Non linear Grav. waves

$$ds^2 = H(u,x,y)\,du^2 + du\,dv - dx^2 - dy^2$$

$dt^2 - dz^2 - dx^2 - dy^2$

$$u = t - z \qquad v = t + z$$

$$\Gamma^i_{jk} \longrightarrow g_{uu}$$

$$\Gamma^u_{ij} = 0 \qquad \Gamma_v{}^i{}_j = 0$$

$$\frac{d^2 u}{ds} = 0 \qquad \frac{d^2 u}{ds^2} + \Gamma^u_{ij}\frac{dx^i}{ds}\frac{dx^j}{ds} = 0$$

$$\Gamma^k_{ik} = 0 \quad \text{an} \quad \underline{\Gamma^k_{i\ell}\,\Gamma^\ell_{jk}}$$

$\underline{\partial_i \Gamma^i_{jk}}$ or $\partial_k \Gamma^i_{ji}$

$R_{ij} = 0$

$$\partial_x^2 H + \partial_y^2 H = 0$$

$\ddot{a}(u)(x^2 - y^2)$, $\ddot{b}(u)(xy)$, $c(u)x$, $d(u)y$.

$$\ddot{a}(u)(x^2 - y^2) = H(u, x, y)$$

Empty Space eqns.

$$h^u_{xx}(t-z) = -h^u_{yy}(t-z)$$

$$ds^2 = du\,d\tilde{v} - f^2 d\tilde{x}^2 - g^2 d\tilde{y}^2 \qquad f(u), g(u)$$

linear $\quad 1+h_{xx}(u) \quad 1-h_{xx}(u)$

$$\tilde{x} = \frac{x}{f} \qquad \tilde{y} = \frac{y}{g}$$

$$\dot{} = \partial_u$$

$$d\tilde{x} = \frac{dx}{f} - x\frac{\dot{f}}{f^2}\,du$$

$$ds^2 = du\,d\tilde{v} - dx^2 + 2x\frac{\dot{f}}{f}\,dx\,du - x^2\left(\frac{\dot{f}}{f}\right)^2 du^2$$

$$- dy^2 + 2y\frac{\dot{g}}{g}\,dy\,du - y^2\left(\frac{\dot{g}}{g}\right)^2 du^2$$

$$= -\left[\left(\frac{\dot f}{f}\right)^2 x^2 + \frac{\dot g}{g} y^2\right] du^2$$

$$+ du \underbrace{\left(d\tilde V + 2\frac{\dot f}{f} x\, dx + 2\frac{\dot g}{g} y\, dy\right)}_{d\Big(\underbrace{\bar V + \frac{\dot f}{f}x^2 + \frac{\dot g}{g}y^2}_{V}\Big) - \left[\dot{\overline{\left(\frac{\dot f}{f}\right)}} x^2 + \dot{\overline{\left(\frac{\dot g}{g}\right)}} y^2\right] du}$$

$$ds^2 = -\left(\frac{\ddot f}{f} x^2 + \frac{\ddot g}{g} y^2\right) du^2 + du\,dv - dx^2 - dy^2$$

$$\ddot a \left(x^2 - y^2\right)$$

$$\frac{\ddot f}{f} = -\ddot a \qquad \frac{\ddot g}{g} = \ddot a$$

to get Vacuum soln.

$$f, g = 1 + (\quad)$$

$$f = 1 - a \qquad g = 1 + a$$

$$\ddot{(f - (1-a))} = +\ddot a\, a \qquad \ddot{(g - (1+a))} = \ddot a\, a$$

$$\ddot{a}\, a = \partial_u(\dot{a}\, a) - \dot{a}^2$$

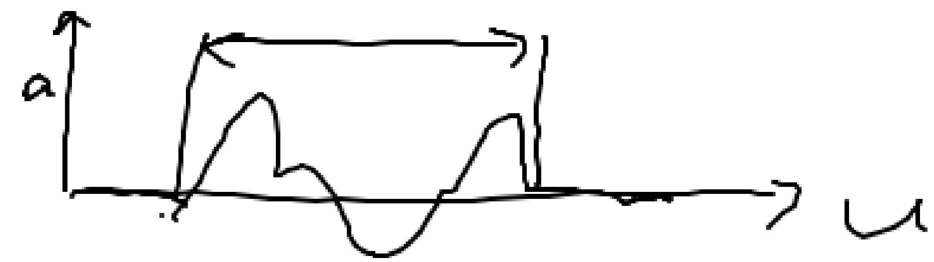
$$= \frac{1}{2}\ddot{\overline{a^2}} - \dot{a}^2$$

$$f = 1 - a + \frac{a^2}{2} - \iint \dot{a}^2 \, du \, du$$

$$g = 1 + a + \frac{a^2}{2} - \iint \dot{a}^2 \, du \, du$$

$$+ O(a^3) \ldots$$

Assum a is nonzero only in finite slab.



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Metric is flat s.t outside the slab. $\ddot{a} = 0$

$du\,dv - dx^2 - dy^2$

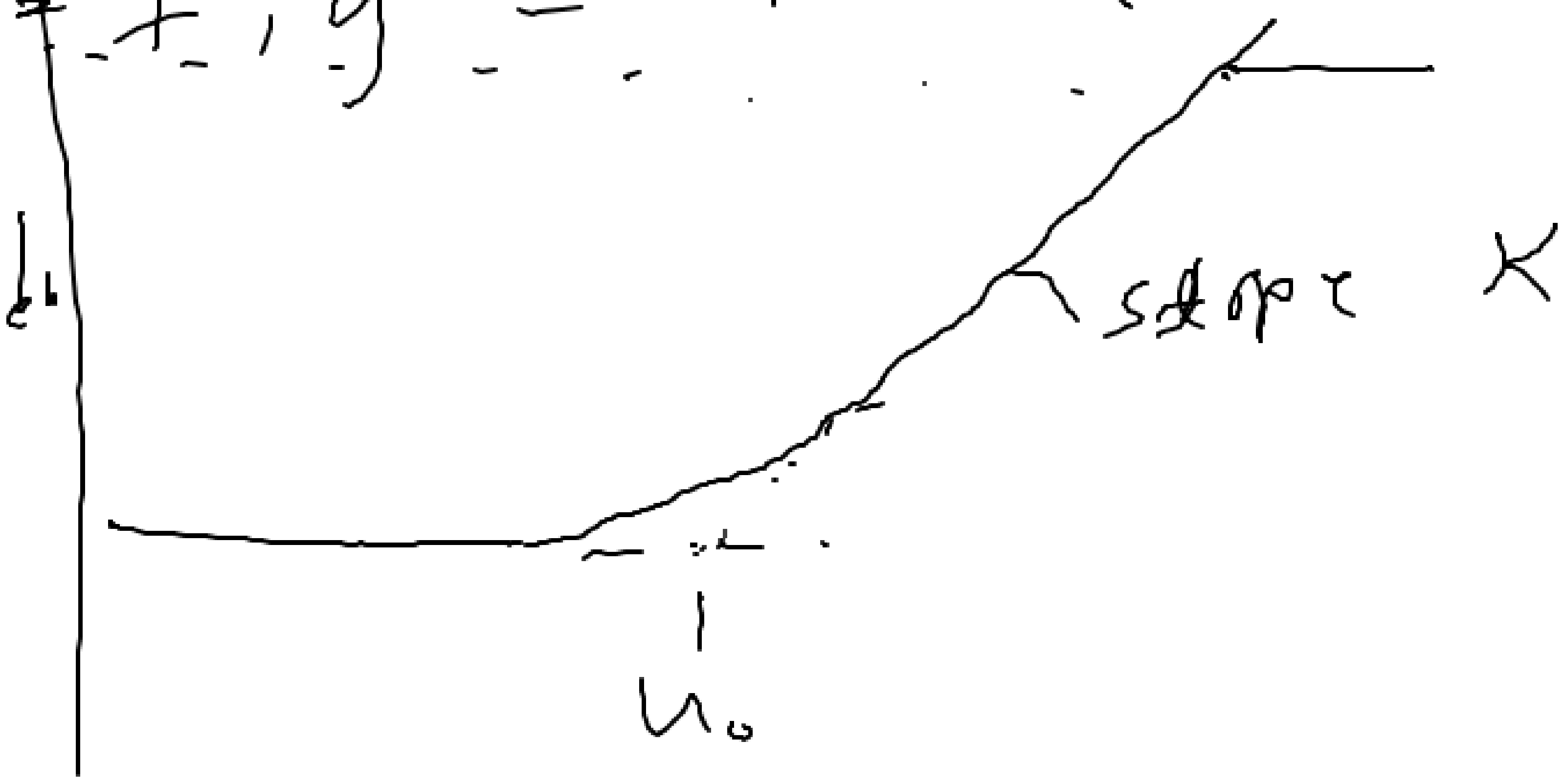
For $u <$ slab, $f = g = 1$

$f, g = 1 - \int\!\!\int \ddot{a}^2 \, du\, du$

$\int \ddot{a}^2 du$

$1 = f, g = 1 - K(u - u_0)$

$\int\int \dot{a}^2 \, dh \, d\ldots$



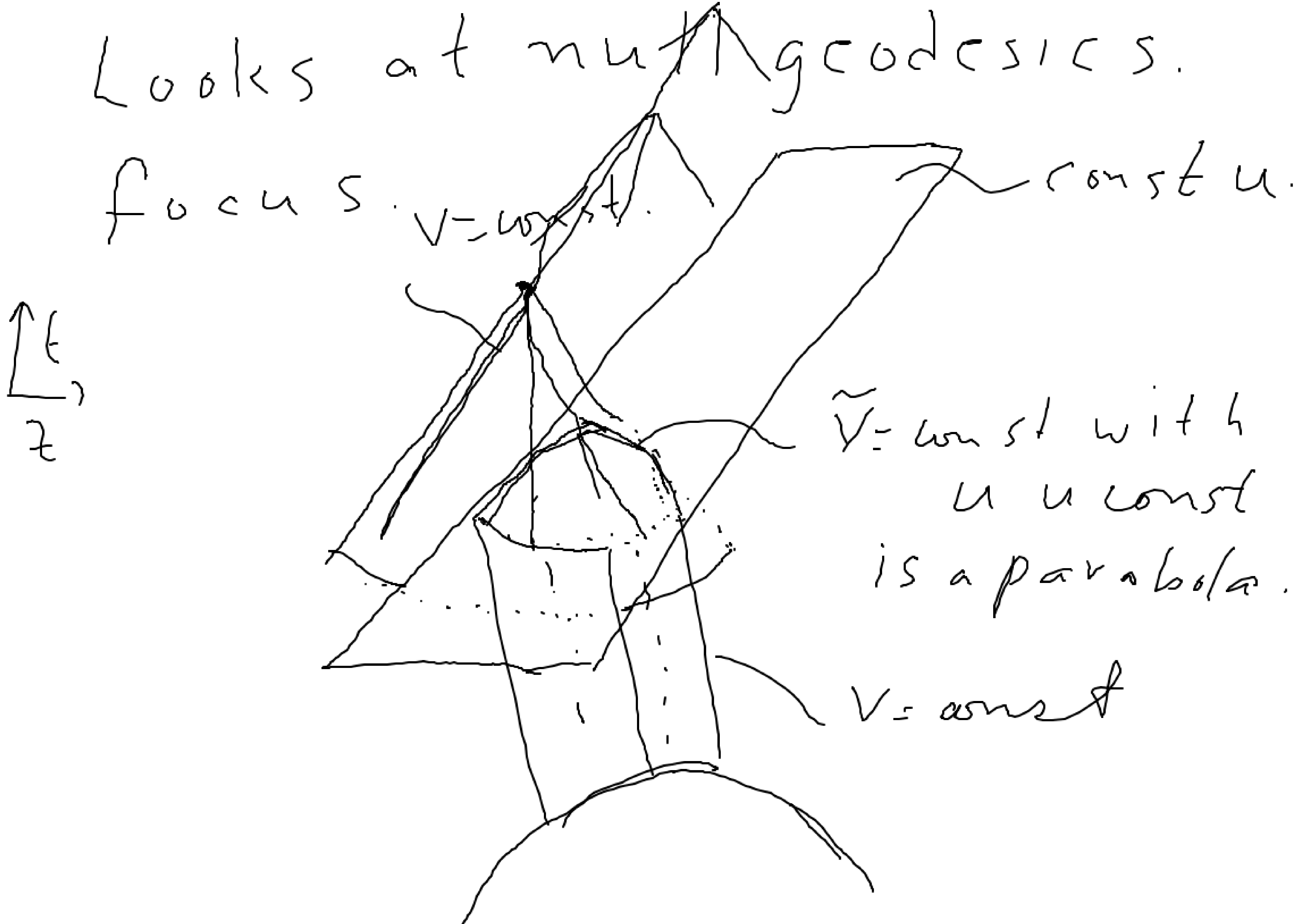
$1 - K(u - u_0) \longrightarrow 0$

$f = g = 0$

Singularity ?? Coord Sing.

x, y, v coord. if $\dot{a} = 0$, then both a flat S.T.

Looks at null geodesics.



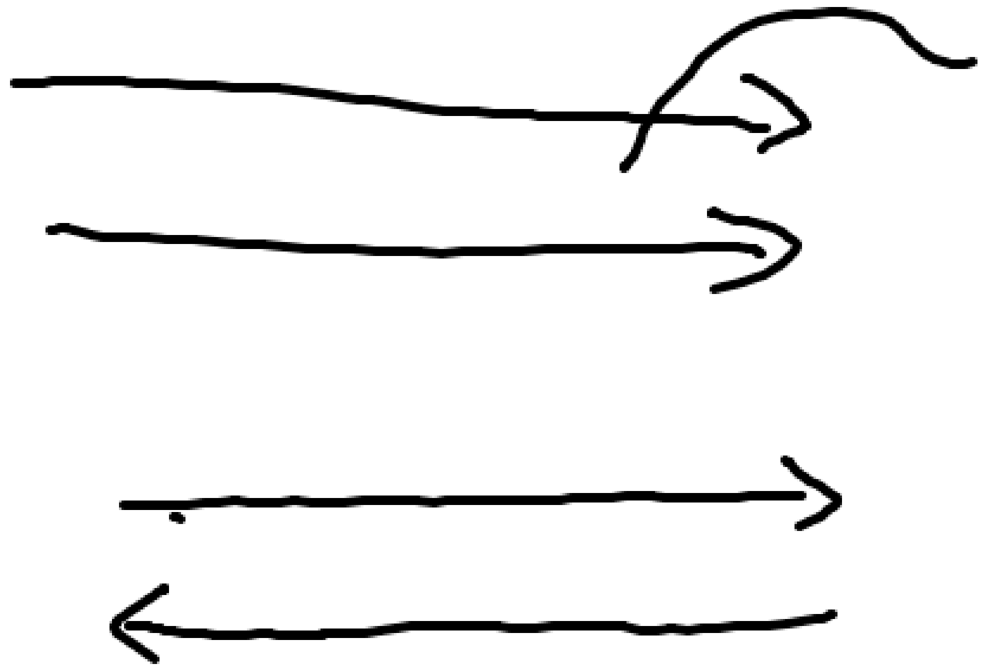
So slab focuses all lightrays to a null line. beyond the grav. wave slab.

Interesting feature.

parallel light rays which cross the slab — focused.

light travelling along the slab (u = const, v changing) do not.

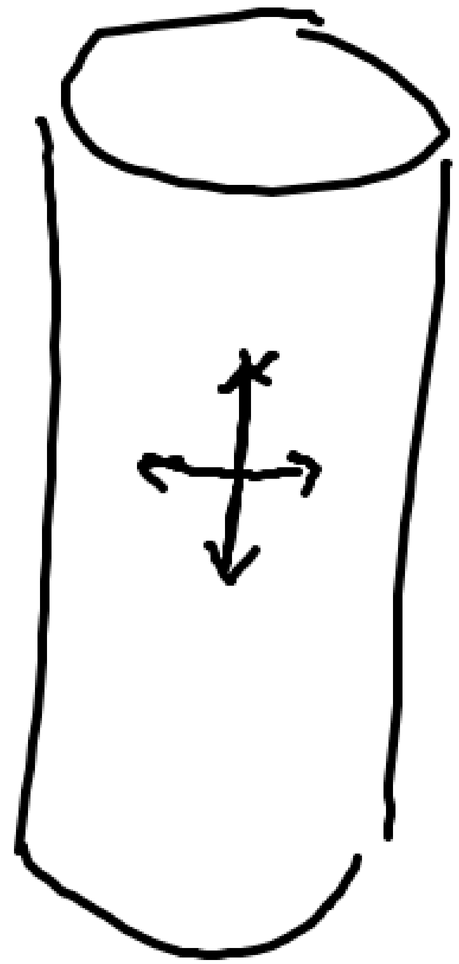
focusing increases with distance. $\Longrightarrow$ "Energ" desity in slab must be const.



no grav. attraction betwee two light in same direction

opp. direction attract.

Fully non linear solution of Einstein eqn's.



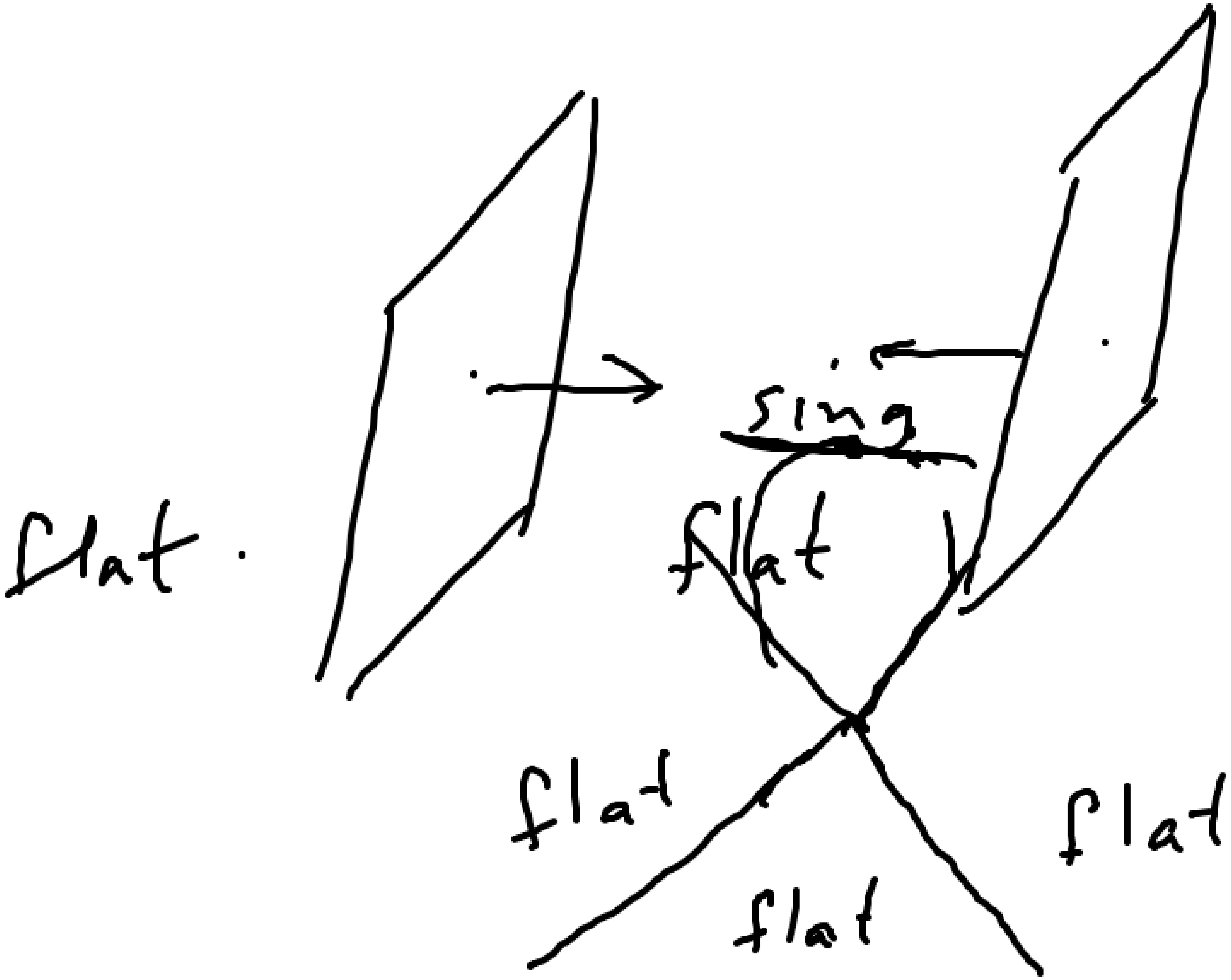
found singularity.

Einstein proves grav. waves impossible

Robertson pointed out to Infeld.

coord. singularity

Penrose studied what happens if two slabs



flat produce a sing. after they collide

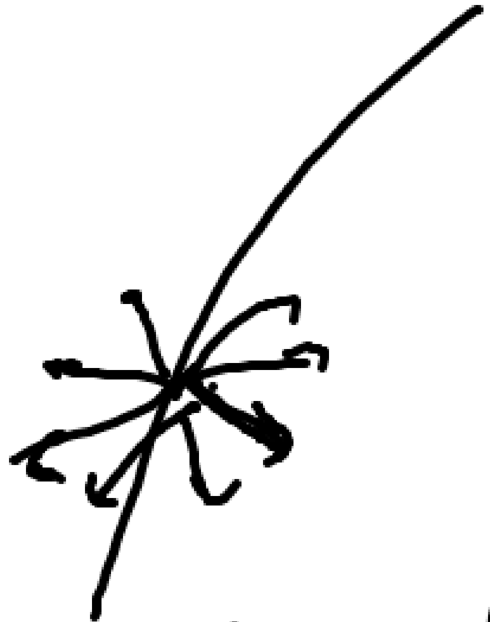
Both waves are focused to a point. - True singularity behind each of the plane waves.

If coord transf gets rid of singularity → coord sing.

1970's → How to characterize singularities.

geodesics which go to the singularity. $\Gamma = 0$

R, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$



$$\boxed{R_{\mu\nu\rho\sigma} R^{\rho\sigma\lambda\delta} R_{\lambda\delta}{}^{\mu\nu}}$$

How to determine if sing. is coord singularity??