1. Geodesics — St lines
Extremum dist bet 2 pts.

$$S = \sqrt{g_{AB}\, \partial_\gamma^A \partial_\gamma^B}$$

$$\int_{\lambda_1}^{\lambda_2} S \, d\lambda \qquad \gamma(\lambda_1) \text{ to } \gamma(\lambda_2)$$

$\gamma(\epsilon, \lambda)$

$$\frac{d}{d\epsilon} \int_{\lambda_1}^{\lambda_2} S \, d\lambda \Big|_{\epsilon=0} = 0$$

$\gamma(\epsilon=0, \lambda) = \gamma(\lambda)$

$\gamma(\epsilon, \lambda_1) = P_1$

$\gamma(\epsilon, \lambda_2) = P_2$

$$\sum_j \frac{d}{d\lambda} \frac{1}{S} g_{ij} \frac{dx^j}{d\lambda} + \frac{1}{2S} \partial_k g_{ij} \frac{dx^k}{d\lambda} \frac{dx^j}{d\lambda} = 0$$

param so that $S = \text{const}$

$S \neq 0$

$\int S \, d\lambda \qquad \gamma(\lambda_1) \neq \gamma(\lambda_2)$

$$\frac{d}{d\epsilon} \int S^2 \, d\lambda = \int S \frac{d}{d\epsilon} S \, d\lambda$$

const.

If S const. on s.ln

$\int S^2 \, d\lambda$ gives same eq as

$\int S \, d\lambda \qquad S^2 \to g_{AB} \partial_\tau^A \partial_\tau^B$

$\underline{\underline{S^2 = \text{const}}}$ on solutions

$$\sum_j \frac{d}{d\cancel{x}} \frac{1}{2S} g_{ij} \frac{dx^j}{d\lambda}$$

$$\sum_j \frac{d}{d\lambda} g_{ij} \frac{dx^j}{d\lambda} =$$

$\int S' d\lambda$ Does not have trouble with $S = 0$

But $S^2 = 0$ Must be imposed

Param. $\sum_j \frac{d}{d\lambda} g_{ij} \frac{dx^j}{d\lambda} - \frac{1}{2} \sum_{kj} \partial_i g_{jk} \frac{dx^j}{d\lambda} \frac{dx^k}{d\lambda}$

Affine param. Null $= 0$

$t = x$, $X = \cos f(x)$, $y = \sin(f(x))$

$z = g(x)$

If $1 - f'^2 - g'^2 = 0$

And. f, g so that

$\rightarrow t(0) = 0$, $X(0) = 1$, $y(0) = 0$

$z(0) = 0$

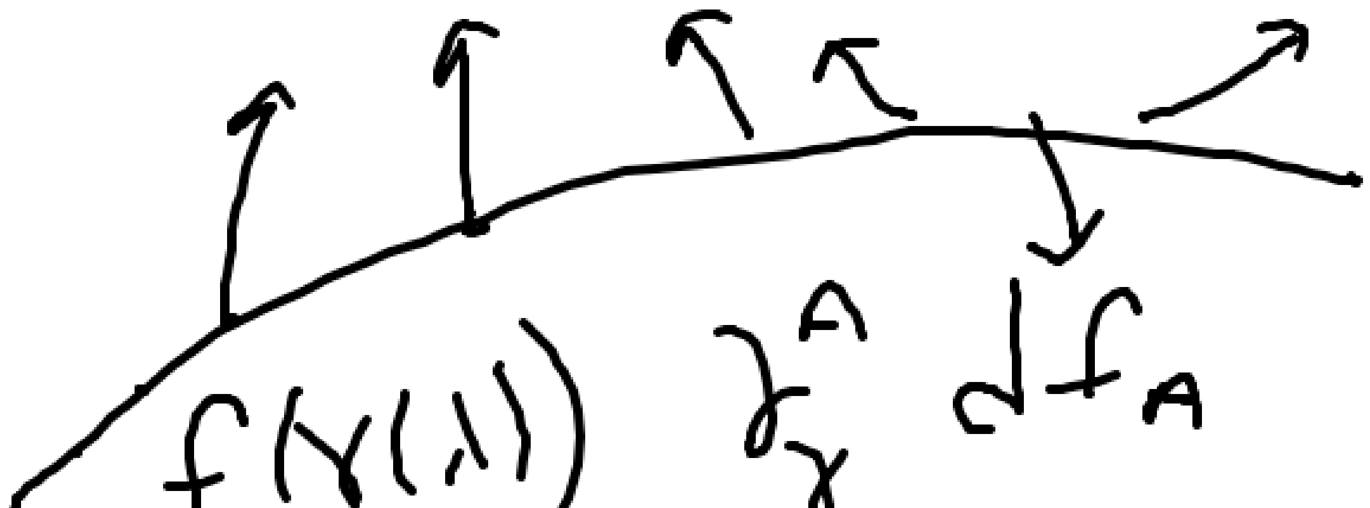
$\rightarrow t(1) = 1$, $X(1) = 1$, $y(1) = 0$

$z(1) = 0$

Derivatives.

2 kinds $\longrightarrow$ Covariant (Parallel) deriv

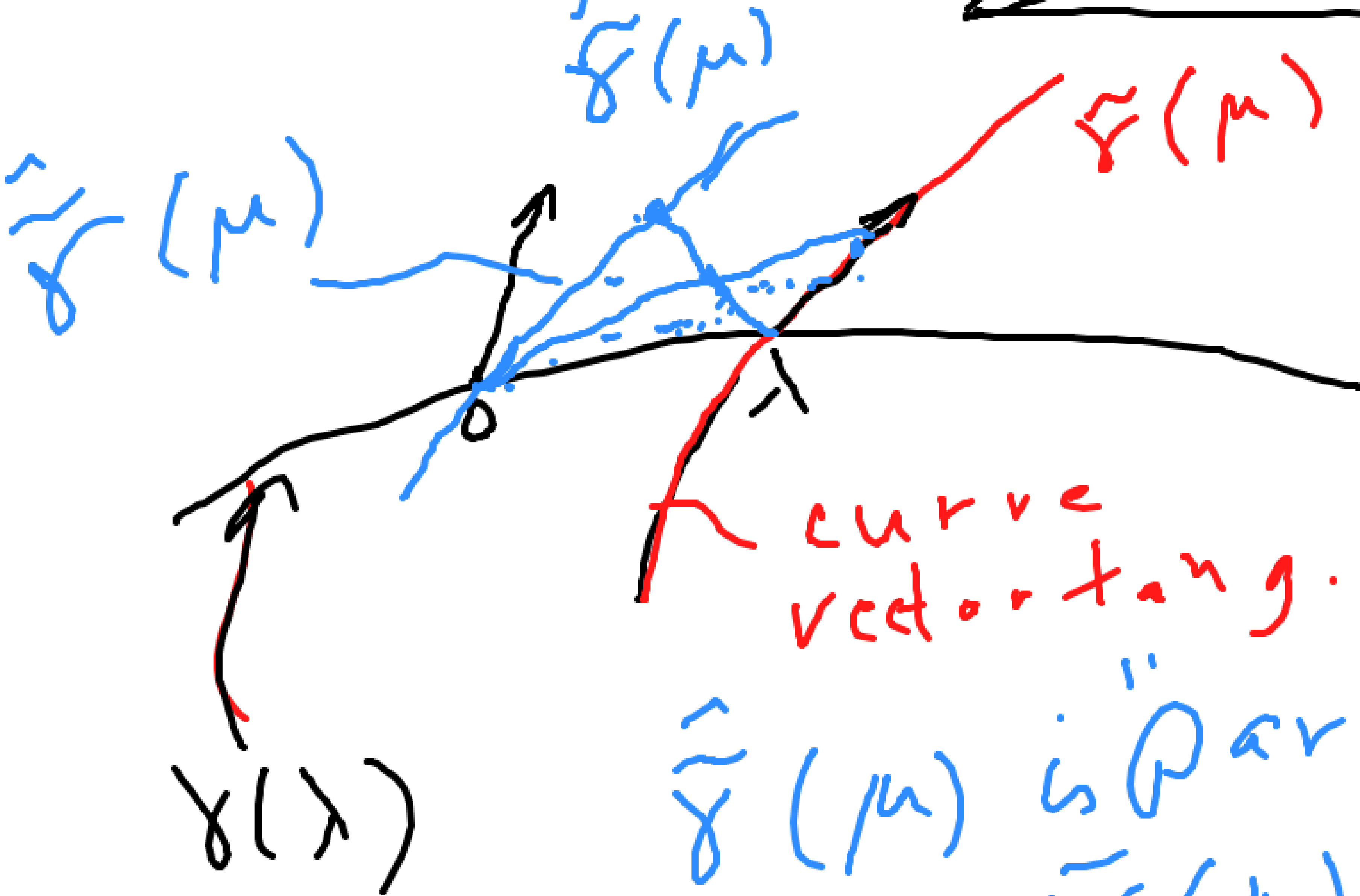
$\longrightarrow$ Lie derivative.



$\gamma(\lambda)$ $\lim_{\Delta\lambda \to 0} \frac{(\cdot)(\lambda+\Delta\lambda)-(\cdot)(\lambda)}{\Delta\lambda}$

$f(\gamma(\lambda))$ $\gamma^A_{\gamma}\, df_A$

Parallism $\longrightarrow$ metric



$$\partial \tilde{\gamma}^A(\gamma(\lambda)) =$$
$$= V^A(\gamma(\lambda))$$

$\hat{\tilde{\gamma}}(\mu)$ is "parallel" (by defn) to $\tilde{\gamma}(\mu)$ but goes through $\partial^A_{\hat{\tilde{\gamma}}}$ at p.

$p = \gamma(0)$

But $\tilde{\hat{\gamma}}(\mu)$ and $\gamma(\mu)$ are curves through same point

$\partial^A_{\tilde{\gamma}}$ and ∂^A_{γ} can be subtracted.

$\frac{D}{D\lambda} V^A$ derivative

$V^A(p)$ is tangent vector to $\partial^A_{\tilde{\gamma}}(p)$

Components of $\dfrac{DV^A}{D\lambda}$

$$\left(\frac{DV^A}{D\lambda}\right)^i dX_A^i = \left(\frac{DV^A}{D\lambda}\right)^i$$

$$\frac{DV^i}{D\lambda} = \frac{dV^i(\gamma(\lambda))}{d\lambda} + \sum_{j,k,l} \frac{1}{2}\left(\partial_k g_{lj} + \partial_j g_{lk} - \partial_l g_{jk}\right) g^{il}\, t^j V^k$$

t^j = tang vector to γ

$$\sum_l g^{il} g_{jl} = \delta^i_j$$

$$\Gamma^i_{jk} = \frac{1}{2}\sum_l g^{il}\left(\partial_j g_{lk} + \partial_k g_{lj} - \partial_l g_{jk}\right)$$

Christophel symbol of 2nd kind.

$V^i = t^i \Longrightarrow$ Eqn is geodesic eqn. $\parallel$

$(\lambda \to s)$

2 defs of geodesic.

$\Gamma^i_{jk} = \Gamma^i_{kj} \longrightarrow 40$ components.

$$\frac{d}{d\lambda}\underbrace{V^i}_{\text{functions}} = t^A\, dV^i{}_A = \underline{\underline{t^j}}\, \partial_j V^i$$

$$\Gamma^i_{jk}\, \underbrace{t^k}\, V^i$$

$\frac{D}{D\lambda}$ is linear in tang vector

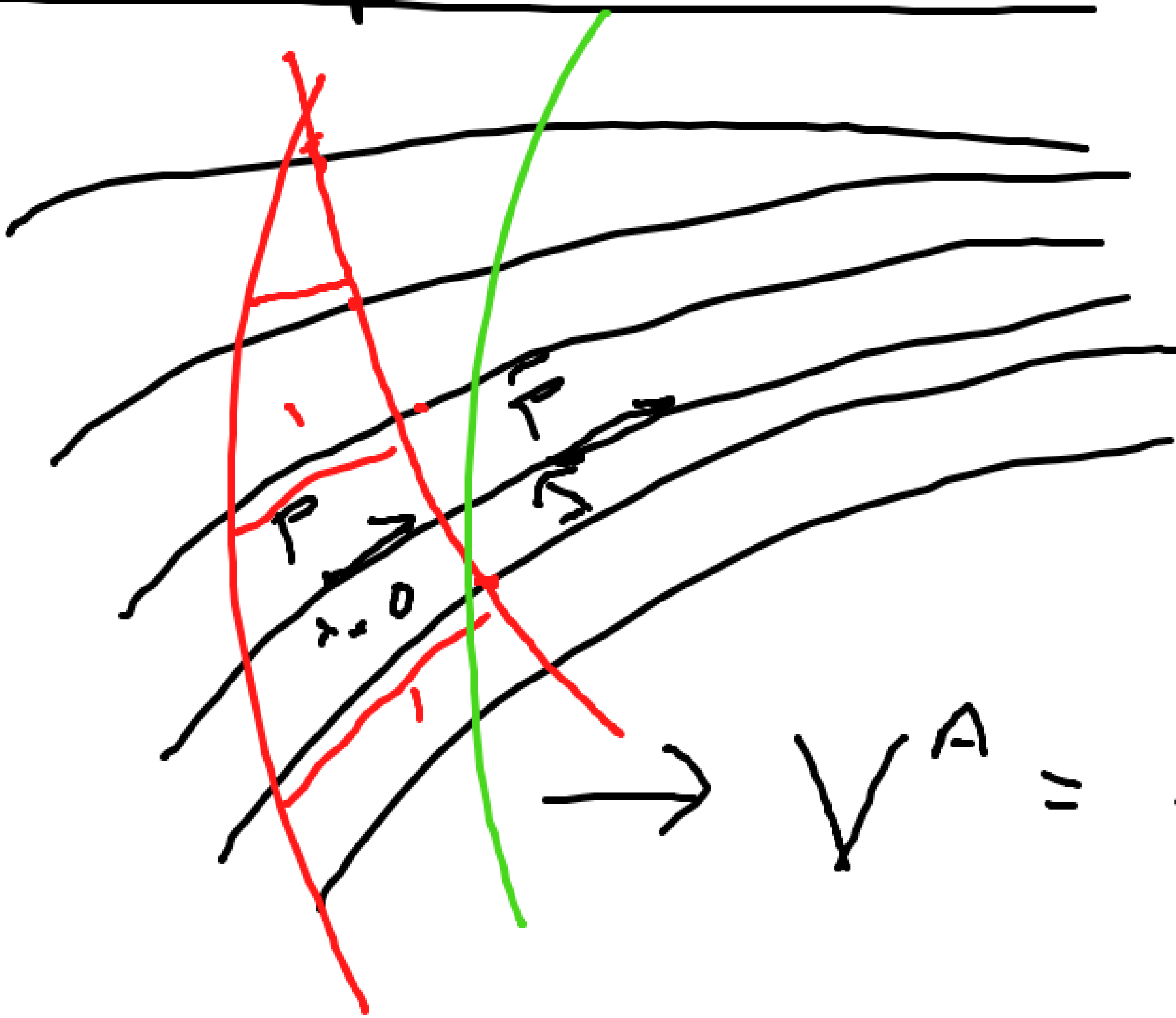
$$\frac{DV^B}{D\lambda} = \underline{t^A \nabla_A} V^B$$

Grad of Vector

Linear in tangvector.

Lie Derivative

More primative. Vectors V^A, W_B



Curves fill space.

$\gamma(p,\lambda)$, $\gamma(p,0)=p$

$\gamma(\tilde{p},\lambda)=\gamma(p,\lambda+\tilde{\lambda})$

if $\tilde{p}=\gamma(p,\tilde{\lambda})$

$\rightarrow V^A = \frac{\partial}{\partial\gamma(p,0)}^{A}$

$\hat{\gamma}(\mu)$

$\tilde{\gamma}(\mu,\lambda)$

$=\gamma(\hat{\gamma}(\mu),\lambda=1)$

Derivate defined by sliding everything over the spacetime by "dist" λ $\rightarrow$ Now compare tang, cot. vectors at same pt + Define deriv.

$$\mathcal{L}_{V^A} f$$

$$\frac{f(\gamma(\lambda)) - f(\gamma(0))}{\lambda} = V^A df_A$$

Cotang Vector W_B

Components: V^i, W_j

$$(\mathcal{L}_V W)_i = \sum_j \left(V^j \partial_j W_i + \underbrace{W_j \partial_i V^j}_{} \right)$$

$U^A W_A \Rightarrow$ function

$$\mathcal{L}_V (U^A W_A) = (\mathcal{L}_V U^A) W_A + (\mathcal{L}_V W_A) U^A$$

$$\sum_i \mathcal{L}_V (U^i W_i) = \sum_j (\mathcal{L}_V U^A)^j W_j + (V^j \partial_j W_i + W_j \partial_i V^j) U^i$$

$$(\mathcal{L}_V U)^i = \sum_j (V^j \partial_j U^i - U^j \partial_j V^i)$$

Antisym in V, U

$$\mathcal{L}_V U = -\mathcal{L}_U V$$ Commutator