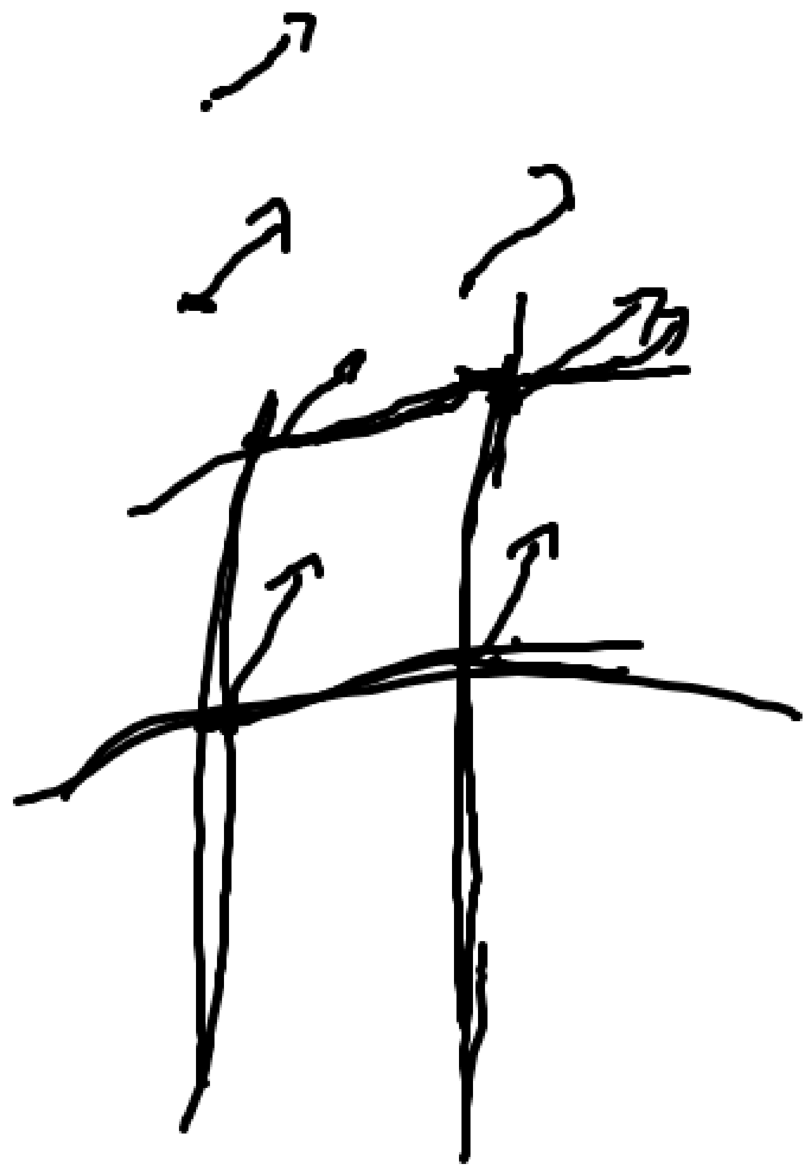
$$\nabla_A \nabla_B V^C - \nabla_B \nabla_A V^C = R_{AB}{}^{C}{}_{D} V^D$$

$$\nabla_A V^C = 0$$

$$t^A \nabla_A V^C = 0$$

$$\eta^A \nabla_A V^C = 0$$



$$\mathcal{L}_{t^A} \eta^A = 0$$

$$\eta^D \nabla_B \left(t^A \nabla_A V^C \right)$$

$$- t^B \nabla_B \left(\eta^A \nabla_A V^C \right)$$

$$= \left(\eta^B \nabla_B t^A - t^B \nabla_B \eta^A \right) \nabla_A V^C + \eta^B t^A \left(\nabla_A \nabla_B V^C - \nabla_B \nabla_A V^C \right)$$

$$\eta^B \nabla_B t^A - t^B \nabla_B \eta^A$$

$$= \mathcal{L}_\eta t^A = 0$$

$$\eta^i \partial_i t^j + \eta^i \Gamma^j_{ik} t^k$$

$$- t^i \partial_i \eta^j - t^i \Gamma^j_{ik} \eta^k \Big] X$$

$$= \boxed{\eta^i \partial_i t^j - t^i \partial_i \eta^j} \quad \mathcal{L}_\eta t^A$$

Parallel derivatives give the diff. in parallelism along 2 diff paths.

$$R_{ij}{}^{k}{}_{l} = \left(\partial_i \Gamma^k_{jl} + \Gamma^k_{im}\Gamma^m_{jl}\right) - \left(i \leftrightarrow j\right)$$

$$R_{ijkl} = g_{km}\left(R_{ij}{}^{m}{}_{l}\right)$$

$$g_{km}\,\partial_i \Gamma^m_{jl} = \partial_i g_{km}\Gamma^m_{jl} - (\partial_i g_{km})\Gamma^m_{jl}$$

$$-(i \leftrightarrow j)$$

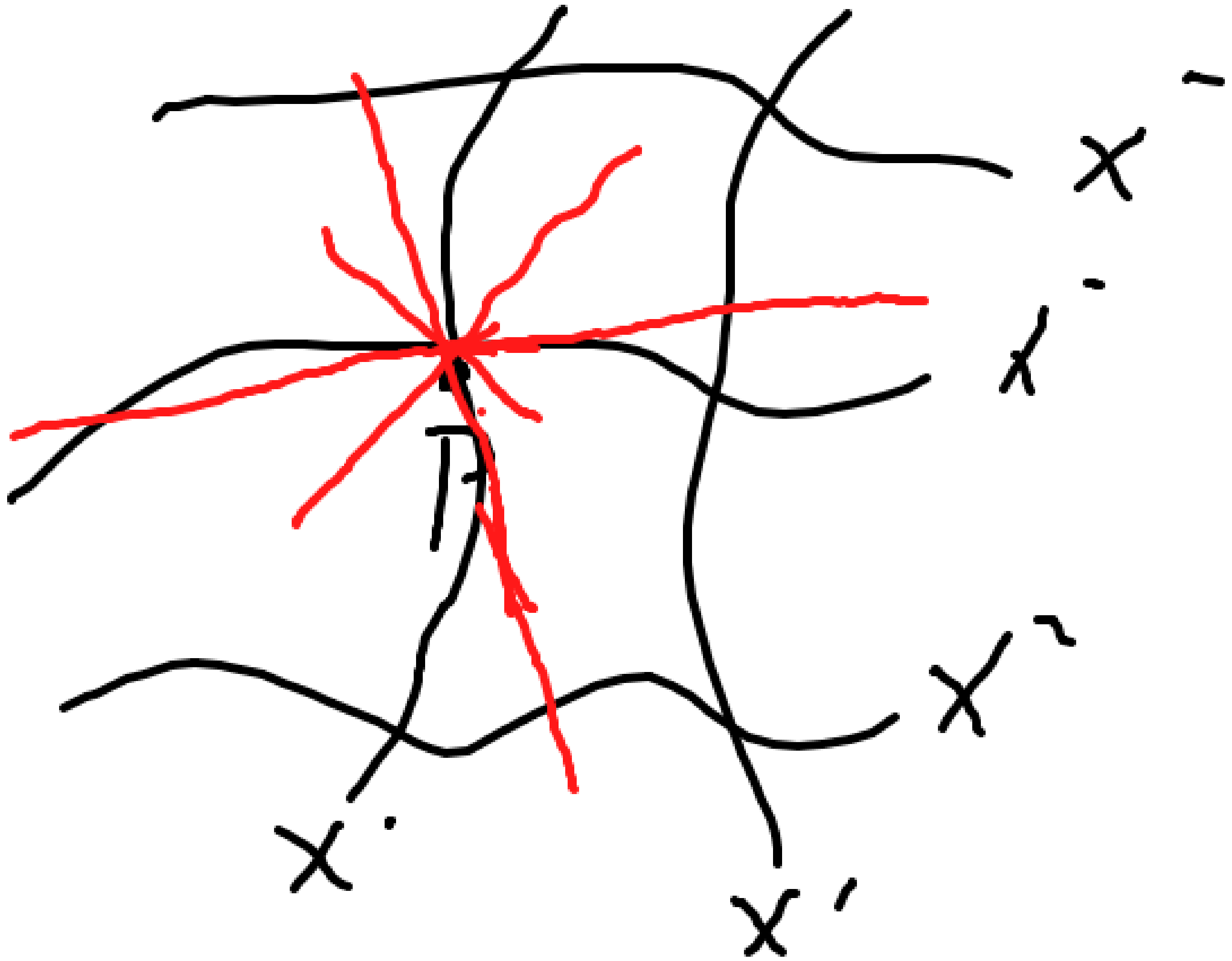
$$g_{km}\Gamma^m_{il} = \Gamma_{kil} = \frac{1}{2}\partial g$$

$$\partial_i\left(g_{km}\Gamma^m_{jl}\right) = \frac{1}{2}\partial\partial g$$

$$\frac{1}{2}\left[\partial_i\partial_k g_{jl} - \partial_i\partial_l g_{jk} - \left(\partial_j\partial_k g_{il} - \partial_j\partial_l g_{ik}\right)\right]$$

$$+ g\partial g\partial g$$

Exponential map.



$$\frac{d^2x^i}{d\lambda^2} + \Gamma^i_{jk}\frac{dx^k}{d\lambda}\frac{dx^j}{d\lambda} = 0$$

$$x^i = t_0^i\lambda - \Gamma_{0jk}^{\ i}\, t_0^j t_0^k \frac{\lambda^2}{2} + O(\lambda^3)$$

$$\tilde{x}^i = t^i\lambda$$

$$x^i = \tilde{x}^i - \Gamma_{0jk}^{\ i}\,\tilde{x}^j\tilde{x}^k/2 + O(\tilde{x}^3).$$

$$\tilde\partial_s \tilde g_{kl}$$

$$\tilde g_{kl} = \overbrace{\frac{\partial x^i}{\partial \tilde x^k}\frac{\partial x^j}{\partial \tilde x^l}}\, g_{ij}\Big|_{\tilde x = 0}$$

(Note assumed that $x^i(p) = 0\ \forall i$

$$x^i = \tilde x^i - \Gamma^i_{mn}\tilde x^m \tilde x^n/2$$

$$\frac{\partial x^i}{\partial \tilde x^k} = \delta^i_k - \Gamma^i_{km} x^m/2 - \Gamma^i_{mk} x^m/2$$

$$= \delta^i_k + \Gamma^i_{km} x^m$$

$$\frac{\partial x^j}{\partial x^l} = \delta^j_l - \Gamma^j_{lm} x^m$$

$$\tilde{g}_{kl} = g_{kl} - \Gamma^{i}_{km} x^m g_{il} - \Gamma^{j}_{lm} g_{kj} x^m$$

$$\tilde{\partial}_s \tilde{g}_{kl} = \underbrace{\underbrace{\tilde{\partial}_s g_{kl}}_{\frac{\partial x^m}{\partial \tilde{x}^s} \frac{\partial g_{kl}}{\partial x^m}}}_{\frac{\partial g_{kl}}{\partial x^s} - \Gamma_{lks} - \Gamma_{kls}} - \Gamma^{i}_{ks} g_{il} - \Gamma^{j}_{ls} g_{jk}$$

$$= 0$$

$$\tilde{\partial}_s \tilde{g}_{kl} = 0 \text{ at } p$$

$$R_{ijkl} = \frac{1}{2}\left(\partial_i \partial_k g_{jl} - \partial_j \partial_k g_{il} - \partial_i \partial_l g_{jk} + \partial_j \partial_l g_{ik}\right)$$

$$R_{ijkl} = -R_{jikl} = -R_{ijlk}$$

$$R_{ijkl} = R_{klij} \; ; \; R_{ijkl} + R_{jkil} + R_{kijl} = 0$$

$$R_{ABCD} = -R_{BACD} = -R_{ABDC} = R_{CDAB}$$

$$R_{ABCD} + R_{BCAD} + R_{CABD} = 0$$

$$\nabla_E R_{ABCD} = \partial(\partial\partial g + \partial g \partial g)$$

$$+ \cancel{\Gamma}(\cdots\cdots)$$

$$0$$

$$\partial\partial g \; \underbrace{\partial g}_{=0}$$

$$\nabla_E R_{ABCD} = \frac{1}{2}\partial_s(\partial_i\partial_k g_{jl} - \partial_j\partial_k g_{il}$$

$$- \partial_i\partial_l g_{jk} + \partial_j\partial_l g_{ik})$$

$$\rightarrow \nabla_E R_{ABCD} + \nabla_A R_{BECD} + \nabla_B R_{EACD} = 0$$

Bianci Identity.

$$\nabla_A g_{BC} = 0 \qquad \nabla_A g^{BC} = 0$$

$$g_{AB} g^{BC} = \delta_A^B$$

$$0 = g^{BD}\left(\nabla_E R_{ABCD} + \nabla_A R_{BECD} + \nabla_B R_{EACD}\right)$$

$$0 = \nabla_E\left(R_{ABCD} g^{BD}\right) + \nabla_A\left(R_{BECD} g^{BD}\right) + \nabla_B\left(R_{EACD} g^{BD}\right)$$

$$R_{AC} = R_{ABCD} g^{BD} = R_{BADC} g^{BD}$$

$$0 = g^{EA}\left(\nabla_E R_{AC} + \nabla_A R_{EC} - \nabla_C R\right) \qquad R = g^{AB} R_{AB}$$

$$g^{AB}\Big(\nabla_A \underbrace{(2R_{BC} - R g_{BC})}_{2G_{BC}}\Big) = 0$$

Einstein Tensor.

G_{BC} has $\partial\partial g$

$$g_{00} = (1 - 2\phi)$$

$$\underline{\nabla^2 \phi} = -4\pi G \rho$$

$$g^{ij}\partial_i G_{jk} \Rightarrow \partial_i G^{ik}$$

$$\partial_j T^{ij} = 0$$

conservation of energy

& momentum.

$$\partial_j \sigma^{ij} = 0$$

$$\boxed{G_{ij} = \kappa T_{ij}}$$

$$\partial_j T^{ij} = 0$$

$$\nabla^2 \phi \propto \rho$$

Linearized gravity and X ?