GRAVITATIONAL WAVES

$$\Box \bar{h}_{\mu\nu} = -8\pi T_{\mu\nu}$$

$$\partial_\mu \bar{h}^{\mu}{}_{\nu} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

(Note : $\partial_\mu \big(\underbrace{\sqrt{|g|}\, g^{\mu\nu}}_{\bar{h}^{\mu\nu}} \big) = 0$

$$h^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\lambda} h_{\rho\lambda}$$

$$\sqrt{|g|} = \frac{1}{2}\eta^{\mu\nu} h_{\mu\nu}$$

$$(\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Harmic coord condition.

$$\nabla_\mu \nabla^\mu \phi = \frac{1}{\sqrt{|g|}} \partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu \phi$$

$$\nabla_\mu \nabla_\nu X^\rho = \frac{1}{\sqrt{|g|}} \partial_\mu \left(\sqrt{|g|} g^{\mu\nu} \underbrace{\partial_\nu X^\rho}_{\delta_\nu{}^\rho} \right)$$

$$= \frac{1}{\sqrt{g}} \partial_\mu \sqrt{|g|} g^{\mu\rho}$$

If X^ρ obeys wave eqn.

$$\partial_\mu (\sqrt{g} g^{\mu\rho}) = 0$$

$$\Box \bar{h}_{\mu\nu} = -8\pi T_{\mu\nu}$$

$$\partial_\mu \bar{h}^{\mu}{}_{\nu} = 0$$

$$x^\mu \to x^\mu + \xi^\mu$$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\bar{h}_{\mu\nu} \to \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \partial_\rho \xi^\rho \eta_{\mu\nu}$$

$$\partial_\mu \bar{h}^{\mu\nu} \to \partial_\mu \bar{h}^{\mu\nu} + \Box \xi^\nu$$

$$\Box \xi^\mu = 0$$

$$h \to h + 2\partial_\mu \xi^\mu$$

I can find ξ^μ such that

$$\partial_\mu \xi^\mu = -\frac{1}{2} h$$

and $\Box \underline{\xi}^\mu = 0$

$h = 0$ $\qquad$ h_{00}, h_{0i}

$$\boxed{\partial_\mu \xi^\mu = 0 = \partial_t \xi_0 - \nabla \cdot \vec{\xi}}$$ preserve $h = 0$

$$\vec{\xi} = (\xi_1, \xi_2, \xi_3)$$

$\Box \xi_\mu = 0$

$$h_{0i} = \partial_0 \xi_i + \partial_i \xi_0$$

$$\partial_t h_{0i} = \partial_0^2 \xi_i + \partial_i \partial_0 \xi_0$$

$$= \nabla^2 \xi_i + \partial_i (\nabla \cdot \xi_0)$$

∂_t^4

$$\sum_i \left(\partial_i \partial_j \partial_t h_{0i} - 2 \underbrace{\partial_i \partial_i}_{\substack{\nabla^2 \\ (\partial_x^2 + \partial_y^2 + \partial_z^2)}} \partial_t h_{0j} \right) = \nabla^4 \xi_i$$

$(\nabla^2)^2$

$$\underline{\xi_j} = \sum_i \iiint (\partial_j \partial_i h_{0i} - 2\,\partial_i \partial_i h_{0j})\, dx\, dx\, dt$$

$$\partial_0 \xi_0 = \sum_i \partial_i \xi_i$$

$$h_{0i} = 0, \quad h = 0,$$

$$\partial_\mu h^{\mu\nu} = 0 \qquad \Box h_{\mu\nu} = 0$$

$$h = \bar{h}$$

$$\partial_0 h^{00} = \sum_i \partial_i \cancel{h^{0i}}^{\,0}$$

$$\partial_0 h^{00} = 0 \quad \Rightarrow \quad h^{00} = 0$$

$$\Box h_{ij} = 0$$

$$h_{0i} = h_{00} = 0$$

$$\sum_i h_{ii} = 0 \qquad h = 0$$

$$\partial_i h_{ij} = 0 \longrightarrow 2 \text{ deg of freedom}$$

If h depends only on z, t.

$$h_{zi} = 0 \qquad h_{xx}, h_{xy}, h_{yy}$$

"wave metric" g $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xx} & h_{xy} & 0 \\ 0 & h_{xy} & -h_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 2 deg of freedom.

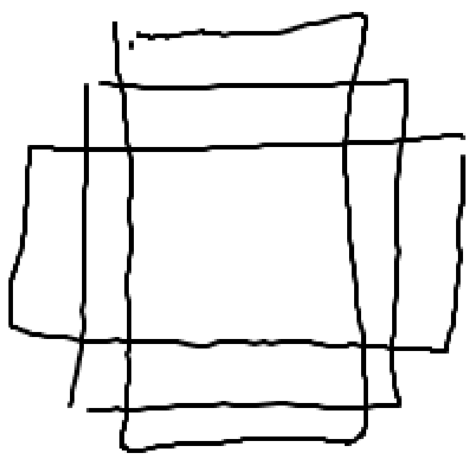
$-h_{yy} = h_{xx}$, h_{xy}

2 polarizations of grav. wave.

"Shears"

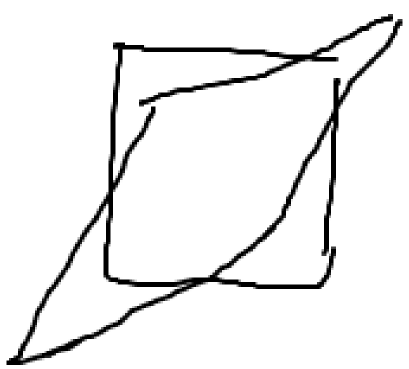
h - rep. "volume" change

shape changes.



dir. prop into page.

Area same.



Midterm.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Then $x = y = z = \text{const.}$

is a geodesic.

Distance changes.

$\left(1 - \frac{h_{xx}}{2}\right)\Delta x$ between 2 part.

Bar type grav. wave detector.

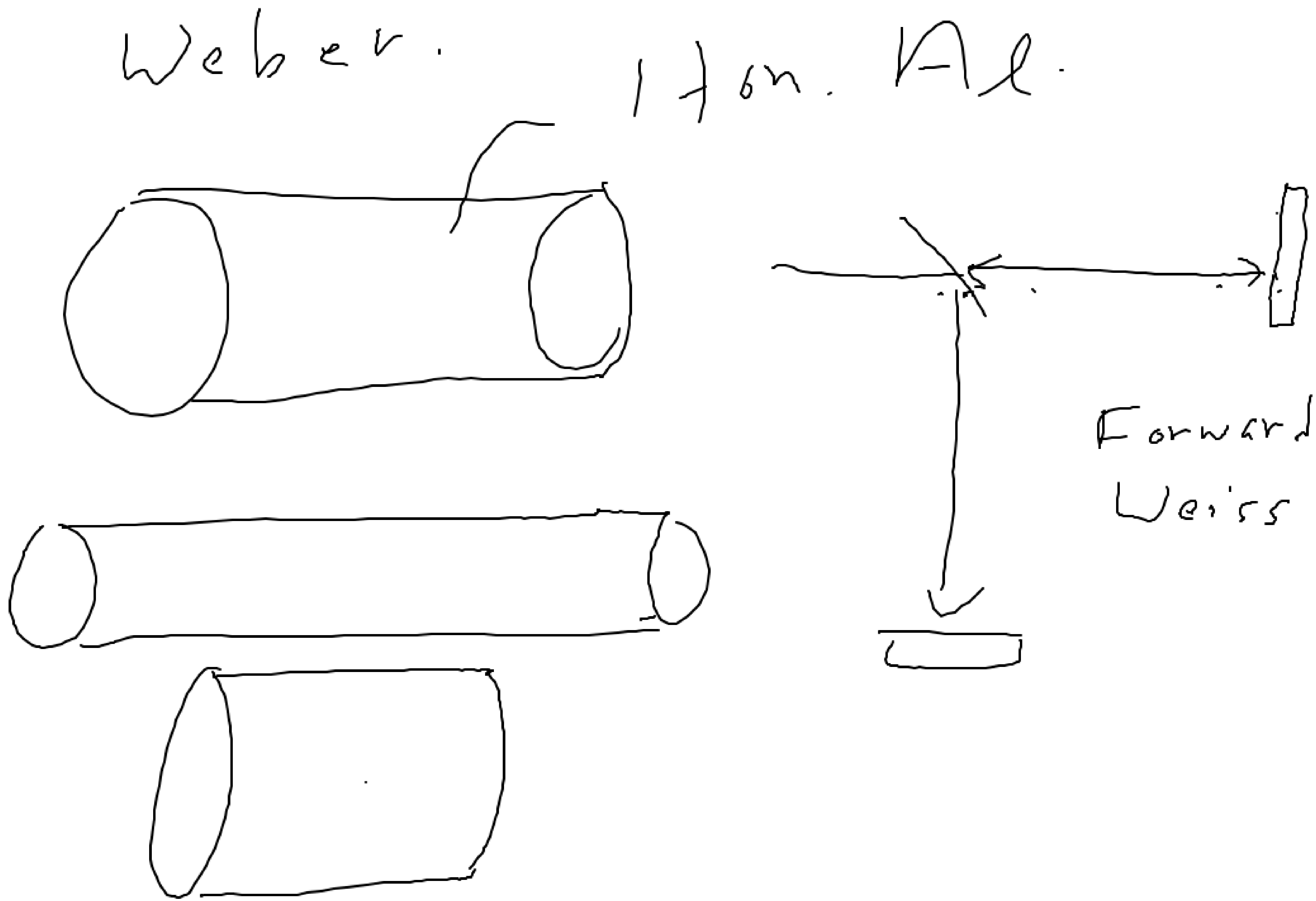
shape changes.

atoms in solid preserve shape. → exert forces on each other.

No compression

Jello v small shear modulus (small force on shape change)

but v. large Bulk modulus.


Weber.
Hon. Al.
Forward
Weiss

Sources of Grav. Rad.

Motion of Energy.

$$\Box \bar{h}_{\mu\nu} = -8\pi T_{\mu\nu}$$

$$\Box \bar{h}_{ij} = -8\pi T_{\mu\nu}$$

$$\bar{h}_{ij} = -\overset{2}{\cancel{8\pi}} \int \frac{T_{\mu\nu}(t', \bar{x}')\, \delta((t-t') - |x-x'|)}{\cancel{4\pi}\, |x-x'|}\, d^3x'\, dt'$$

$$|x-x'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

Lets assume

$|x| \gg |x'|$ for all $\vec{x}'$ where $T^{\mu\nu}(t',x') \neq 0$

Then

$$\bar{h}_{ij}(t,x) = -2 \int \frac{\delta(t-t'-|x|)\, T_{ij}(t',x')\, d^3x'}{|x|}$$

$$\left(\frac{|x'|}{|x|} \simeq 0\right)$$

$\underline{\partial_\mu \bar{h}^{\mu\nu} = 0}$ comes from $\partial_\mu T^{\mu\nu} = 0$ ✓

$$\bar{h}_{ij} \approx -\frac{2}{|x|} \int T_{ij}(t-|x|, x')\, d^3x'$$

$$= -\frac{2}{|x|} \int (\partial_k x_i)\, T_{kj}\, d^3x'$$

$$= +\frac{2}{|x|} \int x_i \underbrace{\partial_k T_{kj}}_{= \partial_0 T_{0k}}\, d^3x'$$

$$\propto \frac{2}{|x|} \int x_i x_j\, \partial_t^2 T_{00}\, d^3x'$$

$$= \frac{2}{|x|} \partial_t^2 \int \underbrace{x_i x_j T_{00}}$$ — quad moment.