Physics 501-20

Vaidman Bomb detector

The cover story is that a country has a large supply of which many are duds. They want to separate out the duds from good one. One way of doing so of course is to try to detonate them. That will tell them which ones were duds and which ones were good. Unfortuantely, it leaves them all of the bombs being useless. Is there a way of doing so¿

The answer is yes, using quantum mechanics. We are going to use a single two level system, and a highly sensitive detector. This detector is attached to the bomb and is so sensitive that if the detector detects, the bomb explodes.

There are three parts to this detector. The first is a two level system. The second is a rotator, R which acts to rotate a two level state so that

$$R\left|\uparrow\right\rangle = \cos(\theta)\left|\uparrow\right\rangle + \sin(\theta)\left|\downarrow\right\rangle \tag{1}$$

$$R\left|\downarrow\right\rangle = \cos(\theta)\left|\downarrow\right\rangle - \sin(\theta)\left|\uparrow\right\rangle \tag{2}$$

Ultimately θ will be very small.

Teh second aspect is a detector D which is sensitive only to the state $\left|\downarrow\right\rangle$. It is attached to the bomb. If the bomb is good, when the state $\left|\downarrow\right\rangle$ comes along, it detonates the bomb for sure which destroys the detector, the two level system and the bomb. If the bomb is a dud, $\left|\downarrow\right\rangle$ is transmitted through the detector without change (including to its phase) as is $\left|\uparrow\right\rangle$.

WE send in the intial state $\left|\uparrow\right\rangle$ first through the rotator, and then through the detector. Assuming that the bomb has not exploded, that two level system is then sent back as input to the rotator, and the detector. This continues N times. We choose the angle θ of the rotator to be $\frac{\pi}{2N}$.

Let us first assume that the bomb is a dud. Then after the nth transit of R the state will be $cos(n\frac{\pi}{2N})\left|\uparrow\right\rangle + sin(n\frac{\pi}{2N})\left|\downarrow\right\rangle$. (proof: by induction

$$R \quad (cos((n-1)\frac{\pi}{2N})\left|\uparrow\right\rangle + sin((n-1)\frac{\pi}{2N})\left|\downarrow\right\rangle) \tag{3}$$

$$= \quad (cos(\frac{\pi}{2N})cos((n-1)\frac{\pi}{2N}) - sin(\frac{\pi}{2N})sin((n-1)\frac{\pi}{2N}))\left|\uparrow\right\rangle \tag{4}$$

$$+ \quad (cos(\frac{\pi}{2N})sin((n-1)\frac{\pi}{2N}) - sin(\frac{\pi}{2N}cos((n-1)\frac{\pi}{2N}))\left|\downarrow\right\rangle \tag{5}$$

$$= \quad cos(n\frac{\pi}{2N})\left|\uparrow\right\rangle + sin(n\frac{\pi}{2N})\left|\downarrow\right\rangle \tag{6}$$

and the formula is certainly true for $n = 1$ when the incoming state is $\left|\uparrow\right\rangle$ which is the expression for the incoming state with n=1.)

After N circuits, the outgoing state is $cos(\pi/2)\left|\uparrow\right\rangle + sin(\pi/2)\left|\downarrow\right\rangle = \left|\downarrow\right\rangle$.

So if the bomb is a dud, the final state of the two level system is $\left|\downarrow\right\rangle$.

If the bomb is live, when the detector sees the $\downarrow$ component, it explodes, if it does not explode the state of the system must be the $\uparrow$ component and so the state after it has gone through the detector is $cos(\frac{\pi}{2N})\left|\uparrow\right\rangle$. When it goes through the next time, it comes out as $cos(\frac{\pi}{2N})^2\left|\uparrow\right\rangle$ and after N times, it is $cos(\frac{\pi}{2N})^N\left|\uparrow\right\rangle$. For large N this is $(1 - \frac{1}{2}\frac{\pi^2}{4N} + O(1/N^3))\left|\uparrow\right\rangle$. Ie, for large N it comes out almost with certainty. Thus the final outstate is $|\downarrow\rangle$ if the bomb is a dud, and $|\uparrow\rangle$ if it is live, but the probability of the bomb having been detonated is ony of order $\frac{\pi^2}{4N}$.