Grav Waves.

$h = 0 \qquad \Box h_{\alpha\beta} = 0 \qquad \partial_\alpha h^\alpha{}_\beta = 0$

$\Box \xi_\alpha = 0 \qquad \Box = \partial_\alpha \eta^{\alpha\beta} \partial_\beta$

$h = 0 \qquad \Box h = \Box \eta^{\alpha\rho} h_{\alpha\rho}$ (const.)

$h \qquad = \eta^{\alpha\rho} \Box h_{\alpha\rho} = 0$

$h \to h + 2\partial_\alpha \xi^\alpha$

$-2\partial_\alpha \xi^\alpha = h \qquad \xi_i = 0$

$-2\partial_0 \xi_0 = h \implies \xi_0 = -\int h \, dt$

$\Box \xi_0 = 0$

$$\Box \xi_\alpha = 0$$

$$h = \partial_\alpha \xi^\alpha = \partial_0 \xi_0 - \underbrace{\partial_i \xi_i}_{\sum_{i=1}^{3} \partial_i \xi_i}$$

$$i = 1, 2, 3$$

$$h_{0i} = 0$$

$$h_{0i} \to h_{0i} + \partial_0 \xi_i + \partial_i \xi_0$$

$$\partial_0 \Rightarrow \quad \partial_0 \cdot \xi_i + \partial_i \xi_\cdot = - h_{0i}$$

$$\partial_0^2 \xi_i + \partial_i \partial_0 \xi_0 = - h_{0i}$$

$$\nabla^2 = \partial_i \partial_i \qquad \nabla^2 \xi_i + \partial_i \partial_j \xi_j = - h_{0i}$$

$$\partial_k \partial_i - 2\nabla^2 \delta_{ki}$$

$$\partial_k \nabla^2 \partial_i \xi_i - 2\nabla^2 \dot{\partial}_k \partial_i \xi_i$$

$$+ \partial_k \nabla^2 \dot{\partial}_i \xi_i - 2\nabla^2 \nabla^2 \xi_i$$

$$= -\partial_k \partial_i \dot{h}_{0i}$$

$$+ 2\nabla^2 \dddot{h}_{0k}$$

$$-2\partial_0^4 \xi_i = -\left(\partial_k \partial_i \dot{h}_{0i} - 2\ddot{h}_{0k}\right)$$

$$\xi_i = \frac{1}{2}\iiiint (\quad)(dt)^4$$

$$= \frac{1}{2}\iiint \partial_k \partial_i h_{0i}\,d\vec{t} - \int h_{0k}\,dt$$

for $h_{\mu\nu}$ = only polynomials.

$h = 0$, $\partial_i \bar{h}^{0i} = 0$ $h = 0$

$\partial_t h_{00} = \partial_i h_{0i} = 0$

$h_{00} = h_{0i} = 0$ (unless h_{00} & h_{0i} are const)

$$\Box h_{ij} = -8\pi \overline{T}_{ij}$$

$$h_{ij} = -2 \int \frac{T_{ij}(t-|x-x'|, x')}{|x-x'|} d^3x'$$

far away. 1st order in $|x|$

$$h^{(1,1)}_{ij} = -2 \int \frac{T_{ij}(t-|x|, x')}{|x-x'|} d^3x'$$

$|x| \gg |x'|$

$$h_{ij} = \frac{-2}{|x|} \int \frac{1}{2} \left(\partial'_k x'^i \, T_{kj} + \partial'_k x'^j \, T_{ik} \, d^3x' \right)$$

$$= \frac{-4}{|x|} \int -\left(x'^i \dot{T}_{0j} + x'^j \dot{T}_{i0} \right) d^4x'$$

$$\partial_\alpha T^\alpha{}_\rho = 0 \qquad \partial_0 T_{0\rho} = \partial_i T_{i\rho}$$

$$= + \frac{1}{|x|} \int \nabla_k x' \, x'^i \, \dot{T}_{0k} + (\nabla_k x'^i) x'^j \dot{T}_{k0}$$

$$h_{ij} = -\frac{1}{|x|}\int x^i x^j \underbrace{\ddot{T}_{00}}_{\text{Energy Density.}} d^3x$$

$$\int x^i x^j T_{ab} d^3x = \mathcal{O}_{ij}$$

$$h_{ij} = \frac{1}{|x|} \ddot{Q}_{ij} \qquad \sum q_{ii}$$

$$h_{ij} = \frac{1}{|x|} \ddot{Q}_{T\,ij} \qquad Q_{Tij} = Q_{ij} - \frac{1}{3} Q \delta_{ji}$$

$$\underline{h_{ij}} = \frac{1}{|x|}$$

Grav Rad is produced by
Trace free Quad moment
of the energy density.
Thorne. ⟹ Extend this to
radn including
grav. energy effects.
Misner-Thorne & Wheeler
Gravitation

$G_{\mu\nu} = -8\pi T_{\mu\nu}$

$T_{\mu\nu}$

$\partial\partial(g_{\mu\nu} - h_{\mu\nu}) + \Gamma\Gamma$

$h\partial\partial h$

call those grav. energy.

only LHS - Linear eqns. for $g_{\mu\nu} - \eta_{\mu\nu}$ RHS → Energy mom of matter plus gravity.

h_{ij} — created by

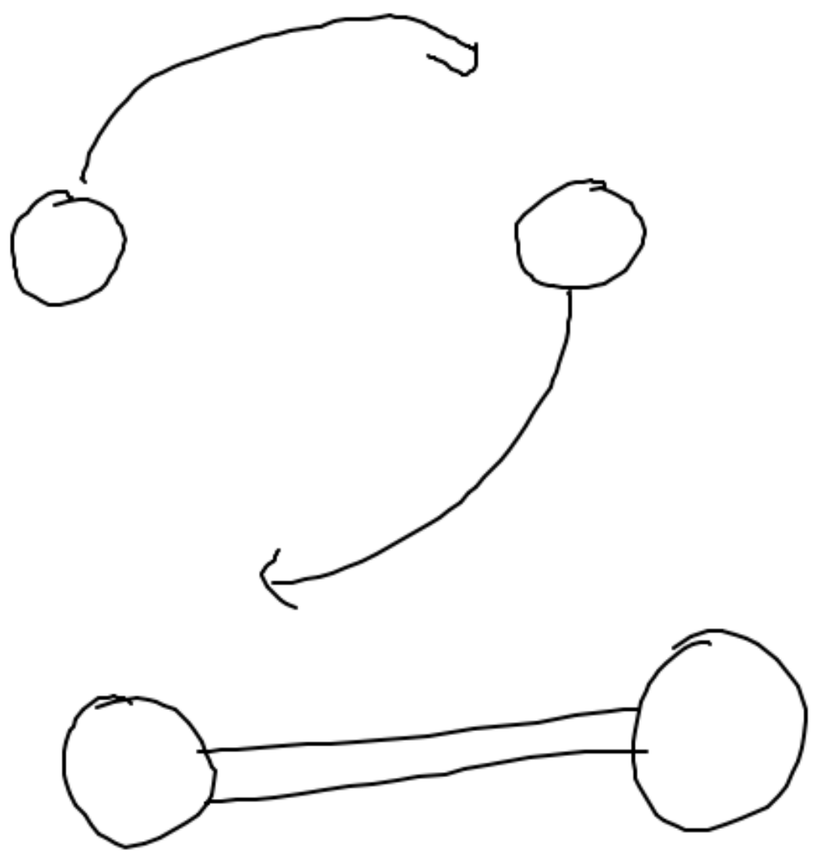
Q_{ij} of $\underline{I_{ij}}$

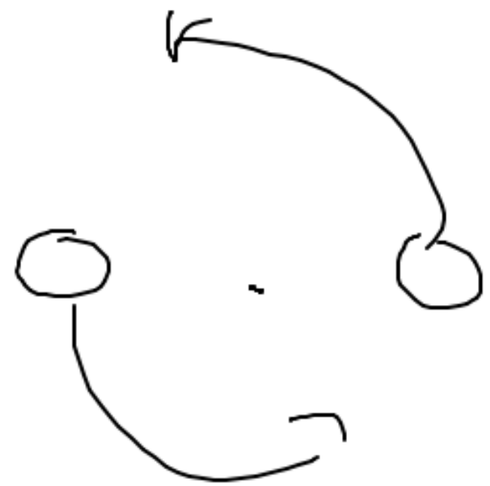
Quadr. formula good even

for grav. energy.

Sources of Grav. Waves.

Quad.-mom. of energy density





at t_1 $\quad Q_{xx} = 2M\left(\frac{d}{2}\right)^2$

$Q_{yy} << Q_x$

t_2 $\quad Q_y = 2M\left(\frac{L}{2}\right)^2$

$Q_{xy} \cong 0$

1st Test

Binary Pulsar

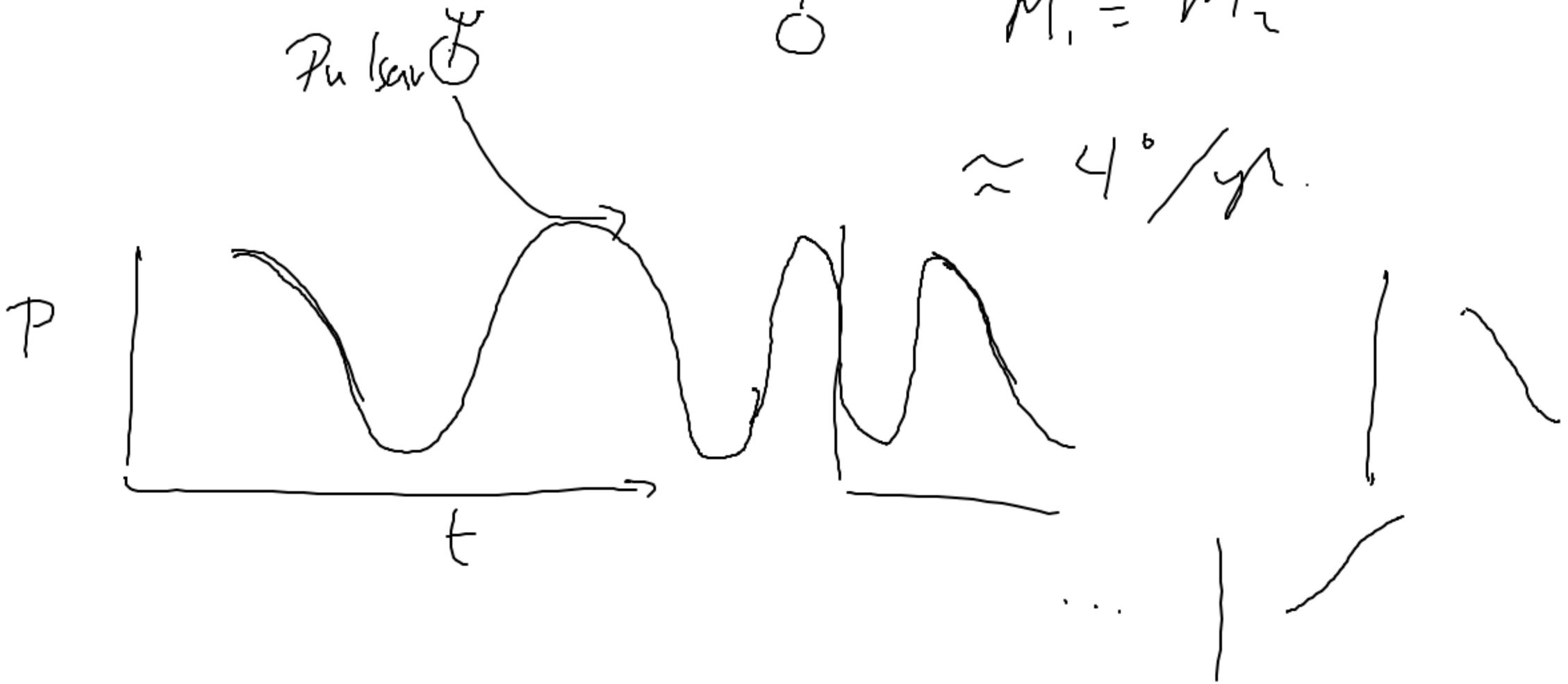
Hulse, Taylor.

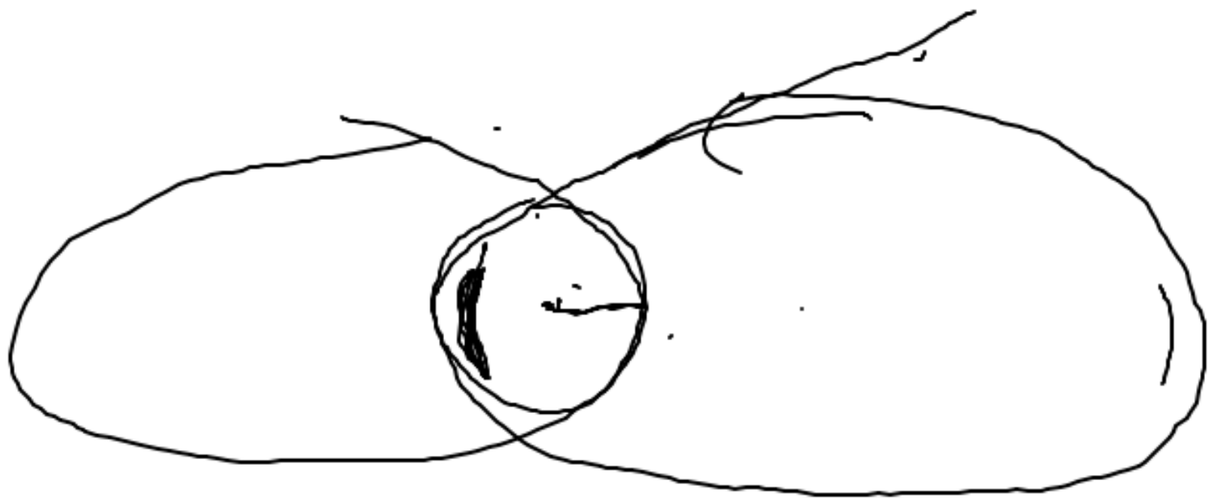
1970 - 1986'

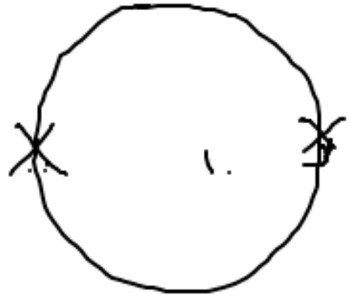
$M_1 = M_2$

Pulsar

$\approx 4^\circ/yr.$







$M_1 = M_2$ by about 15%

1.1 $1.3 M_\odot$

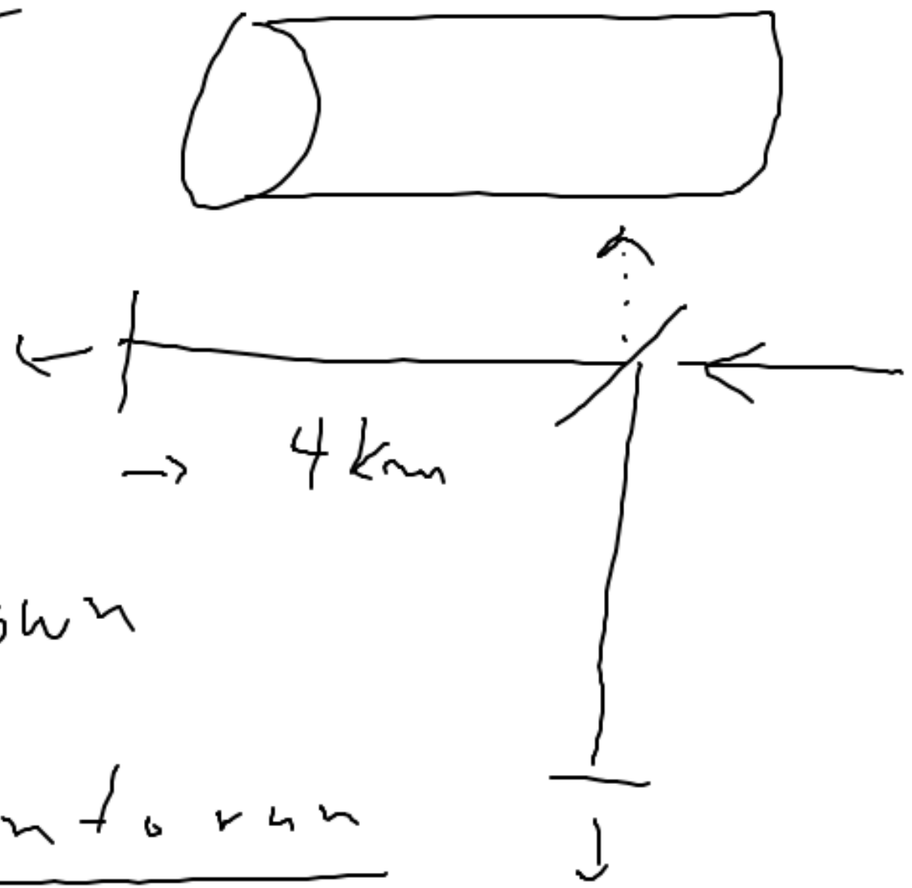
1st detection of effect of grav waves.

1950's Weber

1980's Ligo

1990's

2015 → Shakedown Run.
1 week into run
Saw signal.



30 $M_\odot$

$2 \; 36 M_\odot \longrightarrow 60 M_\odot$

emit $5 M_\odot$ grav. wave energy

$\sim \frac{1}{30}$ sec.

$>$ energy in light from all stars in obs. universe during that time.

$h_{\mu\nu} \simeq \boxed{10^{-22}}$ Nobel prize.

Grav. Radn Exists

Black Holes Exist.

(Public) Lecture on Grav. Radn.

Non Linear. grav waves