Entanglement in Unruh, Hawking and Cherenkov radiation from a Quantum Optical Perspective

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Free quantum field theory in flat space-time is often believed to be well established holding no surprises. We hope, by the end of this paper to have demonstrated that surprises still exist. For example, we will show that a uniformly accelerated atom in Minkowski space-time emits entangled photon pairs in a squeezed state which mimics entanglement of the Minkowski vacuum. Similar emission of photon pairs occurs if an atom is held above the black hole event horizon. Namely, a ground-state atom becomes excited by emitting a "negative" energy photon under the horizon and then spontaneously decays back to the ground state by emitting a positive energy photon outside the horizon, which propagates away from the black hole.

I. INTRODUCTION

The major results reported in this paper are given in Fig. 1. In Fig. 1a we depict a ground-state atom moving in the right Rindler wedge with an acceleration a which becomes excited by emitting a photon into the superposition of a right-propagating Unruh-Minkowski (UM) mode $F_{2\Omega}$, where $\Omega = \omega c/a$ and ω is the atomic transition frequency in the atom's frame, and a left-propagating mode $G_{2\Omega}$. The mode $F_{2\Omega}$ is mostly localized above the $t = z/c$ Rindler horizon, and $G_{2\Omega}$ is mostly localized below the $t = -\frac{z}{c}$ horizon. The excited atom then decays to the ground state by emitting a sum of a right-propagating photon into the Unruh-Minkowski mode $F_{1\Omega}$ which is mostly localized below the $t = z/c$ horizon, and a left propagating mode $G_{1\Omega}$ mostly localized above the $t = -z/c$ horizon. The mode emitted via excitation of the atom, is thus mostly located in regions of the spacetime away from the atom, while the mode emitted during the de-excitation of the atom $F_{1\Omega} + G_{1\Omega}$ is orthogonal to the label 1 mode, and is largest in the region around the atom.

In the following we will concentrate on the calculation of F, the right-propagating modes, but the calculations for the G modes is essentially identical. One could also make the detector sensitive to only the right going modes by coupling the detector to $\frac{1}{2}(\frac{\partial}{\partial_\tau} - \frac{\partial}{\partial_\rho})\phi(t,x)$. instead of to $\frac{\partial\phi}{\partial_\tau}$ where τ is the proper time along the path of the atom, and ρ is the orthogonal spatial coordinate at the atom.

In Fig. 1b we depict a ground-state atom held fixed above the event horizon of a static black hole (BH) in Schwarzschild coordinates t, r. It is uniformly accelerated in the Kruskal–Szekeres coordinates (T, X) through the Hartle-Hawking vacuum (which could also be called the Kruscal, in analogy with the Minkowski vacuum). The atom becomes excited by emitting a right-propagating photon into the Unruh-Schwarzschild (US) mode $F_{2\Omega} + G_{2\Omega}$ which is mostly localized above the $T = X$ line or under the $T = -X$ line, that is behind the BH horizons. The excited atom then decays to the ground state by emitting a right-propagating photon into the US mode $F_{1\Omega} + G_{1\Omega}$ which is mostly localized outside the BH horizon. The right propagating modes (F modes) propagate away from the black hole to infinity while the others propagate into the black hole horizon.

In both the Unruh and Hawking radiation the combination of emission beyond the horizon associated with the $F_{2\Omega}$ mode and spontaneous emission into $F_{1\Omega}$ mode yields entangled states of the two photon pair. This two photon squeezed/entangled state takes the form

$$|\Psi\rangle = \left(1 + \beta \hat{a}^\dagger_{2\Omega}\hat{a}^\dagger_{1\Omega}\right)|0\rangle, \tag{1}$$

where β characterizes small but important correlation between the modes inside ($F_{2\Omega}$) and outside ($F_{1\Omega}$) the horizon associated with the photon creation operators $\hat{a}^\dagger_{2\Omega}$ and $\hat{a}^\dagger_{1\Omega}$ respectively.

From a quantum optical point of view, the Unruh effect can be thought of, and calculated, as a result of virtual photons made real due to accelerating an atom. The probability of finding the atom excited (and a plane-wave photon emitted) is found to be [1]

$$P = \frac{2\pi c g^2}{a\omega}\frac{1}{\exp\left(\frac{2\pi\omega c}{a}\right) - 1}, \tag{2}$$

where g is the atom-field coupling constant and a is the atom's acceleration.

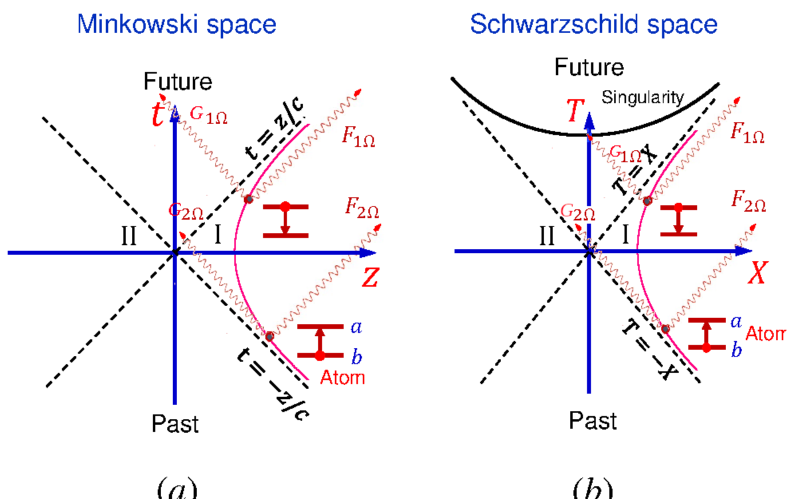

FIG. 1: (a) A ground-state atom accelerated in the wedge I goes to the excited state $|a\rangle$ while emitting a photon into the Unruh-Minkowski (UM) mode $F_{2\Omega}$ which is mostly located in the Future wedge and wedge II by the ratio of the Boltzmann factor for temperature $a/(2\pi c)$. Subsequently the atom spontaneously decays to the ground state $|b\rangle$ emitting a photon into the UM mode $F_{1\Omega} + G_{1\Omega}$ which is mostly located in the same wedge as the atom. (b) A ground-state atom held fixed above the horizon of a Schwarzschild black hole goes to the excited state while emitting a photon into Unruh-Schwarzschild (US) mode $F_{2\Omega}$ which exists mostly in the "Future" wedge below the event horizon. Subsequently the atom spontaneously decays to the ground state emitting a photon into the US mode $F_{1\Omega}$ which is located mostly above the horizon.

But there is more to the story. When a ground-state atom becomes excited by emitting an acceleration radiation photon into, say, the left Rindler wedge, the atom can go back to the ground state by spontaneously emitting another photon into the right Rindler wedge. This entangled two-photon configuration can be (and is) well explained by nothing much more than operator algebra of the type discussed above. One may then well ask if there is a quantum optical explanation for the entangled photon pairs just as we have for the acceleration radiation. The answer is yes and is the subject of the present paper.

To put the present paper in perspective, we recall that the Unruh effect, that accelerated atoms see the vacuum as a thermal state, can be realized without introducing atoms at all. That is using the relation between Minkowski ($\hat{a}_\nu$) and Rindler ($\hat{b}_\Omega$) [2] photon operators as

$$\hat{b}_\Omega = \int_0^\infty d\nu \left[\alpha_{\Omega\nu}\hat{a}_\nu - \beta_{\Omega\nu}\hat{a}_\nu^\dagger\right], \tag{3}$$

where the Bogoliubov coefficients are [2, 4]

$$\beta_{\Omega\nu} = -\frac{c}{2\pi a}\sqrt{\frac{\Omega}{\nu}}e^{-\frac{\pi c\Omega}{2a}}\left(\frac{c\nu}{a}\right)^{i\frac{c\Omega}{a}}\Gamma\left(-i\frac{c\Omega}{a}\right), \tag{4}$$

and $\Gamma(x)$ is the Gamma function. The expectation value of the Rindler photon number operator for the Minkowski vacuum is then found to be

$$\langle 0_M|\,\hat{b}_\Omega^\dagger\hat{b}_\Omega\,|0_M\rangle = \int_0^\infty d\nu|\beta_{\Omega\nu}|^2 = \frac{1}{e^{2\pi c\Omega/a} - 1}. \tag{5}$$

Indeed, the entanglement of photons [5] between right and left Rindler wedges is a hallmark of acceleration radiation having much in common with quantum optics. The Unruh effect can be understood as a process of super-oscillations[16], in which a function, composed purely of modes with a limited range of frequencies can have oscillations outside that range in certain regions. In this case, the limitation is that the frequency is limited to purely negative frequencies (for the modes associated with the creation operator of the fields). On excitation of the atom, the accelerated atom probes that region, and the positive frequencies associated with the creation operators cause an excitation of the field, a particle emission by the atom. The accelerated atom makes a transition from the ground state to an excited state while emitting a photon into a Minkowski vacuum [6]. From another point of view, Unruh radiation can be viewed as (Rindler) photons existing as a thermal bath in a uniformly accelerated reference frame [2, 7]. The Rindler state vacuum can be obtained by operating on the Minkowski vacuum with UM creation operators [5]. From a quantum optics vantage, this amounts to applying a squeeze operator to the Rindler vacuum generating bi-photon pairs of creation operators for photons corresponding to positively and negatively accelerated frames.

This is simply a feature of the Minkowski vacuum, when viewed from the modes naturally associated with the accelerated frame. There is however another effect, intimately involving the atom.

In quantum optics correlated photon pairs can come from, for example, two photon down conversion generating signal photons of frequency ν_1 and idler photon of frequency ν_2. That is, the photon pair operation is governed by the bi-photon operator $\hat{a}_1^\dagger \hat{a}_2^\dagger$, where $\hat{a}_1^\dagger$ ($\hat{a}_2^\dagger$) is the creation operator of the idler (signal) photon.

In the present paper we develop the theory of bi photons generated by atoms emitting acceleration radiation (described by operator $\hat{a}_2^\dagger$) and being excited in the process, and thereafter spontaneously emitting a photon (described by operator $\hat{a}_1^\dagger$). This creates an entangled pair of photons, ordinary Minkowski photons. We apply the same process to generate entangled pairs in Hawking [9] and Cherenkov [10] radiation.

The interplay between aspects of general relativity and quantum optics yields insights and flags open questions. See, e.g., recent work on acceleration radiation from an atom falling into a black hole [11], which is analogous to excitation of a fix atom by a uniformly accelerated mirror in Minkowski space-time [12]. More recently [13] we have emphasized that:

> "Emission of photons by atoms can occur into modes which extend into a region causally disconnected with the emitter. For example, a uniformly accelerated ground-state atom emits a photon into the Unruh–Minkowski mode which is exponentially larger in the causally disconnected region. This makes an impression that photon emission is acausal. Here we show that conventional quantum optical analysis yields that a detector atom will not detect the emitted photon in the region non-causally connected with the emitter. However, joint excitation probability of atoms in the causally disconnected regions can be correlated due to entanglement of Minkowski vacuum and be much large than the product of independent excitation probabilities".

In the present note we show that the vacuum entanglement referred to above can be demonstrated via the simple toy model depicted in Fig. 1a. The message of Fig. 1a is that the atom (coming from infinity) is accelerated through wedge I. That is the atom with high velocity travels along the z axis from infinity, it slows down, stops and and then keeps accelerating back out to infinity. The atom emits an acceleration radiation photon which is predominantly in the wedge II and subsequently spontaneously emits a photon into located dominantly in wedge I. As is shown below these photons are entangled and are in a two mode squeezed state.

In quantum optics [14] the two mode squeezed state is generated by the action of the unitary squeeze operator

$$\hat{S} = e^{\xi^* \hat{a}_1 \hat{a}_2 + \xi \hat{a}_1^\dagger \hat{a}_2^\dagger}$$

on the two photon vacuum state $|0_1 0_2\rangle$, where $\hat{a}_1(\hat{a}_2)$ and $\hat{a}_1^\dagger(\hat{a}_2^\dagger)$ are the usual annihilation and creation operators. In particular, for weak squeezing, such that $\xi \ll 1$, we have

$$|\Psi\rangle = [1 + \xi \hat{a}_1^\dagger \hat{a}_2^\dagger]|0_1 0_2\rangle.$$

The two-mode squeezed state can be generated in a nonlinear crystal by a parametric down conversion process in an optical cavity [15], in which a strong (classical) laser field produces a pair of photons 1 and 2 as described by the Hamiltonian

$$\hat{H} = \hbar g \left(\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_1 \hat{a}_2 \right),$$

where the coupling strength g is the effective Rabi frequency of the driving field. Schrodinger equation yields that the state of the generated field is

$$|\Psi(t)\rangle = e^{-ig(\hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger)t}|0_1 0_2\rangle,$$

which is a two-mode squeezed state. To lowest order, the down conversion process produces the "squeezed" state

$$|\Psi\rangle \cong (1 - ig\tau \hat{a}_1^\dagger \hat{a}_2^\dagger)|0_1 0_2\rangle,$$

where τ is the duration of the short pulse driving field in this simple model. In the squeezed state, the photon numbers in the modes are correlated. Namely, the photon numbers in each mode n_1 and n_2 fluctuate obeying thermal distribution, but the difference $n_1 - n_2$ does not fluctuate. That is, if there are n photons in mode 1 then with unit probability there are n photons in the mode 2.

It is interesting that Minkowski vacuum $|0_M\rangle$ is a squeezed state in terms of Rindler photons, namely, in terms of Rindler states [5]

$$|0_M\rangle = \prod_{\nu>0} \left(1 - e^{-2\pi c\nu/a}\right) e^{\exp(-\frac{\pi c\nu}{a})\left(\hat{b}^\dagger_{R1\nu}\hat{b}^\dagger_{R2\nu} + \hat{b}^\dagger_{L1\nu}\hat{b}^\dagger_{L2\nu}\right)} |0_R\rangle \,, \tag{6}$$

where $|0_R\rangle$ refers to Rindler vacuum, $\hat{b}^\dagger_{R1\nu}$ and $\hat{b}^\dagger_{R2\nu}$ ($\hat{b}^\dagger_{L1\nu}$ and $\hat{b}^\dagger_{L2\nu}$) are creation operators of Rindler photons in the right (left) propagating Rindler modes

$$\phi_{1\nu} = \sqrt{\frac{a}{\nu c}} (\mp z - ct)^{i\frac{\nu c}{a}} \theta(\mp z - ct) \tag{7}$$

and

$$\phi_{2\nu} = \sqrt{\frac{a}{\nu c}} (ct \pm z)^{-i\frac{\nu c}{a}} \theta(ct \pm z). \tag{8}$$

Here $a > 0$ is a parameter which has dimension of acceleration.

Rindler modes $\phi_{1\nu}$ and $\phi_{2\nu}$ are solutions of the wave equation and for $\nu > 0$ have positive norm (defined as the Klein–Gordon inner product). The mode functions (7) and (8) are non-zero in half of the $t - z$ plane and form a complete basis set.

In Section II we present the detailed analysis of the acceleration radiation induced entangling of UM modes $F_{1\Omega}+G_{1\Omega}$ and $F_{2\Omega} + G_{2,\Omega}$ and is essentially a two-mode squeezed state. In Section III we present the analysis of the Hawking radiation which consists of entangled photon pairs localized above and below the BH event horizon. In Section IV we show how "squeezed/entangled" radiation is related to Cherenkov radiation. Section V is a discussion and conclusion.

II. GENERATION OF SQUEEZED PHOTON STATES BY ACCELERATED ATOMS

Next we consider an electrically neutral two-level atom with a transition angular frequency ω which moves along the trajectory $t(\tau)$, $z(\tau)$ in vacuum, where τ is the proper time of the atom. The atom is coupled to the electromagnetic field. For simplicity we approximate the field as a scalar field described by the operator $\hat{\Phi}(t, z)$ and consider dimension $1 + 1$. We will assume the following form of the interaction Hamiltonian between the atom and the scalar field

$$\hat{V}(\tau) = g\left(\hat{\sigma} e^{-i\omega\tau} + \hat{\sigma}^\dagger e^{i\omega\tau}\right) \frac{\partial}{\partial\tau} \hat{\Phi}(t(\tau), z(\tau)), \tag{9}$$

where g is the atom-field coupling constant, and $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ are the atomic lowering and raising operators. Since the atom feels the local value of the field, the operator $\hat{\Phi}$ is evaluated at the atom's position $t(\tau)$, $z(\tau)$. For simplicity we consider only the right-propagating waves. The results can be generalized to include the left-propagating waves in a straightforward way.

In the usual quantization procedure, one splits the solutions to the field equations into two sets of Fourier modes. To make the system as simple as possible, we will examine a massless scalar field ϕ in 1+1 dimensions, although the results, while more complex for massive fields - scalar or vector, apply in higher dimensions as well. The field equation of motion is taken as

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\phi(t, z) = 0. \tag{10}$$

The solutions of Eq. (10) can be written as sums of the Fourier transform modes

$$\phi_k(t, x) = e^{-i(\nu t - kz)} \tag{11}$$

with $\nu^2 = k^2$ or $\nu = \pm k$.

In the usual quantization procedure, one associates the modes with $\nu > 0$ with the annihilation operators for the field $\hat{a}_k$ and $\nu < 0$ with the creation operators $\hat{a}^\dagger_k$, which obey the commutation relations $[\hat{a}_k, \hat{a}^\dagger_{k'}] = \delta_{kk'}$. In order that the field strength ϕ and its conjugate momentum $\pi = c\partial L/\partial\dot{\phi} = (1/c)\partial\phi^*/\partial t$ obey the standard commutation relations, we must have

$$\frac{i}{2c}\int_{-\infty}^{\infty}\left(\phi_k^*\frac{\partial\phi_{k'}}{\partial t} - \frac{\partial\phi_k^*}{\partial t}\phi_{k'}\right)dz = \delta_{kk'}. \tag{12}$$

For our purposes, however, we will choose a different set of modes, often called the Unruh-Minkowski (UM) modes. They were discovered by Unruh in 1976 [2]. The right-propagating UM modes are defined as [5]

$$F_{1\Omega}(t,z) = \frac{|t - z/c|^{i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-\frac{\pi\Omega}{2}\mathrm{sign}(t-z/c)}, \tag{13}$$

$$F_{2\Omega}(t,z) = \frac{|t - z/c|^{-i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{\frac{\pi\Omega}{2}\mathrm{sign}(t-z/c)}, \tag{14}$$

where $\Omega > 0$ is a parameter which is proportional to the photon frequency in the Rindler space [3]. In wedge I, $z > ct$ and the potentially large factor $e^{\pi\Omega/2}$ appears in Eq. (13), which means that the mode $F_{1\Omega}$ is mostly in wedge I (see Fig. 1a) and has positive frequency in wedge II even though it is associated with a creation operator (negative norm). On the other hand, the mode $F_{2\Omega}$ is exponentially larger when $ct > z$, that is in wedge II. The left-moving modes are obtained from Eqs. (13) and (14) by replacing $z \to -z$.

The functions (13) and (14) are the limit for positive $\epsilon \to 0$ of the expression

$$F_{1\Omega}(t,z) = \frac{(t - \frac{z}{c} - i\epsilon)^{i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-\frac{\pi\Omega}{2}},$$

$$F_{2\Omega}(t,z) = \frac{(t - \frac{z}{c} - i\epsilon)^{-i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{\frac{\pi\Omega}{2}}.$$

These functions are analytic and bounded in the upper half complex plane of the complex field argument $t - z/c$ for all values of $\Omega > 0$, just as the functions $e^{-i\nu(t-z/c)}$ are for $\omega > 0$. The factors $1/\sqrt{2\Omega\sinh(\pi\Omega)}$ are the normalization factors under the Klein–Gordon norm.

Expansion of the right-moving part of the field operator in terms of the UM modes (13) and (14) reads [5]

$$\hat{\Phi} = \sum_{\Omega>0}\left(F_{1\Omega}\hat{a}_{1\Omega} + F_{2\Omega}\hat{a}_{2\Omega} + F_{1\Omega}^*\hat{a}_{1\Omega}^\dagger + F_{2\Omega}^*\hat{a}_{2\Omega}^\dagger\right), \tag{15}$$

where $\hat{a}_{1\Omega}$ and $\hat{a}_{2\Omega}$ are the UM photon annihilation operators. $\hat{a}_{1\Omega}$ and $\hat{a}_{2\Omega}$ can be written as sums of the plane-wave annihilation operators $\hat{a}_k$ introduced above. Thus, the vacuum state for the UM photons is the usual Minkowski vacuum $|0_M\rangle$.

We consider a uniformly accelerated atom moving along the trajectory

$$t(\tau) = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right), \quad z(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) \tag{16}$$

in Minkowski space-time. In Eq. (16), τ is the proper time of the atom. If $a > 0$ ($a < 0$), the atom moves in the right (left) Rindler wedge (see Fig. 1a).

The UM modes (13) and (14) are a convenient choice for the description of Unruh acceleration radiation. Namely, a ground-state atom with a transition frequency ω moving in the right Rindler wedge with an acceleration a (see Fig. 1a) in Minkowski vacuum $|0_M\rangle$ can become excited by emitting a right-propagating photon into the single UM mode $F_{2\Omega}$, where $\Omega = \omega c/a$ [5, 17]. The excited atom can then decay to the ground state by emitting a right-propagating photon into the UM mode $F_{1\Omega}$. As a result of these processes, the final state of the field will have two photons in UM modes $F_{1\Omega}$ and $F_{2\Omega}$ which are entangled.

To the lowest order, we find for the final state of the field is

$$|\Psi\rangle = |0_M\rangle - \left(\frac{g}{\hbar}\right)^2\int_{-\infty}^{\infty}d\tau' e^{-i\omega\tau'}\frac{e^{i\alpha\Omega\tau'}e^{\frac{\pi\Omega}{2}}}{\sqrt{2\Omega\sinh(\pi\Omega)}}\int_{-\infty}^{\tau'}d\tau'' e^{i\omega\tau''}\frac{e^{-i\alpha\Omega\tau''}e^{-\frac{\pi\Omega}{2}}}{\sqrt{2\Omega\sinh(\pi\Omega)}}|1_{2\Omega}1_{1\Omega}\rangle, \tag{17}$$

where $\alpha = a/c$. One can write Eq. (17) as

$$|\Psi\rangle = \left(1 + \frac{G}{2\Omega\sinh(\pi\Omega)}\hat{a}^{\dagger}_{2\Omega}\hat{a}^{\dagger}_{1\Omega}\right)|0_M\rangle\,, \tag{18}$$

where G (not to be confused with the modes $G_{\{1,2\}\Omega}$) is a factor containing the time integral

$$G = -\left(\frac{g}{\hbar}\right)^2 \int_{-\infty}^{\infty} d\tau' e^{-i\omega\tau'} e^{i\alpha\Omega\tau'} \int_{-\infty}^{\tau'} d\tau'' e^{i\omega\tau''} e^{-i\alpha\Omega\tau''}. \tag{19}$$

The factor G is large if the resonant condition $\Omega = \omega c/a$ is satisfied.

III. GENERATION OF SQUEEZED PHOTON STATES BY ATOMS HELD ABOVE BLACK HOLE HORIZON

The Schwarzschild metric of a non-rotating black hole in 1+1 dimension is given by

$$ds^2 = \left(1 - \frac{r_g}{r}\right)c^2dt^2 - \frac{1}{1-\frac{r_g}{r}}dr^2, \tag{20}$$

where $r_g = 2GM/c^2$ is the gravitational radius, and t, r are Schwarzschild coordinates.

Kruskal-Szekeres coordinates on a black hole geometry are defined in terms of the Schwarzschild coordinates t, r as

$$T = \sqrt{\frac{r}{r_g} - 1}e^{\frac{r}{2r_g}}\sinh\left(\frac{ct}{2r_g}\right), \tag{21}$$

$$X = \sqrt{\frac{r}{r_g} - 1}e^{\frac{r}{2r_g}}\cosh\left(\frac{ct}{2r_g}\right), \tag{22}$$

for $r > r_g$, and

$$T = \sqrt{1 - \frac{r}{r_g}}e^{\frac{r}{2r_g}}\cosh\left(\frac{ct}{2r_g}\right), \tag{23}$$

$$X = \sqrt{1 - \frac{r}{r_g}}e^{\frac{r}{2r_g}}\sinh\left(\frac{ct}{2r_g}\right), \tag{24}$$

for $0 < r < r_g$.

In 1+1 dimension, in the Kruskal-Szekeres coordinates, the Schwarzschild metric is conformally invariant to the Minkowski metric

$$ds^2 = \frac{4r_g^3}{r}e^{-\frac{r}{r_g}}\left(dT^2 - dX^2\right). \tag{25}$$

where r is a function of $T^2 - X^2$ which has value of r_g when $T^2 - X^2 = 0$. Thus, one can choose mode functions in the Kruskal-Szekeres coordinates as functions of $T \pm X$.

Again, although the atom couples both to the right and left moving modes ($F_{\{1,2\}\Omega}$ and $G_{\{1,2\}\Omega}$) we again concentrate on the right movers.

Here we consider a two-level (a (not to be confused with the acceleration) is the excited level and b is the ground state) atom with transition angular frequency ω near eternal non-rotating BH of mass M. If the atom does not move in the Schwarzschild coordinates, that is atom's trajectory is $r(t) = r_0 > r_g$, then in the Kruskal-Szekeres coordinates the atom is uniformly accelerated along the trajectory

$$T(t) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c}\sinh(\alpha t)\,, \tag{26}$$

$$X(t) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c}\cosh(\alpha t)\,, \tag{27}$$

where $\alpha = c/2r_g$. One can write these equations as

$$T(\tau) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c}\sinh\left(\frac{\alpha\tau}{\sqrt{1 - r_g/r_0}}\right), \tag{28}$$

$$X(\tau) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c}\cosh\left(\frac{\alpha\tau}{\sqrt{1 - r_g/r_0}}\right), \tag{29}$$

where $\tau = t\sqrt{1 - r_g/r_0}$ is the proper time of the atom.

As in the case of the Rindler horizon in the flat Minkowski space-time, the black hole event horizon $T = X$ divides the Schwarzschild space-time for right-running waves into two regions. The outgoing positive frequency modes which are analogous to the right-running UM modes of the previous section on acceleration radiation, are given by [2]

$$F_{1\Omega}(T, X) = \frac{|T - X|^{i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-\frac{\pi\Omega}{2}\mathrm{sign}(T-X)}, \tag{30}$$

$$F_{2\Omega}(T, X) = \frac{|T - X|^{-i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{\frac{\pi\Omega}{2}\mathrm{sign}(T-X)}, \tag{31}$$

where $\Omega > 0$. The corresponding photon annihilation operators we denote as $\hat{A}_{1\Omega}$, $\hat{A}_{2\Omega}$. We assume that state of the field is a vacuum state with respect to the Unruh-Schwarzschild (US) modes (30) and (31). We denote it as $|0_K\rangle$. For simplicity we consider only the outgoing modes.

Along the atom's trajectory (28), (29), the US mode functions (30) and (31) are

$$F_{1\Omega}(T(\tau), X(\tau)) = (2\alpha r_0/c - 1)^{\frac{i\Omega}{2}}\frac{e^{\frac{\pi\Omega}{2}}e^{i\alpha r_0\Omega/c}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-\frac{i\alpha\Omega\tau}{\sqrt{1-r_g/r_0}}}, \tag{32}$$

$$F_{2\Omega}(T(\tau), X(\tau)) = (2\alpha r_0/c - 1)^{-\frac{i\Omega}{2}}\frac{e^{-\frac{\pi\Omega}{2}}e^{-i\alpha r_0\Omega/c}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{\frac{i\alpha\Omega\tau}{\sqrt{1-r_g/r_0}}}, \tag{33}$$

that is from the atom's perspective, the photon described by the mode function $F_{1\Omega}$ has positive frequency, while the photon $F_{2\Omega}$ has negative frequency. The photon is resonant with the atom for $\Omega = \sqrt{1 - r_g/r_0}\omega/\alpha$, where ω is the atomic transition frequency. As a result, the ground-state atom can become excited by emitting a photon into the mode $F_{2\Omega}$. Subsequently the atom can go to the ground state by spontaneously emitting a photon into the mode $F_{1\Omega}$ (see Fig. 1b). As in the case of an atom accelerated in Minkowski space-time, these two processes generate a two-photon entangled state

$$|\Psi\rangle = \left(1 + \frac{G}{2\Omega\sinh(\pi\Omega)}\hat{A}^\dagger_{2\Omega}\hat{A}^\dagger_{1\Omega}\right)|0_K\rangle, \tag{34}$$

where G is a factor containing the time integral

$$G = -\left(\frac{g}{\hbar}\right)^2\int_{-\infty}^{\infty}d\tau' e^{-i\omega\tau'}e^{\frac{i\alpha\Omega\tau'}{\sqrt{1-r_g/r_0}}}\int_{-\infty}^{\tau'}d\tau'' e^{i\omega\tau''}e^{-\frac{i\alpha\Omega\tau''}{\sqrt{1-r_g/r_0}}}. \tag{35}$$

According to Eq. (31), the mode function $F_{2\Omega}$ is exponentially ($e^{\pi a\omega}$) larger in the region $T > X$, and, hence, the probability is high that this photon falls into the black hole singularity rather than propagating to infinity. In contrast, the mode function $F_{1\Omega}$ of the spontaneously emitted photon is exponentially larger by the same factor in the region $X > T$, and, therefore, the spontaneously emitted photon escapes probably escapes from the black hole region to infinity (in thee dimensions, if the frequencyω is less than $\frac{a(L+1)}{2\pi}$ where L is the angular momentum of the photon, the outward directed photon will, with very high probability, reflect back into the black hole, bouncing off the momentum/curvature barrier for photons propagating out from the black hole horizon at $r = r_g$. It is only those photons with a high enough energy to get over those barriers which get out to infinity. It is also important to note that far away from the black hole, the "natural" modes are the equivalent of the Rindler modes, with time dependence $e^{-i\omega t}$ not τ which represent what most would call the natural particle modes. The natural vacuum for the Hartle-Hawking (Kruscal) modes is, for these Schwartzschild modes will be a thermal state, both incoming and outgoing, with temperature $1/8\pi M(\hbar/k_Bc^2)$, the Hawking temperature.

IV. GENERATION OF SQUEEZED PHOTON STATES THROUGH THE CHERENKOV EFFECT

Entangled photon pairs can be generated by a similar mechanism if the ground-state atom is moving through a medium with a constant velocity V greater than speed of light in the medium. For description of Cherenkov radiation it is convenient to choose mode functions as plane waves which in the medium with refractive index n read (in the lab frame)

$$\varphi_k(t,z) = e^{-i\frac{c|k|}{n}t+ikz}, \tag{36}$$

where k is the photon wave vector. Here we consider dimension $1+1$, and k can be both positive and negative. We denote operators of the plane-wave photons (36) as $\hat{c}_k$. At the location of the atom moving with a constant velocity $V>0$ along the trajectory

$$t(\tau) = \frac{\tau}{\sqrt{1-\frac{V^2}{c^2}}}, \quad z(\tau) = \frac{V\tau}{\sqrt{1-\frac{V^2}{c^2}}},$$

the mode function φ_k takes the form $\varphi_k(t(\tau),z(\tau)) = e^{-i\nu\tau}$, that is the atom senses harmonic oscillations of the field with frequency

$$\nu = \frac{\frac{c|k|}{n} - Vk}{\sqrt{1-\frac{V^2}{c^2}}}.$$

From the perspective of the moving atom if $V > c/n$, photons propagating in the same direction as atom's velocity ($k>0$) have negative frequency. This leads to Cherenkov radiation. That is, the atom can become excited by emitting a photon in the forward direction with a wave-vector k_2 such that

$$k_2 = \frac{\omega\sqrt{1-\frac{V^2}{c^2}}}{V-\frac{c}{n}}, \tag{37}$$

where ω is the atomic transition frequency in atom's frame. Negative frequency of the Cherenkov photon in the atom's frame is analogous to the negative frequency of the UM photon in the frame of an accelerated atom.

Photons emitted in the backward direction ($k<0$) have positive frequency. Thus, an excited atom can spontaneously decay to the ground state by emitting a photon in the backward direction with a wave-vector k_1 such that

$$k_1 = -\frac{\omega\sqrt{1-\frac{V^2}{c^2}}}{V+\frac{c}{n}}. \tag{38}$$

As in the case of an atom accelerated in Minkowski space-time, these processes generate a two-photon entangled state

$$|\Psi\rangle = \left(1 + G c^\dagger_{k_2} c^\dagger_{k_1}\right)|0_M\rangle, \tag{39}$$

where G is a factor containing the time integral

$$G = -\left(\frac{g}{\hbar}\right)^2 \int_{-\infty}^{\infty} d\tau' e^{-i\omega\tau'} e^{\frac{\frac{c|k_1|}{n}-Vk_1}{\sqrt{1-\frac{V^2}{c^2}}}\tau'} \int_{-\infty}^{\tau'} d\tau'' e^{i\omega\tau''} e^{\frac{\frac{c|k_2|}{n}-Vk_2}{\sqrt{1-\frac{V^2}{c^2}}}\tau''}. \tag{40}$$

G is large if the resonance conditions (37) and (38) are satisfied. The Cherenkov effect yields entanglement generation between photons propagating in the forward and backward directions.

V. DISCUSSION AND SUMMARY

To put section II in perspective, we recall that Bogoliubov relations allow us to obtain an expression for the Minkowski vacuum $|0_M\rangle$ in terms of excitation states of the Rindler vacuum $|0_R\rangle$. That is we use the following relations between operators for the Rindler modes $\hat{b}_\nu$ and the UM modes $\hat{a}_\nu$ [13]

$$\hat{b}_{1\nu} = \frac{\hat{a}_{1\nu} + e^{-\pi c\nu/a}\hat{a}^\dagger_{2\nu}}{\sqrt{1-e^{-2\pi c\nu/a}}}, \quad \hat{b}_{2\nu} = \frac{\hat{a}_{2\nu} + e^{-\pi c\nu/a}\hat{a}^\dagger_{1\nu}}{\sqrt{1-e^{-2\pi c\nu/a}}}, \tag{41}$$

and the the identities [13]

$$\hat{a}_1 e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger} = e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}\left(\hat{a}_1 + \gamma \hat{a}_2^\dagger\right), \tag{42}$$

$$\hat{a}_2 e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger} = e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}\left(\hat{a}_2 + \gamma \hat{a}_1^\dagger\right), \tag{43}$$

where $\gamma = e^{-\pi c\nu/a}$. Multiplying both sides of Eq. (42) by $|0_M\rangle$, we obtain

$$\hat{a}_1 e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}|0_R\rangle = e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}\left(\hat{a}_1 + \gamma \hat{a}_2^\dagger\right)|0_R\rangle \propto e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}\hat{b}_1|0_R\rangle = 0, \tag{44}$$

That is if $|0_R\rangle$ is the vacuum state for the Rindler photon operator $\hat{b}_1$, then

$$|0_M\rangle = \frac{1}{N}e^{\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}|0_R\rangle \tag{45}$$

is the vacuum state for the UM operator $\hat{a}_1$, where N is a normalization constant. Applying the identity (43), one can show that Eq. (45) is also the vacuum state for the operator $\hat{a}_2$. Multiplying both sides of Eq. (45) by $Ne^{-\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}$, we find

$$|0_R\rangle = Ne^{-\gamma \hat{a}_1^\dagger \hat{a}_2^\dagger}|0_M\rangle. \tag{46}$$

The value of N is determined by the normalization $\langle 0_R|0_R\rangle = 1$, which yields

$$N = \frac{1}{\sqrt{\sum_{n=0}^{\infty}\gamma^{2n}}} = \sqrt{1-\gamma^2}.$$

For $\gamma \ll 1$, Eq. (46) approximately can be written as

$$|0_R\rangle \approx \left(1 - e^{-\pi c\nu/a}\hat{a}_1^\dagger \hat{a}_2^\dagger\right)|0_M\rangle, \tag{47}$$

which has the same exponential factor as Eq. (18) in the limit $\Omega \gg 1$

$$|\Psi\rangle \approx \left(1 + \frac{G}{\Omega}e^{-\pi\Omega}\hat{a}_{R2\Omega}^\dagger \hat{a}_{R1\Omega}^\dagger\right)|0_M\rangle. \tag{48}$$

Having demonstrated in section III that the *Unruh-Schwarzschild* mode expansion applied to the BH problem is very analogous to the acceleration radiation with *Unruh-Minkowski* modes, we conclude by making a key but somewhat subtle point. Namely, the negative energy photon, which is associated with the mode $F_{2\Omega}$, in both cases is localized below the “horizon”. This was already emphasized in [5] That is the $F_{2\Omega}$ mode into which the atom emits the first photon is exponentially large in the left wedge ($t > z/c$) in the case of Unruh acceleration radiation, and is likewise exponentially large below the BH horizon ($T > X$). In the case of Hawking radiation the photon does not so much “fall” into the BH, but rather is created under the event horizon because the mode function $F_{2\Omega}$ is localized in this region.

It would be interesting to look for similar effects in various analogs of the Unruh acceleration radiation [18–20] and Cherenkov radiation. For example, a ground-state atom moving above a metal surface (see Fig. 2a) can become excited by emitting a surface plasmon with wave vector in the direction of the atom's motion [21]. Surface plasmons are collective excitations of the electromagnetic field and metal electrons which propagate along the surface. Electromagnetic field in the surface plasmon exponentially decays away from the surface as shown in Fig. 2a.

From the perspective of the moving atom, the surface plasmon frequency is Doppler shifted and for large wave numbers k the frequency becomes negative (see Fig. 2b). In the moving frame associated with the atom, the ground-state atom becomes excited by emitting surface plasmon with negative frequency which insures energy conservation. The excited atom can then decay to the ground state by emitting a surface plasmon with positive frequency, e.g., with wave-vector opposite to the atom's velocity. Thus generated pair of the surface plasmons is entangled.

We note that this also forms the basis for the Analog gravity program in which the black hole is modelled by a trans-sonic fluid flow, and the quantum field by sound (or other waves) in the fluid. There also, the importance of effects like the Cherenkov discussion above, where the fluid flow is faster then than the wave velocity plays a crucial role.(see for example Schuetzhold and Unruh [22] and diagrams and references therein). This also sheds light on the Landau critical velocity where the frequency of the sound waves in liquid He goes negative like for the Cherenkov situation discussed above. An impurity which couples to the sound waves will create particle pairs (squeezing) which will introduce a quantum friction force on the impurity.

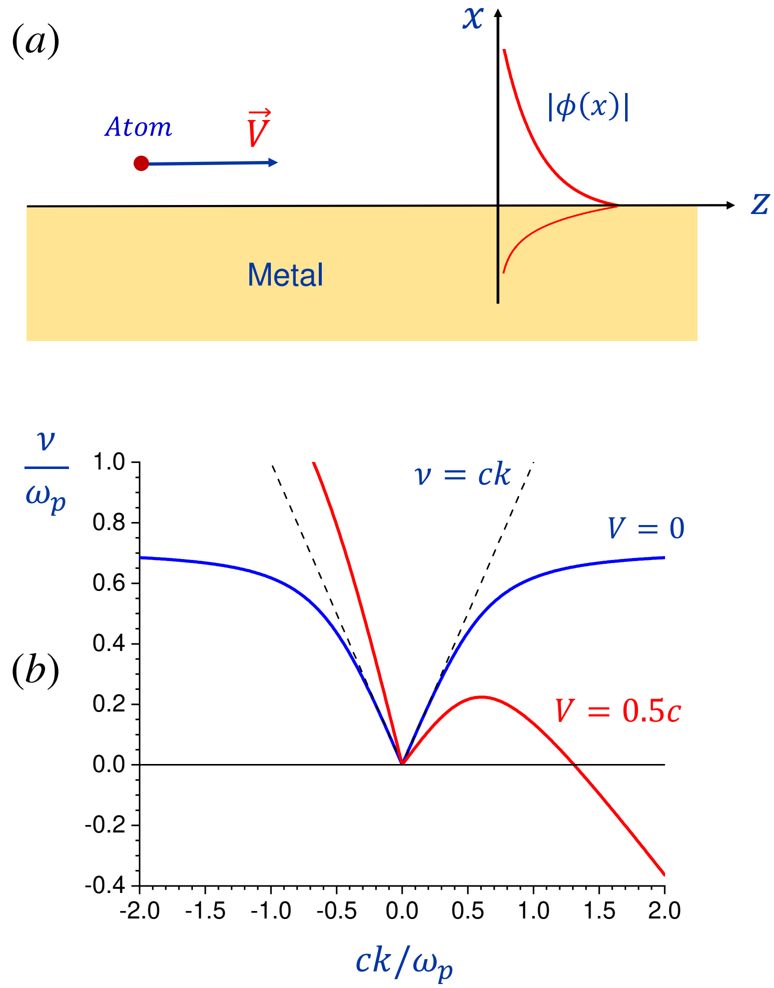

FIG. 2: (a) An atom is moving near metal surface with constant velocity. The atom can become excited by emitting a surface plasmon with negative frequency and then spontaneously decay to the ground state by emitting a surface plasmon with positive frequency. (b) Frequency of surface plasmons in the atom's frame moving with velocity V as a function of the surface plasmon wave vector $\mathbf{k}$ for $V = 0$ and $V = 0.5c$. Frequency of modes with $k > \frac{\omega_p}{V}\sqrt{\frac{c^2-V^2}{2c^2-V^2}}$ is negative, where ω_p is the electron plasma frequency.

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