E M

$F_{\mu\nu} = -F_{\nu\rho}$

Max. Eq.

$$\partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0 \leftarrow$$

$$\partial_\mu F^{\mu\nu} = 0 \leftarrow$$

$$\partial_t E - \nabla \times B = 0 \qquad \nabla \cdot E = 0$$

$$\partial_t B + \nabla \times E = 0 \qquad \nabla \cdot B = 0$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\rho(\cancel{\partial_\mu A_\nu} - \cancel{\partial_\nu A_\mu})$$
$$+ \partial_\mu(\cancel{\partial_\nu A_\rho} - \cancel{\partial_\rho A_\nu})$$
$$+ \partial_\nu(\cancel{\partial_\rho A_\mu} - \cancel{\partial_\mu A_\rho})$$

$$\mathcal{L} = \frac{1}{4}\int (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)\, d^4x$$

Vary w.r.t. A_μ

$$\longrightarrow \quad \nabla \cdot E = 0 \qquad \frac{\partial B}{\partial t} + \nabla \times E = 0$$

$$\partial_0 A_0 \qquad \frac{\delta \mathcal{L}}{\delta \dot{A}_0} = 0$$

$$\mathcal{L} = \frac{1}{4}\int A_0\, \partial_i (\partial^0 A^i - \partial^i A^0) + (\partial_i A_j - \partial_j A_i)(\partial^i A^j - \partial^j A^i) + (\partial^0 A^i - \partial^i A^0)\, \underline{\partial_0 A_i}$$

$$\frac{\delta \mathcal{L}}{\delta \partial_0 A_i} = \vec{E}$$

E is Momentum conjugate to A_i

$$\mathcal{L} = \frac{1}{4}\int 2(E\cdot E - B\cdot B) \qquad \vec{\nabla}\times\vec{A}$$

$$H = \frac{1}{2}\int \Big(\underset{\pi\cdot\pi}{E\cdot E} + B\cdot B\Big) + A_0 \underline{\nabla\cdot E}$$

$$\underline{\nabla\cdot\pi = 0}.$$

Constraint on intial data.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Gauge trans. $\underline{\Phi}$ $\quad A_\mu \to \underbrace{\partial_\mu \underline{\Phi} + A_\mu}_{\tilde{A}}$

$\tilde{A}$ will also satisfy the Field eqns.

$$B = \cancel{\partial_i}\,\nabla \times \vec{A} = \nabla \times (\bar{A} - \nabla \Phi) = \nabla \times \bar{A}$$

Similarly, E is ~~left~~ invar.

$\to$ 2 A_i, 2 Π_i are dynamical
2 degrees of freedom at each pt in s.t.

Initial formulation.

$\vec{E}$, A_i $\quad \nabla \cdot E = 0 \to$ arbitrary trans. on A_i

Place a constraint on A_i
- Gauge Fixing.

A_i chosen "arbit" an π^i but
Get constraint on π^i, and
must impose a constraint
on A_i $\rightarrow$ Initial value
which lead to unique soln
of equations.

Quantization:

① → Impose constraints classically + then Quantize.

② → Try to impose them afterwards. Problem

$\nabla \cdot \vec{E} = 0$

E_x, E_y, E_z are indep operators
an $\nabla \cdot \vec{E} = 0$ → relation between indep ops.

$[A_i, E_j] = \delta_{ij}\, \delta(x, x')$ $\quad [A_i, \int \phi\, \nabla \cdot E]$

$$\left[A_i, \int \phi \nabla \cdot E\right] = 0$$

$$= \left[A_i, \int \nabla \phi \cdot \vec{E}\right]$$

$$= \int \partial_i \phi(x') \delta(x, x') \, dx'$$

$$= \underline{\partial_i \phi} = 0 \, ?$$

Can only smear with const.

Hilbert space for all

A_i, E_i $\quad |\Psi\rangle$

Physical Hilbert space

$$\underbrace{\nabla \cdot E}\, |\Psi\rangle_p = 0.$$

$$\int \phi \nabla \cdot E \, |\Psi_p\rangle \rightarrow -\int \underline{\vec{\nabla}\phi} \cdot \vec{E}\, |\Psi\rangle_p = 0.$$

? Difficult to find $|\Psi\rangle_p$.

Gen Rel.

Lagrangian

$$\mathcal{L} = \int \sqrt{|g|}\, \underline{R}\, d^3x$$

Scalar (Ricci scalar) curvature.

Defined in term metric.

metric physical. — distaces.

Hamiltonian Formulation.

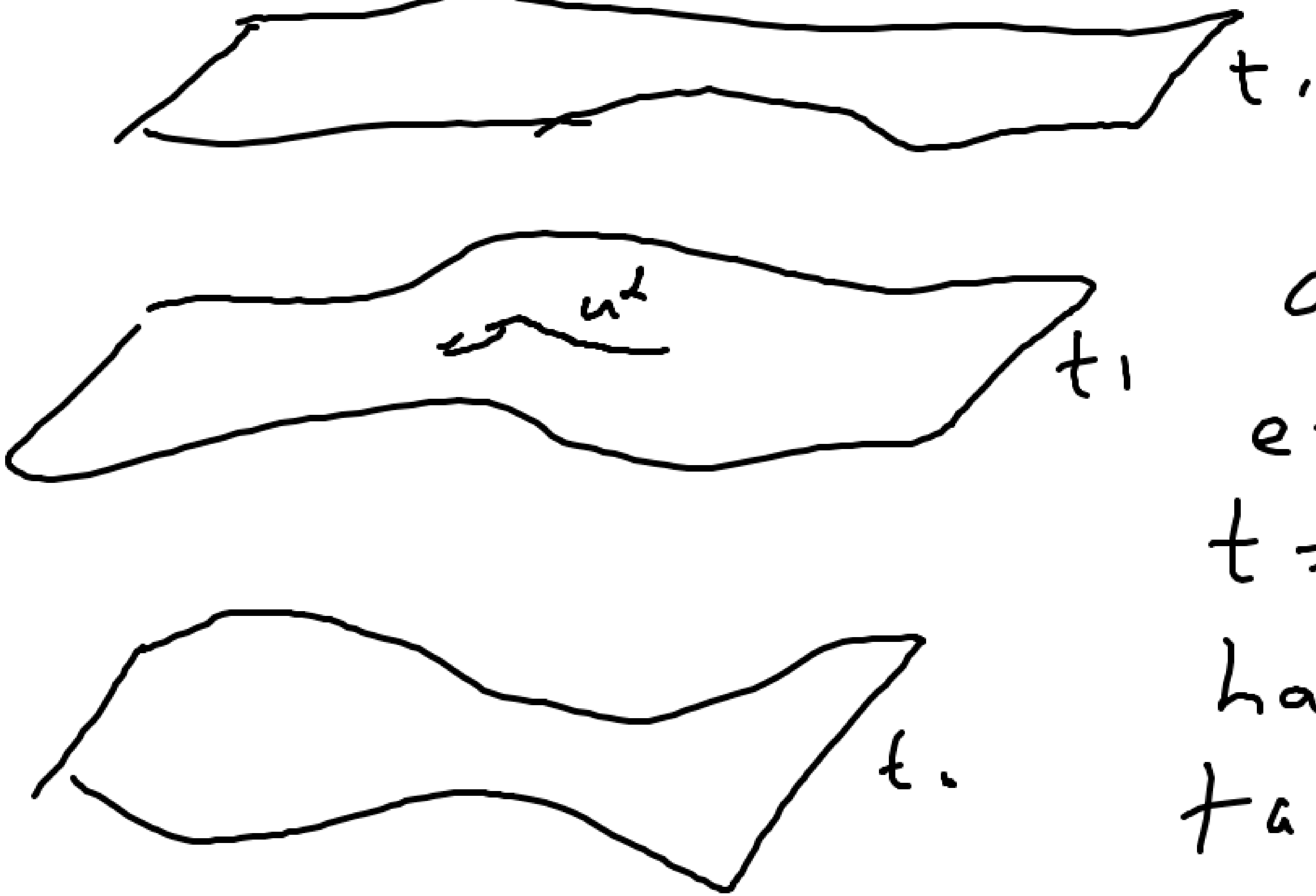
① $$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$\Downarrow$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

time development. from one time to next.

- Arbitrary foliation of S.T. by spacelike sheets.

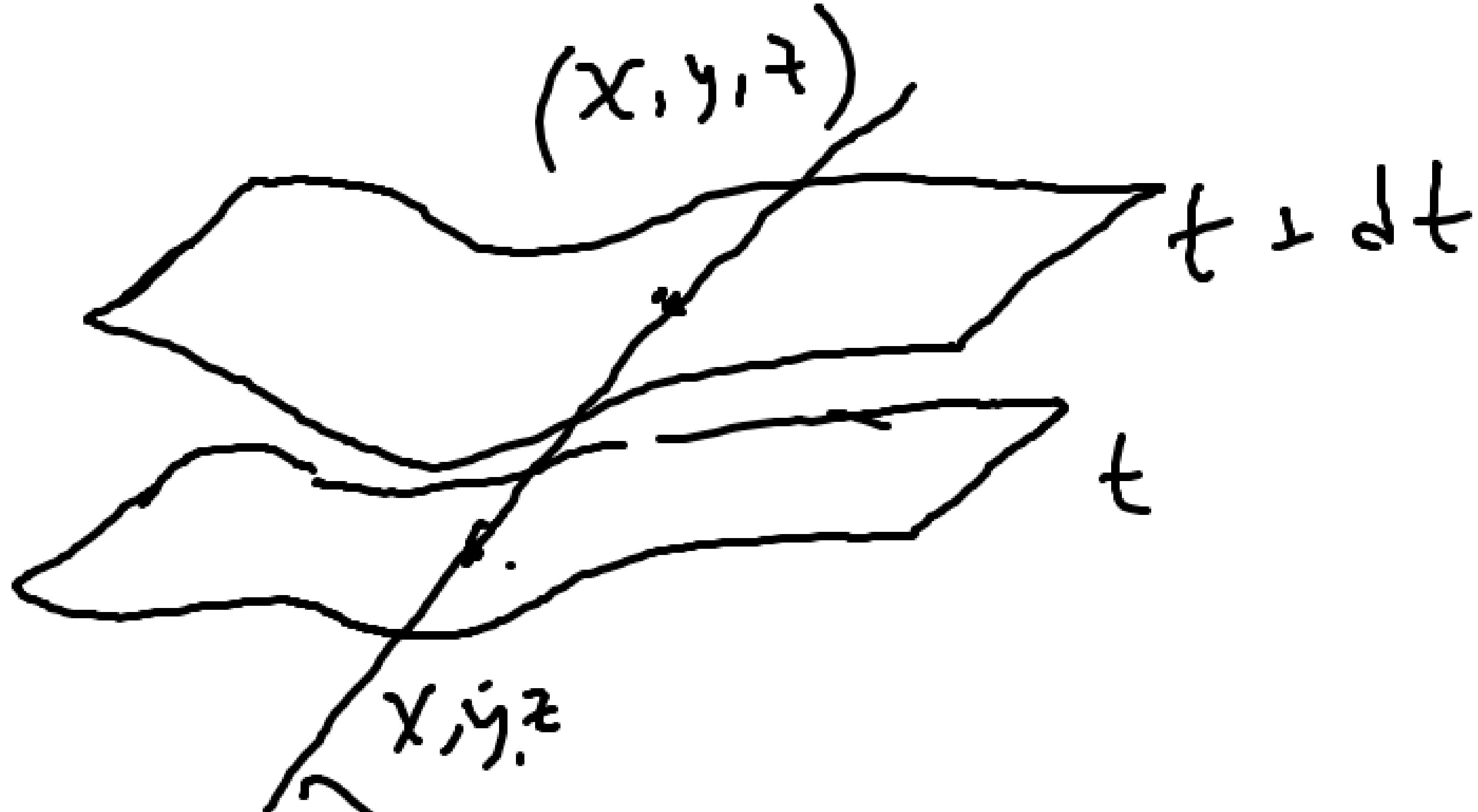


any curve entirely in $t = \text{const}$ sheet has spacelike tangent vectors.

$$g_{\alpha\beta} u^\alpha u^\beta > 0$$

3) → if I refer to coor. inside sheet I use Latin indices, 4vectors use Greek

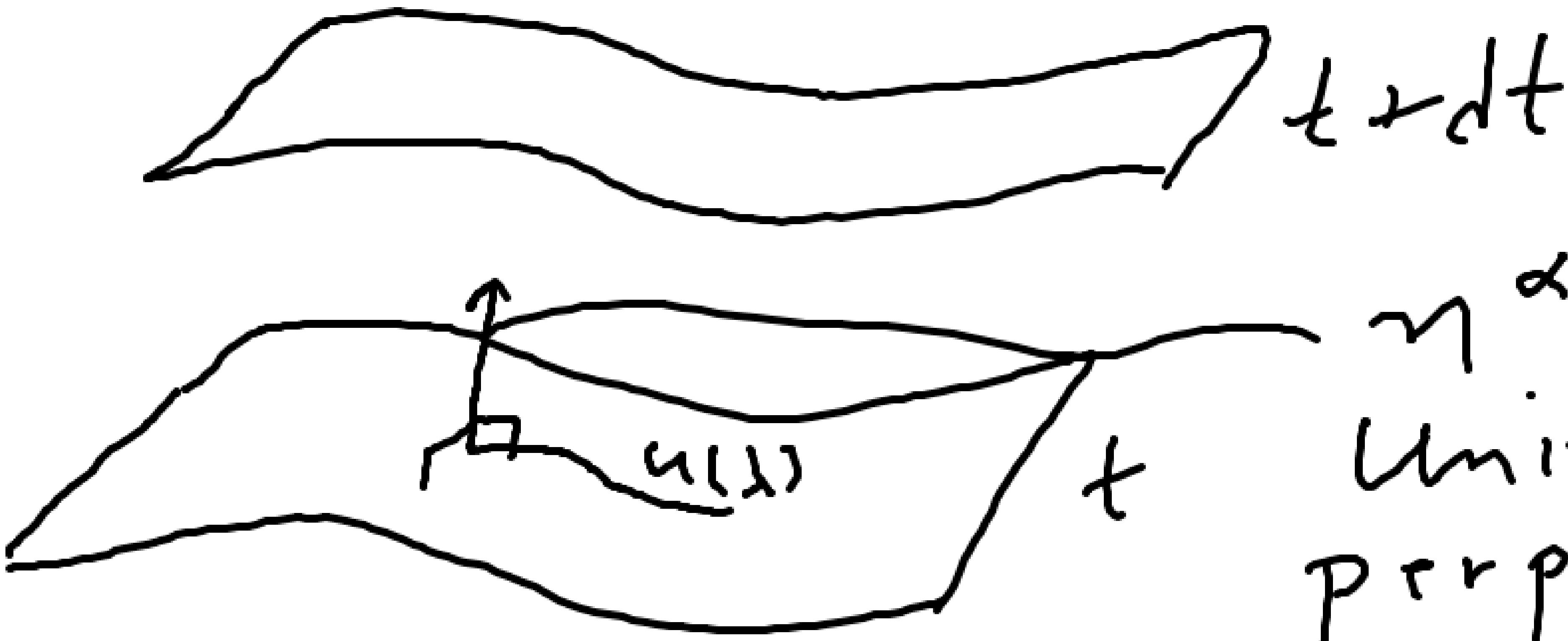
t, x, y, z



line of const (x, y, z)

Parameter on curve will be t.

tang. vector is $(\partial_t)^\alpha dt$



Unit vector perp. to every curve in surface

$(dt)_\alpha \qquad t(u(\lambda)) = \text{const}$

$(dt)_\alpha (\partial_\lambda)^\alpha = 0$

$=$ $(dt)^\alpha$ is $\perp$ to every curve in surface

$n^\alpha = -\frac{1}{\alpha} dt^\alpha \qquad (\; dt_\alpha = (1,0,0,0)$

length of $(dt_\alpha) = g^{00}\,1\,1 = -\frac{1}{\alpha^2}$

$\alpha = \sqrt{-g^{00}}$

$\underline{\beta^i\, dt}$

$(\partial_t)^\alpha\, dt$

$\alpha n^\alpha\, dt$

g

length of tang vectors inside sheet is $g_{ij} \equiv \gamma_{ij}$

$$g_{0i} = \gamma_{ij}\beta^j$$

$$g_{00} = -\alpha^2 + \beta^i\beta^j\gamma_{ij}$$

$$g_{ij} = \gamma_{ij}$$

Use γ_{ij} as my dynamic var.

α, β^i will be Lagrange multipliers.

(no dynamics).

(Pretorius thesis)

laplace.physics.ubc.ca/ People/matt/ Doc/Theses/phd/pretorius.pdf
(2002)

Extrinsic Curvature. α - lapse.
$\beta^i \to$ shift vector.

$\gamma_{ij} \to R_{ijkl}$

$$K_{ij} = \nabla_i n_j \propto \Gamma^0_{ij}$$

$$= -\frac{1}{2\alpha}\left[\partial_t \gamma_{ij} - \hat{\nabla}_i \beta_j - \bar{\nabla}_j \beta_i\right]$$