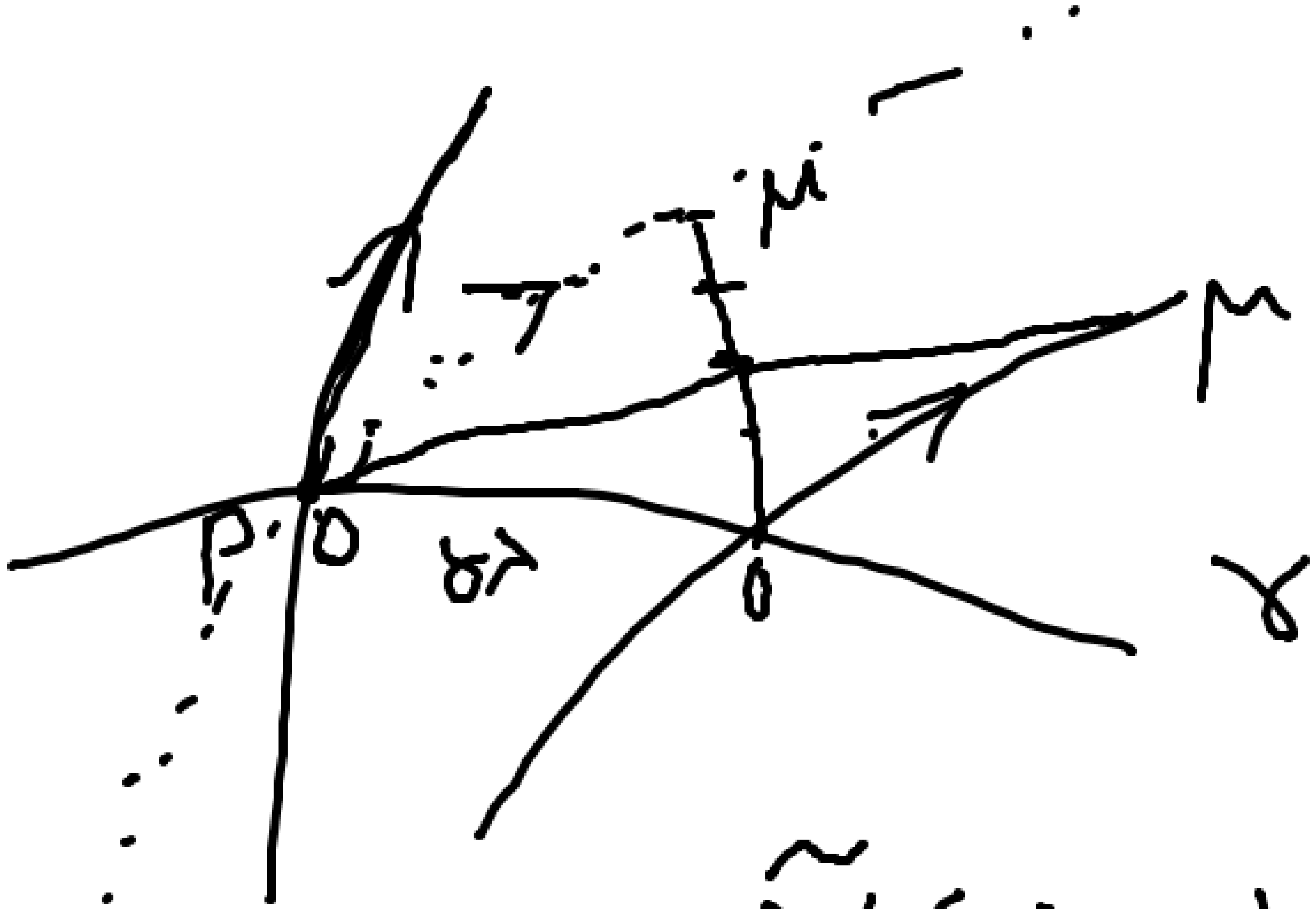

$$\lim_{\delta\lambda \to 0} \frac{\tilde{V}(\lambda+\delta\lambda) - V(\lambda)}{\delta\lambda}$$

$$\frac{D V^A}{D\lambda} = \text{Linear in } \partial_\gamma^A$$

$$= (\nabla_B V^A)(\partial_\gamma^B) = \frac{D}{D\lambda} V^A$$

$$\nabla_B W_A \implies \nabla_B (W_A V^A) = (\nabla_B W_A) V^A + W_A \nabla_B V^A$$

$$d(W_A V^A)_B$$

$$\partial_j \left(\sum_i W_i V^i \right)$$

$$(\nabla_B W_A)_{ij} = \partial_i W_j - \sum_k \Gamma_{ij}^{k} W_k$$

$T(V^A, U_B, W^C)$, linear

$T(\alpha V^A, U_B, W^C) = T(V^A, \alpha U_B, W^C)$

$= T(V^A, U_B, \alpha W^C) = \alpha T(V^A, U_B W^C)$

$T(V^A + Q^A, U_B, W^C) = T(V^A, U_B, W^C)$
$+ T(Q^A, U_B, W^C)$

sim for 2nd + 3rd arg.

Notation: T

$W^A T_A$ $\quad W(.)$ $\quad W^A \Rightarrow$ functn of cot vect.

W^A either Lang v. or funcn of cot vectors overload symbols.

$$T(V^A, U_B, W^C)$$

$$T_A{}^B{}_C V^A U_B W^C \quad \text{Means.}$$

$$(T_A{}^B{}_C \, \partial_i^A \, dx^j_B \, \partial_k^C) \equiv T_i{}^j{}_k$$

$A, B, C \longrightarrow$ Tensor fun.

i, j, k lower components.

$$\xi^A \nabla_A V^B \longrightarrow \xi^A \text{ Tang vector to curve.}$$

$$\frac{D}{Dx} V^B \qquad (\nabla_A V^B)\, \xi^A$$

$$\underbrace{T(V^A, u_B, W^C) + R(V^A, u_B, W^C)}$$

$$T_A{}^B{}_C + R_A{}^B{}_C$$

$$T(V^A, u_B) + S(V^A)$$

$$T(V^A, u_B + \tilde{u}_B) + S(V^A)$$

Not linear

$$= T(V^A, u_B) + T(V^A, \tilde{u}_B) + S(V^A)$$

$$\neq \left(T(V^A, u_B) + \cancel{S(V^A)}\right) + \left(T(V^A, u_B) + S(V^A)\right)$$

To add Tensors. Must have same arg.

(Must have same super + sub scripts)

$g(V^a, W^B)$ — metric.

linear.

g_{AB} $\quad \bar{g}(W_A, W_B)$ — g^{AB}

$\delta_A{}^B(V^A, W_B) = V^A W_A$

$$T^{AB} \;,\; R_C{}^D$$

$$P^{AB}{}_C{}^D \equiv T^{AB} R_C{}^D$$

$$P(V_A, u_B, X^C, Y^D) = T(V_A, u_B) R(X^C, Y_D)$$

Contraction

$$V^A W_B \Rightarrow V^A (W_A)$$

$$T^{A}{}_{B}{}^{C} \Longrightarrow T^{A}{}_{A}{}^{C}$$

$$V^{A} W_{B} \longrightarrow \sum_{i} \left(\cancel{V}^{i} W_{i} \right)$$

$$T^{A}{}_{B}{}^{C} = \alpha R_{1}{}^{A} S_{1B} U_{1}{}^{C} + \beta R_{2}{}^{A} S_{2B} U_{2}{}^{C} + \gamma \cdots\cdots$$

$$T^{A}{}_{A}{}^{C} = \alpha \left(R_{1}{}^{A} S_{1A} \right) U_{1}{}^{C} + \beta R_{2}{}^{A} S_{2A} U_{2}{}^{C}$$

$$\perp \ldots$$

$$T^{i}{}_{j}{}^{k} \qquad T^{A}{}_{A}{}^{B} \rightarrow \underbrace{\sum_{i} T^{i}{}_{i}{}^{k}}_{\text{Trace.}}$$

Tensor. Indep of how one breaks up the tensor into vectors.

$$g_{AD}\, g^{CD} \Rightarrow g_{AD}\, g^{BD} = \delta_{A}{}^{B}$$

$$g^{CD} W_D = W^C$$

$$g_{AB} T^B = T_A$$

$$W^C T_C = g^{CD} W_D \, g_{CB} T^B$$

$$= g^{CD} g_{CB} \, W_D T^B$$

$$= W_C T^C$$

$$g^{CD} g_{CB} T^B = T^C$$

$$\delta^D{}_B T^B = T^C$$

$$V^A W_A = \sum_i V^i w_i$$

Einstein summation convention

If up index in components & down index are repeated Then sum over them.

convention.

Write tensors in S.R. indices.

$\Downarrow$

Tensors.

$$\partial_i \Rightarrow \frac{\partial}{\partial x^i} \Rightarrow \nabla_A$$

sprel. metric $\underbrace{(-1,+,+,+)}_{\text{metric}}$

Take physics of S.R. g_{AB}, g^{AB}
to G.R.

$F_{ij} \Rightarrow F_{0i} \rightarrow E_i$

$F_{jk} \Rightarrow B_i$

(ijk are perm (even) 123)

F_{AB} $\partial_t E = \nabla \times B$

$\nabla_A g^{ac} F_{cB} = 0$

$\nabla_A F_{BC} + \nabla_D F_{CA} + \nabla_C F_{AB} = 0$

$$\partial_i \partial_j V_k - \partial_j \partial_i V_k = 0$$

$$\nabla_A \nabla_B V_C - \nabla_B \nabla_A V_C \neq 0$$

$$\nabla_A \nabla_B V^C - \nabla_B \nabla_A V^C$$

$$= R_{AB}{}^{C}{}_{D} V^{C}$$

Curvature Tensor.

$$\nabla_a \nabla_b V^c - \nabla_b \nabla_a V^c$$

$$\to \nabla_a \left(\partial_j V^k + \Gamma^k_{jl} V^l \right) \quad -\Gamma \quad k \leftrightarrow m$$

$$\to \left[\partial_i \left(\quad \right) + \Gamma^k_{im} \left(\quad \right) - \Gamma^m_{ij} \left(\partial_m V^k + \Gamma^k_{ml} V^l \right) \right] -$$

$$- \left[\quad \right]_{i \leftrightarrow j}$$

$$\Rightarrow \left(\_ \right) V \qquad R \to \partial\partial g$$

$$(1 - 2\Phi) \Rightarrow \nabla^2 \phi \cdot \rho$$