$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 \pm 2\sqrt{\frac{2m}{r}}\,dt\,dr - r^2\,d\Omega^2 - dr^2$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{array}{c|cccc} t & 0 & 1 & & \\ r & 1 & 0 & & \\ \theta & & & r^2 & \\ \phi & & & & r^2\sin^2\theta \end{array}$$

$$\left(1-\frac{2m}{r}\right)dt^2 \pm \frac{4m}{r}\,dt\,dr - \left(1+\frac{2m}{r}\right)dr^2 - r^2\,d\Omega^2$$

Penrose 1960's.

$$u = t - r^* \qquad r^* = r + 2m\ln\frac{r-2m}{2m}$$

$$\left(1-\frac{2m}{r}\right)du^2 + 2\,du\,dr - r^2\,d\Omega^2$$

$$v = t + r^*$$

$$\left(1-\frac{2m}{r}\right)dv^2 - 2\,dv\,dr$$

$$u = t - r^*$$

$$\left(1 - \frac{2m}{r}\right) du^2 + 2\, du\, dr - r^2 d\Omega^2$$

$$\begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & 0 \\ & 0 & r^2 & \\ & & & r^2\sin^2\theta \end{pmatrix}$$

$$v = t + r^*$$

$$\left(1 - \frac{2m}{r}\right) dv^2 - 2\, dv\, dr - r^2 d\Omega^2$$

$u = \text{const}$, θ, ϕ const is a null line

$v = \text{const}$ " " "

$$v - u = 2r^* = 2\left(r + 2m \underbrace{\ln\left(\frac{r-2m}{2m}\right)}\right)$$

∞ at $r = 2m$.

v const, $r \to 2m$, $u \to -\infty$

u const, $r \to 2m$, $v \to +\infty$

2 diff $r = 2m$.

1956 - Synge

1960 $\rightarrow$ Kruscal, Szekers

$$u = t - r^* \qquad du = dt - \frac{dr}{\left(1 - \frac{2m}{r}\right)}$$

$$v = t + r^* \qquad dv = dt + \frac{dr}{\left(1 - 2m/r\right)}$$

$$v - u = 2r^* = 2\left(r + 2m \ln\left(\frac{r-2m}{2m}\right)\right)$$

$$e^{\left(\frac{v-u}{4m}\right)} = e^{r/2m + \ln\left(\frac{r-2m}{2M}\right)} = \left(\frac{r-2m}{2M}\right) e^{r/2m}$$

$$\left(1 - \frac{2m}{r}\right) du\, dv - r^2 d\Omega^2$$

$$\frac{r/2m - 1}{r/2M} \; \underbrace{e^{-u/4m} du}_{d\left(-4m e^{-u/4m}\right) \atop U} \; \underbrace{e^{+v/4m} dv}_{d\left(4m e^{v/4m}\right) \atop V}$$

$$ds^2 = \frac{e^{-r/2M}}{r/2M}\, dU\, dV - r^2\, d\Omega^2$$

$$UV = -(4M)^2 e^{\frac{v-u}{4M}} = -(4M^2)\frac{(r-2M)}{2M} e^{\frac{r}{2M}}$$

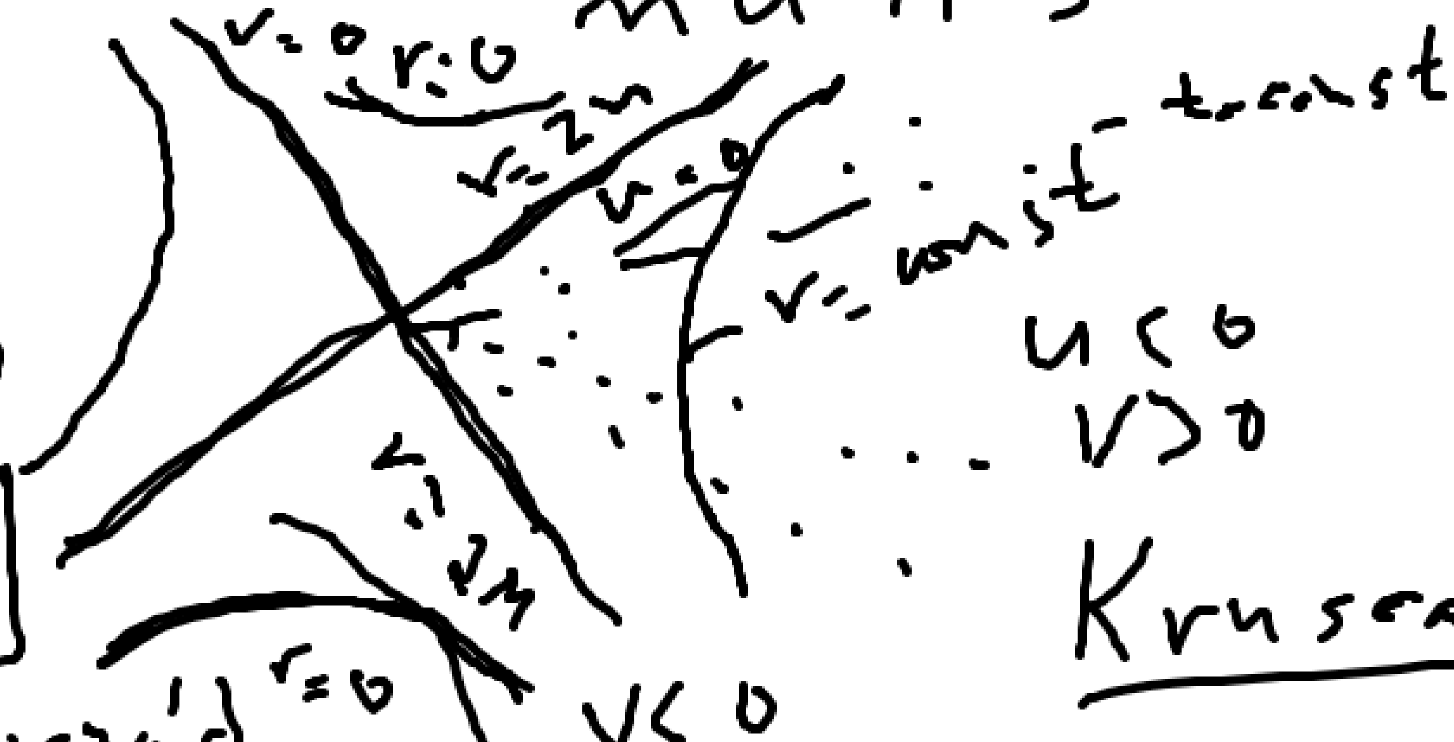
$r(UV)$ $\quad r = 2M \to UV = 0$

$U = 0, \; V = 0$

U n V const are null surfaces.

Plot

$V<0$
$U>0$



$\boxed{\text{UNIQUE}}$

Birkoff 1920's

$U<0$
$V>0$

$V<0$
$U<0$

$$\frac{V}{U} = e^{\frac{t}{2M}}$$

$UV = 4M^2 \quad r = 0$

Kruskal extension

Matter $r < R$

outside

$$\left(1-\frac{2m}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2m}{r}} - r^2 d\Omega^2$$

U, V

$r(UV)$

$$\left(\frac{r}{2m}-1\right) e^{r/2m} = UV$$

Kruscal is unique analytic extension of Sch.

$G_{\mu\nu} = 0$

$f(x)$


Sch.
flat
Sl
M
Sch.
flat
Schw

TESTS of GR.

Perihelion of Mercury