$$\nabla_E R_{ABCD} + \nabla_A R_{BECD} + \nabla_B R_{EACD} = 0$$

$$R_{AC} = R_{ABCD}\, g^{BD} = R_{BADC}\, g^{BD}$$

$$= -R_{ABDC}\, g^{BD}$$

$$R = g^{AC} R_{AC}$$

$$\nabla_A g_{BC} = 0 \qquad g_{BC}\, g^{CD} = \delta_B^{\ D}$$

$$\nabla_A g^{CD} = 0$$

$$g^{BD} \qquad \nabla_E R_{AC} + \nabla_A(-R_{EC}) + \nabla^B R_{EACB} = 0$$

$$g^{AC} \qquad \nabla_E R - 2\nabla^A R_{AE} = 0$$

$$\left(G_{AB} = R_{AB} - \frac{1}{2} R g_{AB}\right)$$

$$\nabla^A T_{AB} = 0$$

$$\nabla^A G_{AB} = 0$$

$$G_{AB} = \kappa T_{AB}$$

$$ds^2 = (1+2\phi)\, dt^2 - dx^2 - dy^2 - dz^2$$

$$\frac{d^2 x}{d\lambda^2} = -\partial_x \phi$$

$$\nabla^2 \phi = 4\pi G \rho_{\text{mass}}$$

energy density

$$g_{ij} = \underbrace{\eta_{ij}}_{\begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}} + h_{ij}$$

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$
$$= \eta_{ij}\, dx^i dx^j$$

Neglect h^2

$\partial\partial h$

$\cancel{\partial h\ \partial h}\ \ \eta + h$

$$R_{ijkl} = \underbrace{\partial\partial h}\ \ \partial\partial g - {}^{-1}\ \partial g\, \partial g\, g$$

$$R_{ik} = \underbrace{g}_{\eta}{}^{jl} R_{ijkl}$$

$$R = g^{ik} \underbrace{R_{ik}}$$
$$= \eta^{ik} R_{ik}$$

$$R_{ij}{}^{kl} = \frac{1}{2}\left(\partial_i \partial_k g_{jl} \cdots \right.$$

$$= \frac{1}{2}\left(\partial_i \partial_k h_{jl} + \partial_j \partial_l h_{ik} - \partial_j \partial_k h_{il} - \partial_i \partial_l h_{jk} \right)$$

$$R_{ik} = \eta^{jl}\left(R_{ij}{}^{kl}\right) = R_i{}^l{}_{kl}$$

$$R_{ik} = \frac{1}{2}\left(\overset{\Box}{\underbrace{\partial_i \partial^j}} h_j{}^{j} + \partial_i \partial_k h - \partial_i \partial^j h_{jk} - \partial_k \partial^j h_{ji} \right)$$

$$R = g^{ik} R_{ij} = \frac{1}{2}\left(\Box h + \Box h - \partial^i \partial^j h_{ij} - \partial^i \partial^j h_{ij} \right)$$

$$\bar h_{ij} = h_{ij} - \frac{1}{2} h \eta_{ij}$$

$$G_{ij} = \frac{1}{2}\left(\Box \bar h_{ij} - \partial_i \partial^k \bar h_{kj} - \partial_j \partial^k \bar h_{ki} + \partial^k \partial^l \bar h_{kl} \eta_{ij} \right)$$

$$\bar x^i = x^i + \xi^i(\bar x)$$

ξ^i is small (neglect $\xi\xi$)

$$= x^i + \xi^i(x)$$

$$x^i = \bar x^i - \xi^i(\bar x)$$

$$\frac{\partial x^i}{\partial \bar x^k} = \delta^i{}_k - \bar\partial_k \xi^i$$

$$\bar g_{kl} = \frac{\partial x^i}{\partial \bar x^k} \frac{\partial x^j}{\partial \bar x^l} g_{ij}$$

$$= g_{kl} + g_{ki}\,\bar{\xi}^i$$

$$\tilde{g}_{kl} = g_{kl} - g_{il}\partial_k\xi^i - g_{ik}\partial_l\xi^i + O(\xi\xi)$$

$$\eta_{kl} + \tilde{h}_{kl} = \eta_{kl} + h_{kl} - \eta_{ik}\partial_l\xi^i - \partial_k(\xi^i\eta_{il})$$

$$= \eta_{kl} + h_{kl} - \partial_k\xi_l - \partial_l\xi_k$$

$$\xi_k = \eta_{ki}\xi^i$$

$$\tilde{h}_{kl} = h_{kl} - \partial_k\xi_l - \partial_l\xi_k.$$

Choose ξ^i so that $\partial^k \bar{h}_{kl} = 0$

$$\partial^k \tilde{\bar{h}}_{kl} = \partial^k \bar{h}_{kl} - \Box \xi_l$$

$$\rightarrow \Box \xi_l = \partial^k \bar{h}_{kl} \text{ then}$$

$$\partial^k \bar{\bar{h}}_{kl} = 0$$

$$\boxed{G_{ij} = +\frac{1}{2} \Box \bar{h}_{ij}}$$

$$\bar{h}_{ij} = h_{ij} - \frac{1}{2} h \eta_{ij}$$

$$\frac{1}{2} \Box h_{ij} = K T_{ij} \} T_{00}$$

$$\bar{h} = h - \frac{1}{2} h 4 = -h$$

$$T_{00} \gg T_{ij} \quad i,j \neq 0,0$$

∂_t are small.

$$\underbrace{\Box}\bar{h}_{00} = 2\kappa T_{ij}$$

$$\cancel{\partial_t^2} - \nabla^2$$

$$-\nabla^2 \bar{h}_{00} = 2\kappa T_{00} \qquad \nabla^2 \phi = 4\pi G\rho$$

$$\bar{h}_{00} \propto \phi \qquad \bar{h}_{ij} = h_{ij} - \frac{1}{2} h \eta_{ij}$$

$$h_{00} = 2\phi \qquad h_{ij} = \bar{h}_{ij} + \frac{1}{2} h \eta_{ij}$$

$$= \bar{h}_{ij} - \frac{1}{2} \bar{h} \eta_{ij}$$

$$\bar{h}_{00} = 4\phi \qquad \kappa = -8\pi G \qquad 2\phi = h_{00} = \bar{h}_{00} - \frac{1}{2} \bar{h}_{00}$$

$$= \frac{1}{2}\bar{h}_{00}$$

$$G_{AB} = -8\pi G_N T_{AB}$$ 1915

Newton's const

$$\bar{h}_{00} = +4\phi \qquad h_{ij} = 0$$

$$h_{00} = 2\phi$$

$$h_{xx} = \bar{h}_{xx} - \frac{1}{2}\bar{h}\eta_{xx} = 2\phi$$

$$h_{yy} = h_{zz} = 2\phi$$

$$g_{00} = 1 + 2\phi \qquad ds^2 = (1+2\phi)dt^2 - (1-2\phi)(dx^2 + dy^2 + dz^2)$$

$$g_{xx} = -1 + 2\phi$$

$$g_{yy} = -1 + 2\phi$$

$$g_{zz} = -1 + 2\phi$$

Grav. spher. $\phi = -\frac{M}{r}$ $(G_N = 1)$

$$ds^2 = \left(1 - 2\frac{M}{r}\right)dt^2 - \left(1 + 2\frac{M}{r}\right)(dx^2 + dy^2 + dz^2)$$

$$dx^2 + dy^2 + dz^2 \longrightarrow \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right)$$

$$\left(1 - \frac{2m}{r}\right) dt^2 - \left(1 + \frac{2M}{r}\right)\left(dr^2 + r^2 d\Omega^2\right)$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

ρ is $\frac{\text{circ}}{2\pi}$ $\quad \rho^2 = \left(1 + \frac{2m}{r}\right) r^2 \approx \left(\left(1 + \frac{m}{r}\right) r\right)^2$

$$= (r + m)^2$$

$$1 - \frac{2m}{r} \approx 1 - \frac{2m}{\rho}$$

$$\boxed{\left(1 - \frac{2m}{\rho}\right) dt^2 - \left(\left(1 + \frac{m}{\rho}\right) d\rho\right)^2 - \rho^2 d\Omega^2}$$

$$\left(1-\frac{2m}{\rho}\right)dt^2-\left(1+\frac{2m}{\rho}\right)d\rho^2-\rho^2 d\Omega^2$$

$$=\left(1-\frac{2m}{\rho}\right)\left(dt^2-\frac{1+\frac{2m}{\rho}}{1-\frac{2m}{\rho}}d\rho^2\right)-\rho^2 d\Omega^2$$

$$=\left(1-\frac{2m}{\rho}\right)\left(dt^2-\left(\left(1+\frac{2m}{\rho}\right)d\rho\right)^2+\ldots\right.$$

$$\rho^* = \int\left(1-\frac{2m}{\rho}\right)d\rho = \rho-2m\ln\rho$$

$$\left(1-\frac{2m}{\rho}\right)\left(dt^2-d\rho^{*2}\right)$$

$u = t - \rho^*$ $\quad du = dt - d\rho^*$

$$ds^2 = \left(1 - \frac{2m}{\rho}\right)\left(du^2 + 2\,du\,d\rho^*\right) + \rho^2 d\Omega^2$$

$$= \left(1 - \frac{2m}{\rho}\right)\left(du^2 + 2\left(1 + \frac{2M}{\rho}\right)du\,d\rho\right) + \rho^2 d\Omega^2$$

$$\left(1 - \frac{2M}{\rho}\right)du^2 + 2\,du\,d\rho\Big) + \rho^2 d\Omega^2$$

Exact Soln of FULL EINSTEIN Eqns.

Exact Soln. $\rightarrow$ Schw. found.