

# Simplification of eqn (12)

$$e^{i2\pi kx/2^N}$$

N

$$\boxed{x_{N-1} | x_{N-2} | \dots | x_1 | x_0}$$

$x_{N-1}$

binary Repn of number x

$$\boxed{k_0 | k_1 | \dots | k_{N-2} | k_{N-1}}$$

reverse binary of k

$$e^{i2\pi \frac{kx}{2^N}} = \sum_{m=0}^{N-1} k_m 2^m \sum_{n=0}^{N-1} x_{N-m-1} 2^{N-m-1} 2^{-N}$$

$$= 2\pi \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} k_n x_{N-m-1} 2^{n-m-1}$$

If  $n-m-1 \geq 0$  phase is  $2\pi 2^k$

$$2^{-2^k} \quad m < n$$

$$= 2\pi \sum_{n=0}^{N-1} \sum_{m=n}^{N-1} (k_m x_{N-1-m}) 2^{n-m-1}$$

$$= \sum_n \left[ \pi k_n x_{N-1-n} + \sum_{m>n} k_m x_{N-1-m} 2^{n-m} \right]$$

$$e^{i\pi k_n x_{N-1-m}} = \begin{cases} -1 & \text{if } k, x = 1 \\ 1 & \text{otherwise.} \end{cases}$$

$$(\sigma_x + \sigma_z)$$

converts  $|x\rangle$  to

$$|1\rangle + |0\rangle \quad \text{if } |x\rangle = 0$$

$$|0\rangle - |1\rangle \quad \text{if } |x\rangle = 1.$$

eqn (12)

Each of the terms convert  
to  $e^{i \frac{\pi}{2^r}}$  if  $n-m = -r$

Control 0.

Start at  $n=0$  Do  $\sigma_x - \sigma_x$  on  
1st bit. Do control phase  
on 1st bit and each successive

$|00\rangle \rightarrow |100\rangle$   
 $|01\rangle \quad |101\rangle$   
 $|10\rangle \quad |110\rangle$   
 $|11\rangle \quad e^{i \frac{\pi}{2^{m-n}}} |111\rangle$

for each odd bit.

Go to next bit. Do  $\sigma_x - \sigma_x$  on it  
then step through rest + do  
control phase.

Note phases get really small

$\frac{\pi}{2^r}$ . Can neglect say  
 $r > \#$