Penrose Diagrams.

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

length.

$$d\bar{s}^2 = \phi^2(x,y,z,t)\, g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}$$

Null curves, null geodesics of both. is the same.

Flat S.T.

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$= dt^2 - dr^2 - r^2 d\Omega^2$$

$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$

$u = t - r$ $\qquad r = \frac{v-u}{2}$

$v = t + r$

$$= du\,dv - \left(\frac{v-u}{2}\right)^2 d\Omega^2$$

$U = f(u) \qquad dU = f'(u)\,du$

$V = g(v) \qquad dV = g'(v)\,dv$

$ds^2 = \frac{dU\,dV}{f' \, g'} \qquad du = \frac{d(U)}{f'(f^{-1}(U))}$

$$u = \tan(U) \qquad -\infty < u < \infty \qquad -\frac{\pi}{2} < U < \frac{\pi}{2}$$

$$v = \tan(V)$$

$$du = \frac{1}{\cos^2(U)}\, dU \qquad dv = \frac{1}{\cos^2(V)}\, dV$$

$$ds^2 = \frac{1}{\cos^2 U \cos^2(V)}\, dU\, dV - \left(\frac{\tan(V) - \tan(U)}{} \right)^2 d\Omega^2$$

$$r = \tan V - \tan U = \frac{\sin V}{\cos V} - \frac{\sin(U)}{\cos U}$$

$$= \frac{\sin(V-U)}{\cos V \cos U}$$

$$ds^2 = \frac{1}{(\cos V \cos U)^2}\left[dU\, dV - \frac{\sin(V-U)^2}{4}\, d\Omega^2 \right]$$

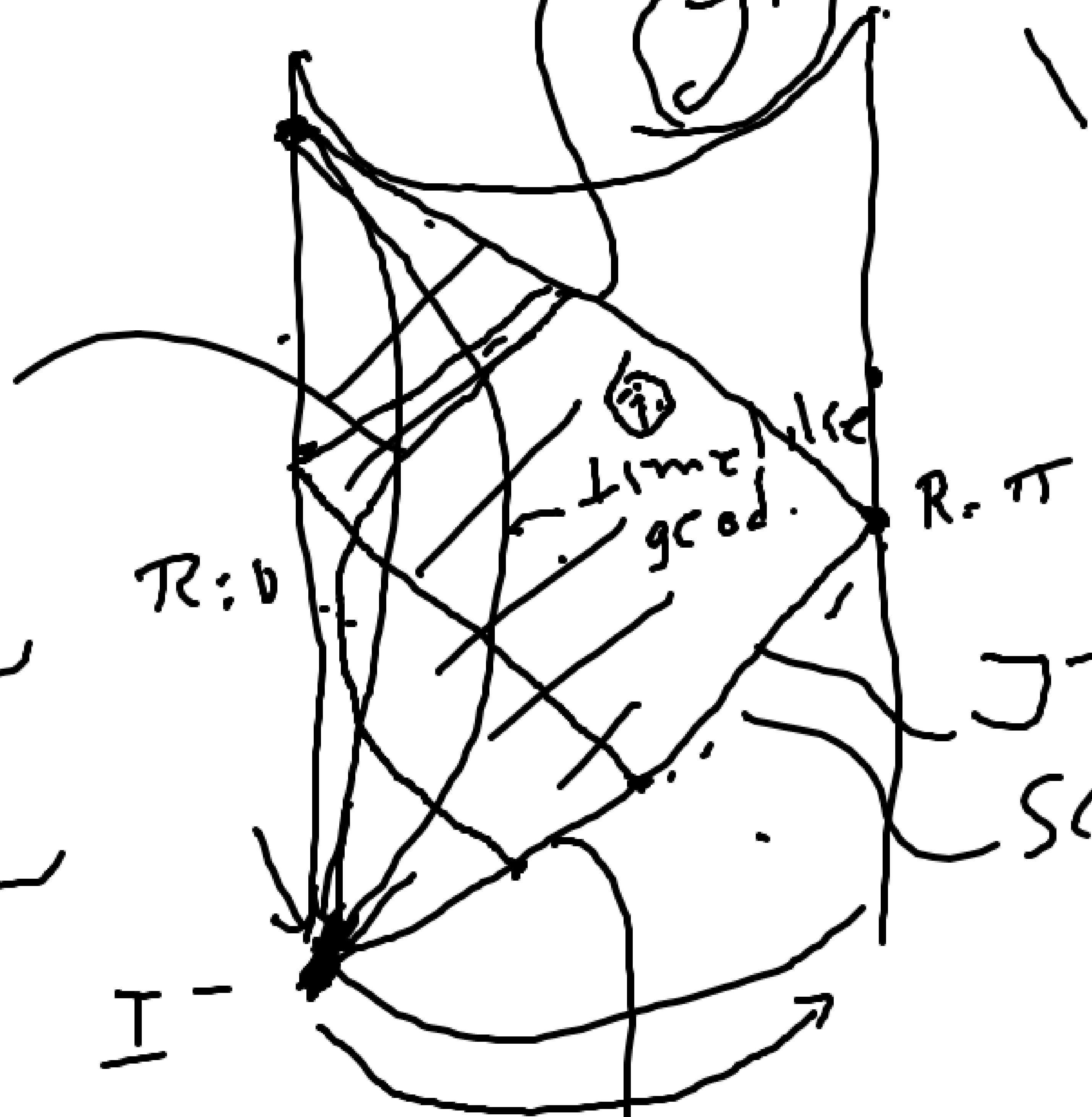
$u = T - R$

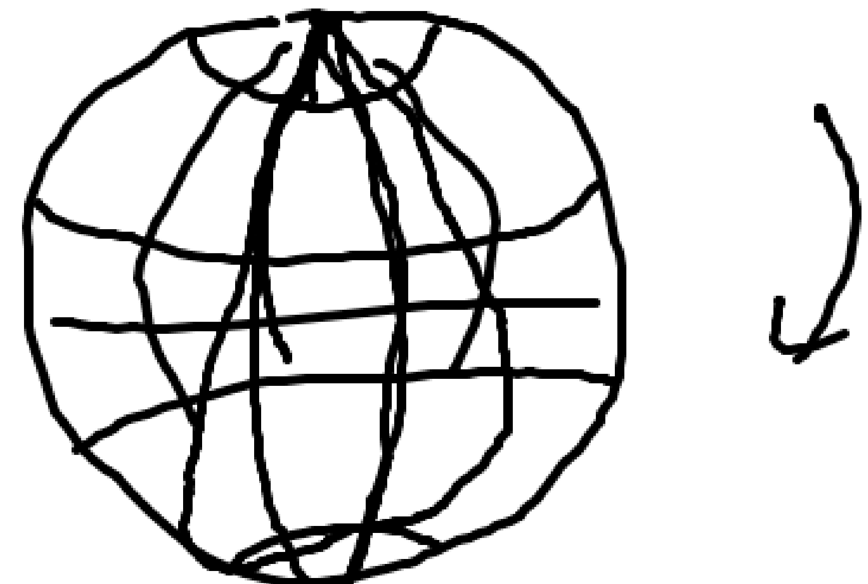
$v = T + R$

$$ds^2 = \frac{1}{(\cos v \cos u)^2}\left(dT^2 - dR^2 - \frac{\sin^2 2R}{4}\, d\Omega\right)$$

Metric of a 3 sphere surface
(sphere in 4-D)

Scrit $\mathcal{J}^+$ \Cal J





$U=\pi/2$ $V=\pi/2$

$U=\pi/2$ $V=$ anything.

I^- – origin of all timelike geodesics.

$I^+ \to$ goal of all timelike geod.

$I^0 \to$ origin & goal of all spacelike geodesics

$\mathcal{J}^- \to$ origin of all null geod. $\to$ sphere.

$\mathcal{J}^+ \to$ goal of all null geod.

Schwar. geom.

$$\left(1-\frac{2m}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2m}{r}} - r^2 d\Omega^2$$

$r \to \infty$, metric becomes flat S.T.

$$u = t - r^* \qquad r^* = \int \frac{dr}{1-\frac{2m}{r}} = r + 2m \ln\left(\frac{r}{2m}-1\right)$$

Tortoise coord.

$$v = t + r^*$$

$$ds^2 = \left(1-\frac{2m}{r}\right) du\, dv - r^2 d\Omega^2$$

$$u = \tan U \qquad v = \tan V$$

$$\hat{u} = -4m\, e^{-u/4m}$$

$$\hat{v} = +4m\, e^{v/4m}$$

$$ds^2 = \frac{e^{-r/2m}}{r/2m}\, d\hat{u}\, d\hat{v} - r^2\, d\Omega^2$$

$$uv = \left(\frac{r}{2m} - 1\right) e^{r/2m}$$

$\bar{u} = \hat{u}$ near $\hat{u} = 0$

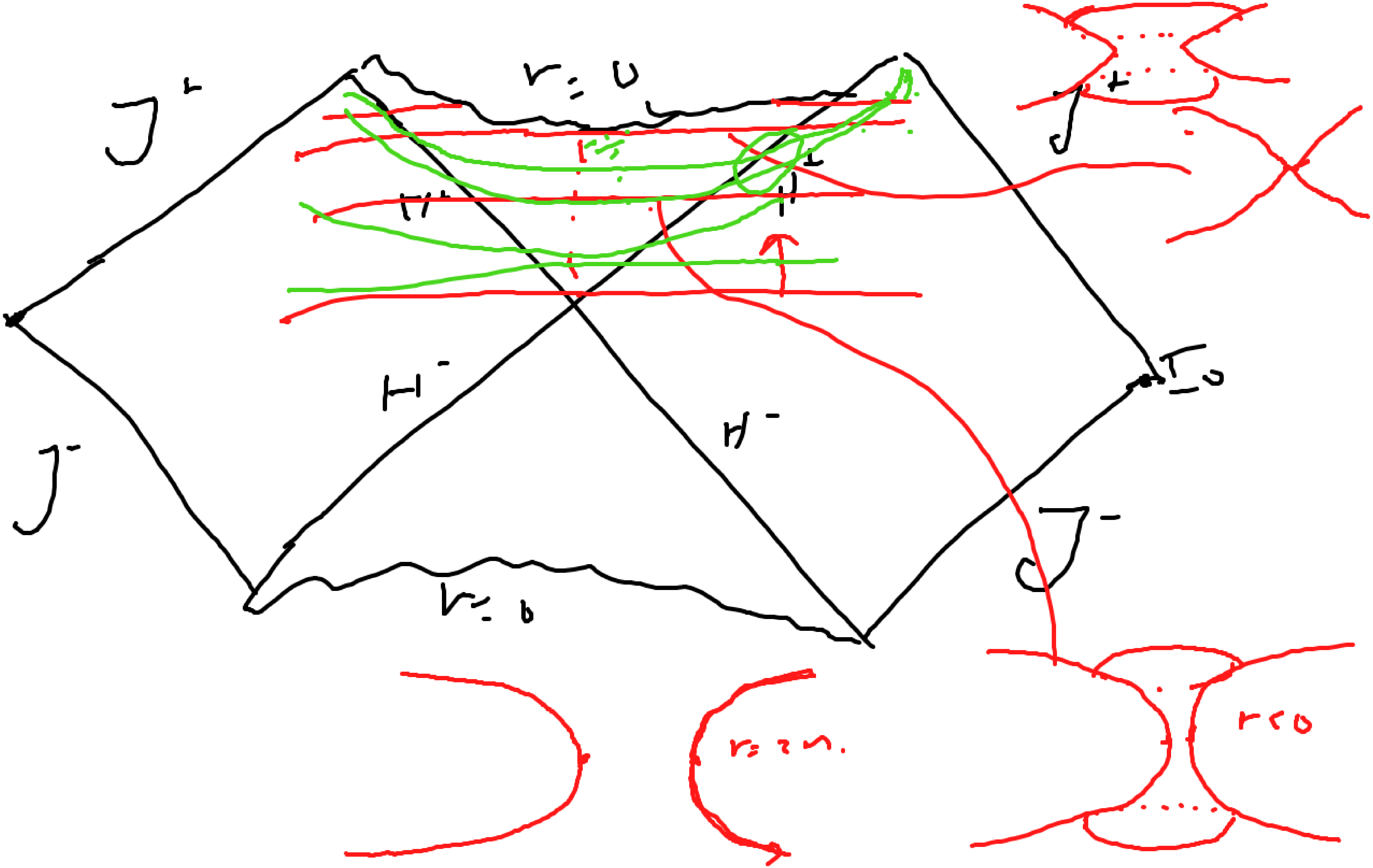
$= u$ for larg $\hat{u}$ ~ $u \to -\infty$

~ $u \to -\infty$

$$\tilde{U} = \tan U$$
$$\tilde{V} = \tan V.$$

$\cos^2(U)\cos^2(V)$ conformal factor.

$$-\frac{\pi}{2} < U < \frac{\pi}{2}$$
$$-\frac{\pi}{2} < V < \pi/2.$$


J+
r = 0
H+
H-
H-
J-
I0
J+
J-
r = 0
r = 2m.
r < 0

Vaidja metric.

$$ds^2 = \left(1 - \frac{2M(v)}{r}\right)(dv^2) - 2\,dv\,dr - r^2 d\Omega^2$$

$$M(v) = \Theta(V)$$

Masless "dust" shell collapsing to B.H

Penrose diagram.