.

Condition on apparent horizon is nonlocal condition.

Black Holes.

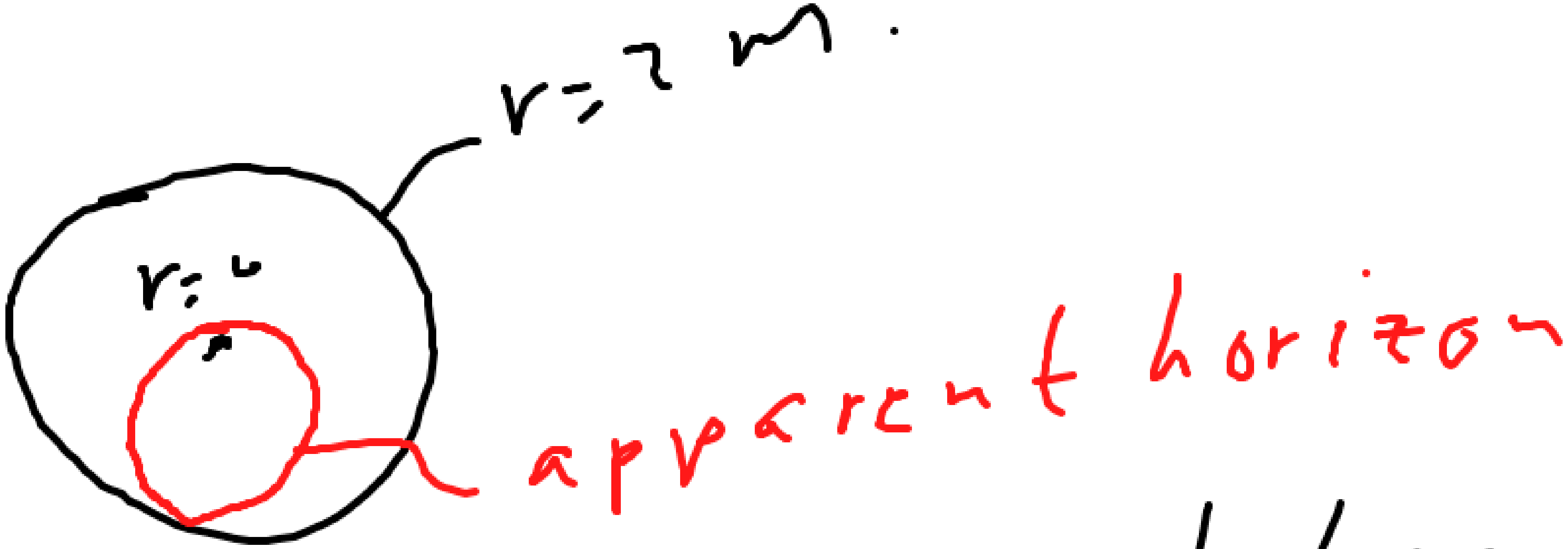
$r=2M$ is an apparent horizon for eternal B.H.

Wald has shown that we can choose a time slicing of E.F. coor. or Kruscal cord.

$$ds^2 = \left(1-\frac{2M}{r}\right) dv^2 - 2\,dr\,dv - r^2 d\Omega^2$$

If we choose $V = f(\theta,\phi)\, r$ as time.

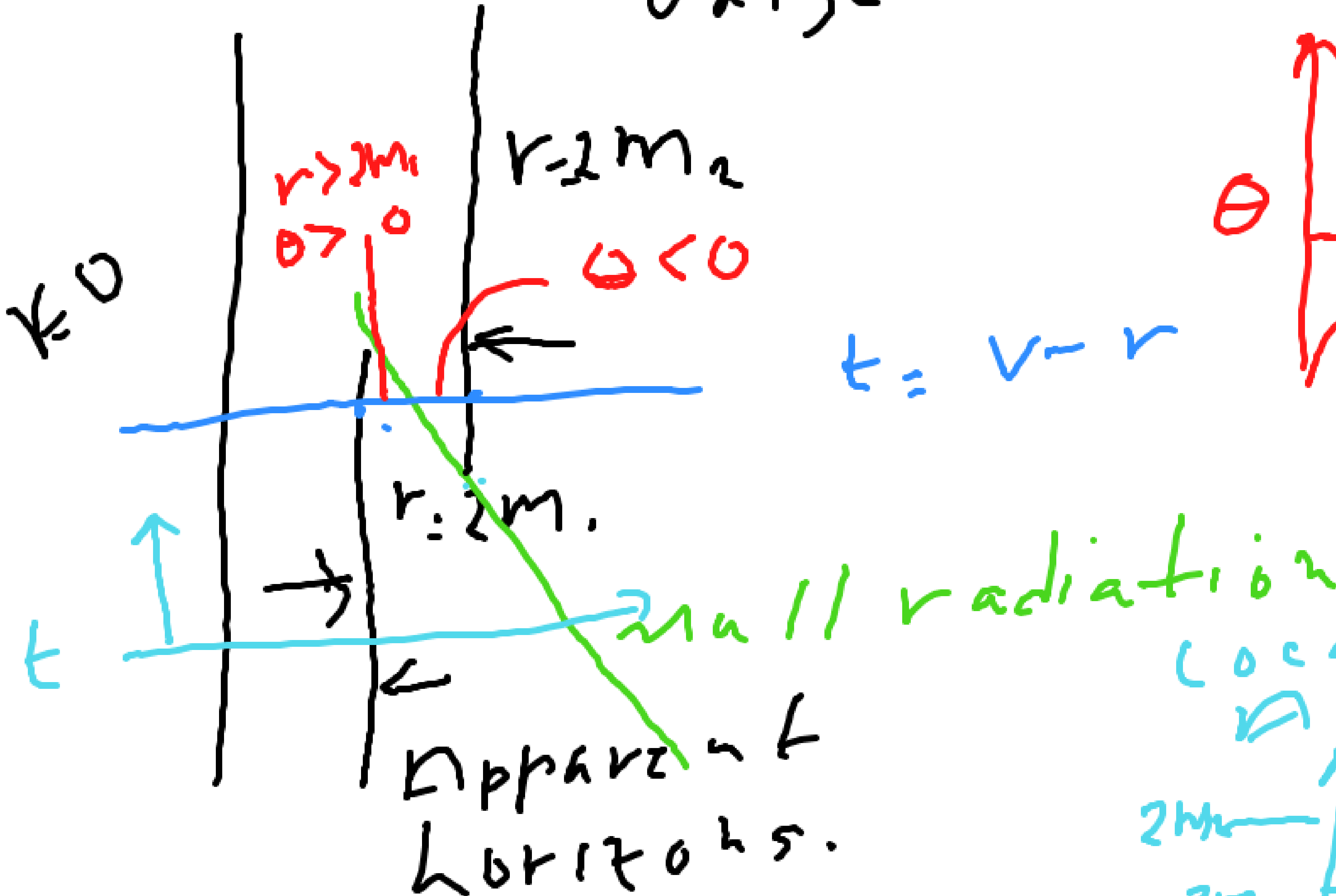
- Can have apparent horizons which touch the singularity



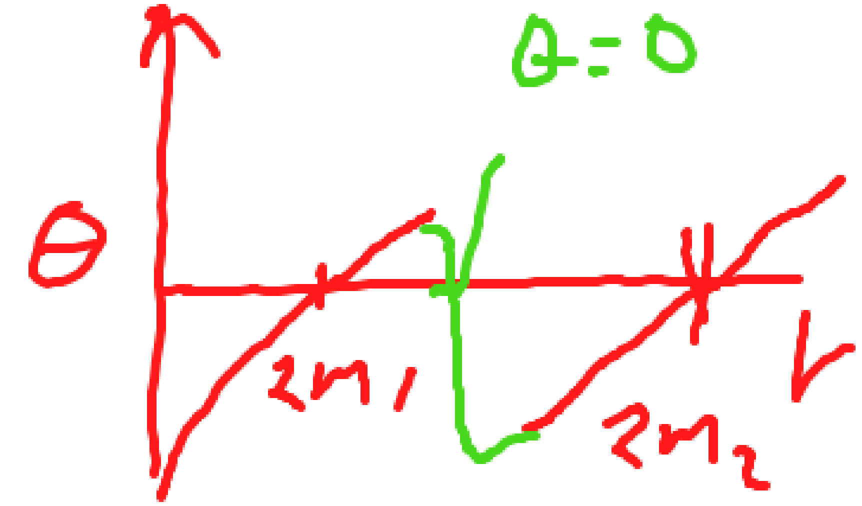
If Null energy condition is satisfied Then θ always decreases on any geodesic null surface. $\rightarrow$ always ends up at Singularity. (Penrose Singularity Thm).

Apparent Horizons always lie inside real Horizon.

Vaidya metric.



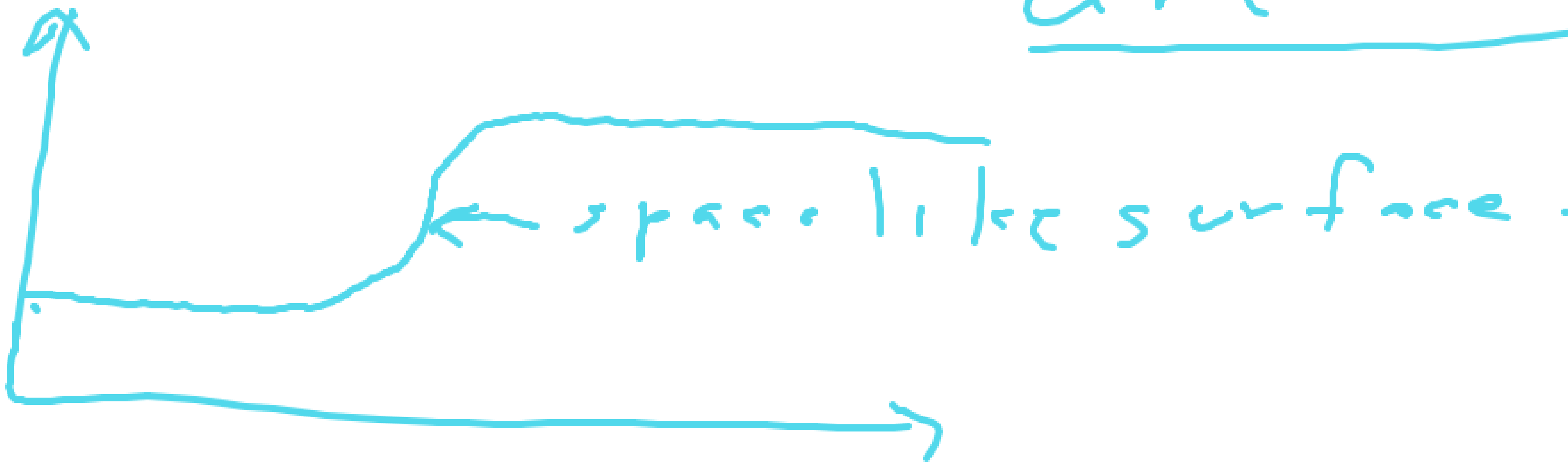
$t = v - r$



location of appr hor.

Locus of App Hor

Apparent Horizons are weird.



True Horizon is teleological.

B.H. excision

- find location of apparent horizon.

throw away anything inside apparent horizon.

(coords falling into B.H.)

Difficult.

(change r.h.s of the condition

$\partial_\alpha \sqrt{g}\, g^{\alpha\beta} = H^\beta(\gamma, \pi)$ or $\underline{H(\gamma, K)}$)