Locality and Quantum Mechanics

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It is argued that it is best not to think of quantum mechanics as non-local, but rather that it is non-realistic.

The work in this paper is based on prior papers by myself[1] [2] and L Hardy[3].

What does one mean by locality in quantum mechanics? There is a sense in which quantum mechanics is clearly non-local. The property of the position of a particle is clearly non-local. It is represented by an operator which has all possible positions of the particle as possible eigenvalues. Any operator which does not commute with it, will make the particle's position be spread all over the place– i.e., the particle does not really have a locality. But this is clearly not what one means in saying that quantum mechanics is non-local, especially since we know that quantum mechanics as the theory of the world is wrong. Only quantum field theory is consistent with special relativity.

In quantum field theory, the operators associated with the field have positions. $\phi(x)$ is an operator associated with the field at the position x in spacetime. Furthermore, the quantum fields obey equations which are strictly local and causal. As an example, $\Box\phi + m^2\phi = 0$ is the Heisenberg equation for a free massive scalar quantum field, and it is local and causal. If one causes a change in ϕ localized at location x, then $\phi(y)$ changes only for y in the causal future light cone of that location x. It is the retarded Green's function which determines how ϕ changes at some location because of changes made to ϕ at some other location.

But again, this is not the sense in which people say that quantum mechanics is non-local.

In 1955 Bell[4] argued that if one placed two conditions on a theory, that of realism and of relativistic locality, one could not obtain a theory which was consistent with the predictions of quantum mechanics. Ever since, this has led many people to argue that quantum mechanics must therefor be a non-local theory. Unfortunately this is unhelpful in deriving an intuition of how quantum theory works. This paper will largely follow an earlier paper which I published in which I argued that locality in Bell's argument does not play the central role that many ascribe to it. Locality plays the role in Bell's argument of making the classical system more similar to quantum theory, rather than differentiating it from quantum theory. Furthermore, entanglement plays a key role in Bell's argument leading many to believe that the essence of the difference between quantum and classical theory is the presence of entanglement. I will also argue that this is not the case by presenting a system modelled on one first described by Hardy[3]. While entanglement plays a role, it is when entanglement is small that the difference between classical and quantum becomes greatest. In particular the difference between the quantum and the classical system is greatest in the limit that the two systems are in a product state and un-entangled (although in that limit the conditions necessary to carry out the argument do not apply).

In examining Bell's argument I used the Clauser-Horne-Shimony-Holt[5] version of the argument, in which one has two "spin 1/2" particles (ie, two two-level systems). One makes a series of measurements of properties, A and B for the first system, and C and D for the second in various combinations. Since the measurement on the one system may alter measurements of other properties on the same system, the observer always makes measurements of attributes on the first and the second system. All four properties are such that they can take only two values, which I will designate by values +1 and -1. Since quantum mechanics is a probabilistic system, the values obtained in any experiment will also be probabilistic. There can, however, be correlations between the measurements on the two systems because of the way the two systems were prepared. Such correlations are important to the argument both in classical and in quantum systems. We now carry out a vast series of measurements on the two systems, and calculate from those measurements what the various correlations are between the outcomes on the two systems. For simplicity we assume that however they were prepared, the mean value of the measurements of any single property is 0. Note that 0 is not one of the possible outcomes of any one of the measurements. This implies that for any of the properties the probabilities of the the individual values must be 50%.

Given the measurements made, we can divide them into four classes, those in which A was measured on the first, and C on the second, those in which A was on the first and D on the second, those with B on the first and C on the second and those with B on the first and D on the second. In each of these classes , the mean value of A or B and C or D is found to be 0 to within the statistical accuracy expected (i.e., $1/\sqrt{N}$ for the standard deviation of the

deviation from 0). In each of these samples we calculate the correlation function by taking the value measured for the measured quantity, A or B on 1 and multiplying it with the measured value on the other, C or D. We calculate the correlation function $\{AC\}, \{AD\}, \{BC\}, \{BD\}$ where

$$\{AC\} = \sum \frac{a_i c_i}{N} \tag{1}$$

where a_i is the value recorded for the measurement of A in the ith trial, and N is the total number of trials in which both A and C were measured. Note that since a_i and c_i take values of only ± 1, this correlation function must lie between -1 and 1.

The interesting correlation function is the sum

$$\mathcal{K} = \{AC\} + \{AD\} + \{BC\} - \{BD\} \tag{2}$$

In quantum mechanics, this correlation function is easily calculated. The properties A, B, C, D are represented by operators, and, if the experiment is designed so that the same state $|\phi\rangle$ is used in each run, then

$$\{AC\} = \langle\phi|\mathbf{AC}|\phi\rangle \tag{3}$$

where $\mathbf{A}$ is the operator (matrix) associated with the property A (and similarly for the other properties). Then

$$\begin{aligned} \mathcal{K}_Q &= \langle\phi|\mathbf{AC}|\phi\rangle + \langle\phi|\mathbf{AD}|\phi\rangle + \langle\phi|\mathbf{BC}|\phi\rangle - \langle\phi|\mathbf{BD}|\phi\rangle & (4)\\ &= \langle\phi|\mathbf{AC} + \mathbf{AD}|\phi\rangle + \langle\phi|\mathbf{BC} - \mathbf{BD}|\phi\rangle & (5)\\ &= \langle\phi|\mathbf{A}(\mathbf{C} + \mathbf{D})|\phi\rangle + \langle\phi|\mathbf{B}(\mathbf{C} - \mathbf{D})|\phi\rangle & (6) \end{aligned}$$

where the second and third lines follow from the linearity of quantum mechanical theory.

However one cannot assume that the above procedure is possible for the correlation function $\mathcal{K}_C$ for the classical case. It is in order to derive the same expression for the classical theory that Bell made the assumption especially of locality. Ie, it is in order to argue that the classical system behaves similarly to the quantum system that the assumption of locality is introduced: namely that in each case the expectation value of the sum is the sum of the expectation values. (Peres and Zurek[7] make the stronger assumption that the all of the operators actually have values in each set of measurements, and that those values are the same as would have been measured had the unmeasured attbributes been measured. This is a stronger assumption than the Bell locality assumption)

This is where the locality plays its role in two different ways. The first is that we assume that the values delivered by the measurements depend only on the physics local to the system. Thus the measurements on system 1 depend only on the physics local to system 1. This means that the outcome of the measurements cannot depend on what is measured at system 2. Technically this means that we assume that the correlation functions are such that they do not depend on which subset of possible measurements one chooses. If one had measured A and D rather than A and C in the subset in which one measured A and C, one would have gotten the same result for the correlations of A and D as one got in the subset one did measure, i.e., that

$$[AD] = \sum_{AC} a_i \tilde{d}_i / N = \{AD\} \tag{7}$$

where $\tilde{d}_i$ would have been the outcome if instead of C one had measured D on the ith measurement of A and C. We can thus write

$$C_C = \sum_i (a_i c_i + a_i \tilde{d}_i)/N + \sum_i b_i (c_i - \tilde{d}_i)/N = \{A(C + D)\} + \{B(C - D)\} \tag{8}$$

Now since if $|c_i + d_i|$ is 2, then $c_i - d_i$ is zero, and if $|c_i + d_i|$ is 0 then $|c_i - d_i|$ is 2, then for any i, the sum is bounded by -2 and +2, and the average must also be bounded by them. However, if one uses an entangled quantum state, the $\mathcal{K}_Q$ can be made equal to $2\sqrt{2}$, we find that the classical system can never mimic the quantum. The role that the locality assumption has played is to make the classical system more like the quantum system than it would have without that assumption. Locality makes classical more like quantum, not less.

In this argument, it seems that entanglement plays a crucial role. It is the entangled quantum state which leads to the violation of the Bell inequality. Thus entanglement is critical to the quantum violation of the Bell bound. This would seem to imply that the difference between quantum and classical systems depends on entanglement. However, one can set up a situation in which the more un-entangled the quantum system is, the more completely it violates the classical laws of logic. This example, discovered by Hardy, shows that quantum mechanics violates the Einstein

realism enunciated in the paper with Rosen and Podolsky[6]. *"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding lo this physical quantity."*

Consider again two two-level quantum systems as above. Consider again four specially selected properties of the two systems, A and B of system 1 and C and D of system 2. Consider the four propositions:

$\mathcal{A}$ = The measurement of property A gives value +1.

$\mathcal{B}$= The measurement of property B gives value +1

$\mathcal{C}$ = The measurement of property C gives value +1

$\mathcal{D}$= The measurement of property D gives value +1

What we will generate is an appropriate choice of the properties A and B of system 1 and C and D of system 2 so that

If $\mathcal{A}$ then it is always true that $\mathcal{C}$ is true.

If $\mathcal{C}$ then always $\mathcal{B}$ is true

If $\mathcal{B}$ then always $\mathcal{D}$ is true.

or

$$\mathcal{A} \rightarrow \mathcal{C} \rightarrow \mathcal{B} \rightarrow \mathcal{D} \tag{9}$$

where the arrows denote implication. For any realistic system this would seem to imply that

If $\mathcal{A}$ then always $\mathcal{D}$: $\mathcal{A} \rightarrow \mathcal{D}$.

However for the quantum system, this last will turn out to be false, in fact the closer the state is to a product state, a completely un-entangled state, the lower is the probability that if $\mathcal{A}$ then $\mathcal{D}$.

To simplify the argument, I will choose the state such that we can choose properties A,B,C,D such that

$$|\phi\rangle = (\sin(\theta)|1\ 1\rangle + \sin(\theta)|-1\ 1\rangle)/\sqrt{2} + \cos(\theta)|-1\ \ -1\rangle \tag{10}$$

where $|ij\rangle$ represents the state with the eigenvalue i for $\mathbf{A}$ and eigenvalue j for operator $\mathbf{C}$. One can always do this as long as the reduced density matrix of one of the particles has a smallest eigenvalue that is less than $(2-\sqrt{2})/4$ (if the smallest eigenvalue were 1/2 the state would be a maximally entangled state, a Bell state). One can carry out a similar argument for an arbitrary state with a different choice of properties.

We choose the operator corresponding to property B $\mathbf{B}$ so that $|B+\rangle_1 = (|1\rangle_1 + |-1\rangle_1)/\sqrt{2}$ is its eigenstate with +1 eigenvalue. Ie, if $\mathbf{A} = \sigma_z$, then $\mathbf{B} = \sigma_x$ where σ are the Pauli spin matrices. If we choose the operator $\mathbf{D}$ so that $|D+\rangle = (\sin(\theta)|1\rangle_2 + \left(\cos(\theta)/\sqrt{2}\right)|-1\rangle_2)/R$ is its eigenvector with eigenvalue +1 where $R^2 = \sin(\theta)^2 + \cos(\theta)^2/2$. ($|\psi\rangle_i$ designates the state ψ for the i*th* particle.) $\mathbf{D}\approx -\sigma_z$ if $\mathbf{C}= -\sigma_z$.

Then, defining $|A+\rangle = |1\rangle_1$ and $|C+\rangle = |1\rangle_2$, we can write $|\phi\rangle$

$$\begin{aligned} |\phi\rangle &= \left[\sin(\theta)|A+\rangle|C+\rangle/\sqrt{2}\right] + |A-\rangle(\sin(\theta)/\sqrt{2}|C+\rangle + \cos(\theta)|C-\rangle) && (11)\\ &= [\sin(\theta)|B+\rangle|C+\rangle] + \cos(\theta)|A-\rangle|C-\rangle && (12)\\ &= [\sin(\theta)|B+\rangle|D+\rangle] + |B-\rangle|D-\rangle && (13) \end{aligned}$$

The square bracket on the first line makes it clear that if one measures A and C, then $\mathcal{A}$ always implies $\mathcal{C}$. The second line makes it clear that if $\mathcal{C}$ then always $\mathcal{B}$. The third makes it clear that if $\mathcal{B}$ then always $\mathcal{D}$. But $|D+\rangle$ is almost never the same as $|C+\rangle$. The probability of finding $|C+\rangle$ if one has $|D+\rangle$ is proportional to $\sin(\theta)^2$. Thus $\mathcal{A}$ almost always implies that $\mathcal{D}$ is false if A and D are measured.

In the limit as $\theta \rightarrow 0$, the state becomes a product state $|-1\ \ -1\rangle$, and the probability that "if $\mathcal{A}$ then $\mathcal{D}$" is never true. Of course, in that limit, the condition "if $\mathcal{A}$" also never is satisfied. No measurement ever gives $\mathcal{A}$. However, for small θ, $\mathcal{A}$ is rarely satisfied and $\mathcal{D}$ is almost always false if A is measured.

It thus becomes clear that, despite the implication chain which would lead one to believe that $\mathcal{D}$ is true if $\mathcal{A}$ is true, one cannot ascribe a value to D if it is not measured. Quantum mechanics is not a realistic theory in the EPR sense. It is realism, not locality which fails.

There is of course one way in which one could generate classical non-local theory which mimics quantum mechanics. Regard the wave function as the hidden variable. The mechanics of the universe then takes the wave-functions, carries out exactly the classical calculations which we do to evaluate the probabilities of any outcome, and then runs a random number generator to decide on the outcome of the particular experiment carried out by the physicist. This would certainly be a non-local theory, primarily because in this case the locality loses all meaning. The theory is defined only on configuration space (the space of all functions on spacetime in the case of a field theory), not on spacetime. I at least do not regard this as a reasonable interpretation. It is, however hard to see how it could be disproven.[8]

If I discount that theory, then my conclusion is that Bell's theorem, rather than implying the loss of locality, rather implies that the treatment of the world as a realistic system, in the sense enunciated by Einstein, is what is wrong, rather than that quantum mechanics is non-local.

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[8] See Navid Siami, "An investigation of No-Go theorems in Hidden variable models of Quantum Mechanics" Master's Thesis, University of British Columbia, Mar 2016 who looks at the implications of such a model (amongst others).