W. G. Unruh
CIfAR Cosmology and Gravity Program
Dept. of Physics
University of B. C.
Vancouver, Canada V6T 1Z1
email: unruh@physics.ubc.ca

What is a Particle in QFT?

A particle is what a particle detector detects.

What is a particle detector? A particle detector is a quantum system, which, if it starts in its lowest energy state, when it interacts with a quantum field, it ends up in a higher energy state. It has absorbed on quantum of energy, and that carrier of energy is a particle.

Exampls : 2 level system with states $|\downarrow\rangle$, $ket\uparrow$ which are the eigenstates of the Hamiltonian

$$H = \frac{E}{2}\sigma_3 \tag{1}$$

This is the Heisenberg Hamiltonian for the evolution of the detector in the interal proper time along the path of the detector.

Another operator we will be using is $\sigma_1(\tau)$ which obeys the Heisenberg representation equations

$$i\partial_t\sigma_1 = [\sigma_1, H_d] \tag{2}$$

which can be written as

$$\sigma_1(t) = \sigma_- e^{-iE\tau} + \sigma_-^\dagger e^{iE\tau} \tag{3}$$

As a simple example of a quantum field, we take a free field $\Phi(t,\vec{x})$. This field is a solution to the free-field equations. (Ie, we are treating the field as being in the Heisenberg repreentation.

Coupling:

$$H_I = \mu_0 e^{-\frac{\tau^2}{2T^2}}\sigma_1(\tau)\partial_\tau\Phi(t(\tau),\vec{x}(\tau))) \tag{4}$$

where $t(\tau),\vec{x}(\tau)$ is the path of the detector/atom in the spacetime.

The usual quantization of the field is given by introducing Annihilation and Creation operators and plane wave modes.

$$\Phi(t,\vec{x}) = \int \frac{1}{\sqrt{(2\pi)^D 2\omega(\vec{k})}}\left(A_{\vec{k}}e^{-i(\omega(k)t-\vec{k}\cdot x)} + A_{\vec{k}}^\dagger e^{i(\omega(\vec{k})t-\vec{k}\cdot\vec{x})}\right) \tag{5}$$

In these notes I will take the simplest case, with D the spatial dimensions of space, to equal 1, and the spatial coordinate to be z. We will also assume that the equations of motion of the free field are those of a massless scalar field. Using a massive field or a guage field (eg electromagnetism) would simply make the algebra slightly messier, but the fundamental principles would be the same.

$$\partial_t^2\Phi(t,\vec{x}) - \nabla^2\Phi(t,\vec{x}) = 0 \tag{6}$$

One question regarding the expression for the quantized field is: " Where does the $\frac{1}{\sqrt{2\omega(\vec{k})}}$ come from?"

The answer is that one wants the the quantum field to obey the commutation relations

$$[\Phi(t,\vec{x}),\Pi(t,\vec{x}')] = i\hbar delta^D(\vec{x}-\vec{x}') \tag{7}$$

Cionsider two solutons to the classical Heisenberg equations of motion $\phi(t,\vec{x}),\pi(t,\vec{x})$ and $\tilde{\phi}(t,\vec{x}),\tilde{\pi}(t,\vec{x})$. Define a Norm of these two complex solutions

$$<\phi;\tilde{\phi}> \equiv <\phi(t,\vec{x}),\pi(t,\vec{x});\tilde{\phi}(t,\vec{x}),\tilde{\pi}(t,\vec{x})> = i\int^D \phi^*(t,\vec{x})\tilde{\pi}(t,\vec{x}) - \pi^*(t,\vec{x})\phi(t,\vec{x})d^Dx \tag{8}$$

Choosing a set of solutions ϕ_α, π_α such that

$$< \phi_\alpha, \phi_\beta >= \delta_(\alpha, \beta) \quad (9)$$
$$< \phi^*_\alpha, \phi^*_\beta >= -\delta_(\alpha, \beta) \quad (10)$$
$$< \phi_\alpha, \phi^* \beta >= 0 \quad (11)$$

We can write

$$\Phi(t, \vec{x}) = \sum_\alpha A_\alpha \phi_\alpha(t, \vec{x}) + A^\dagger_\alpha \phi^*(t, \vec{x}) \quad (12)$$
$$\Pi(t, \vec{x}) = \sum_\alpha A_\alpha \pi_\alpha(t, \vec{x}) + A^\dagger_\alpha \pi^*(t, \vec{x}) \quad (13)$$

Where the A obey $[A_\alpha, A^\dagger_\beta] =< \phi_\beta, \phi_\alpha >= \delta_(\alpha, \beta)$ and $[A_\alpha, A_\beta] =< \phi^*_\alpha, \phi_\beta >= 0$ arising directly from the norm of the solutions. Ie, these operators obey exactly the commutation relations of the annihilation and creation operators one usually uses.

The number operator is $\sum_\alpha A^\dagger_\alpha A_\alpha$ and is minimized by the state $A_\alpha|0\rangle = 0 \ \ \forall\alpha$

Note that the above also applies if α is a continuum label (lke $\vec{k}$). All that happens is that the sum becomes integrals, and the Kroneker delta becomes Dirac delta functions.

If the Hamiltonian is $\frac{1}{2}\int(\pi^2 + (\nabla\phi \cdot \nabla\phi) + m^2\phi^2)d^Dx$ one of the equations of motion is

$$\partial_t \phi = \pi \quad (14)$$

and the norm becomes

$$< \phi, \tilde{\phi} >= i \int \phi^* \partial_t \tilde{\phi} - \partial_t \phi^* \phi d^D x \quad (15)$$

which is usually called the Klein gordon Norm. Ie, the KG norm is simply a special case of the above norm for a specific form of the Hamiltonian.

If we choose modes such that, at time t

$$\partial_t \phi_\alpha = -i\omega\phi_\alpha \quad (16)$$
$$\partial_t \pi_\alpha = -i\omega\pi_\alpha \quad (17)$$

then the norm is $2\omega\phi^*_\alpha\phi_\alpha$ which is positive, and to normalize we need to divide by $\frac{1}{\sqrt{2\omega}}$ That is where that term in the usual normalisation comes from.

1+1 Dimensions

$$\partial^2_t \phi - \partial_z \phi^2 = 0 \quad (18)$$

Normalized modes

$$\phi_{R\omega} = \frac{1}{\sqrt{4\pi\omega} e^{-i\omega(t-z)}} \quad (19)$$
$$\phi_{L\omega} = \frac{1}{\sqrt{4\pi\omega} e^{-i\omega(t+z)}} \quad (20)$$

In the following I will concentrate on the $\phi_{R\omega}$ terms but the $\phi_{L\omega}$ terms follow the identical arguments.

Then

$$\Phi_L = \int_0^\infty \frac{1}{\sqrt{2\pi\omega}} (A_{R\omega} e^{-i(\omega(t-z)} + A^\dagger_{R,\omega} e^{i(\omega(t-z)} d\omega \quad (21)$$

and, with the detector located at $z = 0$

$$|\phi_1\rangle = \int (\mu(t)\sigma^\dagger_- e^{iEt} A^\dagger_{R\omega} e^{i\omega t} A^\dagger_{R\omega} d\omega dt| \downarrow\rangle|0\rangle = \int \mu(t) e^{i(Et+\omega t)} dt| \uparrow\rangle A^\dagger_{R\omega}|0\rangle \quad (22)$$

As long as $\mu(t)$ has a fourier transform which is concentrated narrowly around 0, the fourier transform $\mu(E+\omega) = \int \mu(t)e^{i(E+\omega)t}dt$ will be zero for all positive ω.

For example, if

$$\mu(t) = \mu_0 e^{-\frac{t^2}{2T^2}} \tag{23}$$

then

$$\mu(\omega + E) = \sqrt{\pi}T e^{-(\omega+E)^2 T^2/2} \tag{24}$$

which for large $T >> \frac{1}{E}$ is effectively 0 for all $\omega > 0$.

Now let us introduce some new coordinates, $t,\ \rho,\ \tilde{\rho}$, so that

$$t = \frac{1}{a}e^{a\rho}\sinh(a\tau); z = \frac{1}{a}e^{a\rho}\cosh(a\tau) \ \ ; \ \ z > |t| \tag{25}$$

$$t = \frac{1}{a}e^{-a\rho}\sinh(a\tau); z = -\frac{1}{a}e^{-a\tilde{\rho}}\cosh(a\tau) \ \ ; \ \ z < -|t| \tag{26}$$

or to extend it

$$t - z = -\frac{1}{a}e^{-a(\tau-\rho)} \ \ ; \ \ t - z < 0 \tag{27}$$

$$t - z = \frac{1}{a}e^{a(\tau-\tilde{\rho})} \ \ ; \ \ t - z > 0 \tag{28}$$

$$t + z = -\frac{1}{a}e^{-a(\tau+\rho)} \ \ ; \ \ t + z < 0 \tag{29}$$

$$t - z = \frac{1}{a}e^{a(\tau+\tilde{\rho})} \ \ ; \ \ t + z > 0 \tag{30}$$

Note that all of $\frac{dt}{d\tau}$, $\frac{dz}{d\rho}$, and $\frac{dx}{d\tilde{\rho}}$ are positive. Ie, all these coordinates increase in the same direction.

Instead of using the above plane wave modes, let us look look at the modes

$$\hat{\phi}_{R\Omega} = \lim_{\epsilon\to 0^+}(a(t-z) - i\epsilon)^{i\Omega/a} \tag{31}$$

The argument is proportional to $e^{i\Omega/a\ln(t-z-i\epsilon)}$ which has a branch cut starting at $t-z = i\epsilon$ Let us choose the branch cut so that it runs to infinity confined to $Im(t-z) > 0$so the function is analytic along Im(t-z)=0. This gives

$$\hat{\phi}_{R\Omega} = \theta(t-z) + e^{-\pi\Omega/a}\theta(-(t-z))(a|(t-z)|)^i\Omega/a \tag{32}$$

and choosing the layer for the logarithm such that

$$|(a|(t-z)|)^i\Omega/a| = 1 \tag{33}$$

To find the norm of $\phi_{R\Omega}$ we do

$$i\int \phi^*_{R\Omega}\partial_t\phi_{R\Omega'} - \phi_{R\Omega'}\partial_t\phi^*_{R\Omega'} \tag{34}$$

$$= -i\int \phi^*_{R\Omega}\partial_z\phi_{R\Omega'} - \phi_{R\Omega'}\partial_z\phi^*_{R\Omega'} \tag{35}$$

$$dz \tag{36}$$

$$= -i\int \phi^*_{R\Omega}\partial_\rho\phi_{R\Omega'} - \phi_{R\Omega'}\partial_\rho\phi^*_{R\Omega'}d\rho \tag{37}$$

$$\phi^*_{R\Omega}\partial_{\tilde{\rho}}\phi_{R\Omega'} - \phi_{R\Omega'}\partial_{\tilde{\rho}}\phi^*_{R\Omega'}d\tilde{\rho} \tag{38}$$

$$= +i\int \phi^*_{R\Omega}\partial_\tau\phi_{R\Omega'} + \phi_{R\Omega'}\partial_\tau\phi^*_{R\Omega'}d\rho \tag{39}$$

$$-\phi^*_{R\Omega}\partial_\tau\phi_{R\Omega'} - \phi_{R\Omega'}\partial_\tau\phi^*_{R\Omega'}d\tilde{\rho} \tag{40}$$

$$= 2\Omega(e^{2\pi\Omega/a} - 1)2\pi\delta(\Omega - \Omega') \tag{41}$$

This is positive for all values of Ω, positive or negative., and thus these modes are positive norm modes for all values of Ω. To normalize the mode we have to divide by the square root of the norm.

Furthermore the only singularity is that branch cut. Thus with this choice of branch cut, the function $\hat{\phi}$ is analytic in the whole half plane $Im(t-z)<0$.. If we look at $\int_0^\infty \alpha_\omega e^{-i\omega(t-z)}d\omega$ and look at $Im(t-x)<0$ we get

$$\int \alpha_\omega e^{-i\omega Re(t-z)} e^{\omega(Im(t-z))} d\omega \tag{42}$$

The multiplier is less than 1 for all $\omega>0$ and goes to 0 as $\omega\to\infty$. Thus if for $Im(t-z)$ is regular and bounded, Then it will be analytic for all values of $Im(t-z)<0$. Ie, linear combinations of positive frequency functions are analytic in the lower half imaginary t-z plane, while linear combinations of negative frequency Minkowski functions are analytic in the upper half imaginary (t-z) plane. But $(t-z)^i\Omega$ with the above definition of the branch cut is analytic in the lower half imaginary plane, and thus it must be of the form $\int_0^\infty e^{-i\omega(t-z)}d\omega$. Ie, it must be a positive norm mode in the same sense as the usual plane wave modes are positive norm. The annihilation operators $A_{R\Omega}$ must be linear combinations of the usual Minkowski postive norm annihilation operators $A_{R\omega}$. The vacuum state $A_{R\Omega}|0\rangle=0$ is the same state as the $A_{R\omega}|0\rangle=0$.

The mode $\hat{\phi}_{R\Omega}$ can be written in therms of $\tau,\ \rho$ as

$$\hat{\phi}_{R\Omega} = \frac{e^{\pi\Omega/a}}{\sqrt{e^{2\pi\Omega/a}-1}} e^{-i\Omega(\tau-\rho)} \tag{43}$$

But this is the postitive norm state associate with the annihilation operator, which annihilates the vacuum. The mode associated with the creation operator is the complex conjugate of this or

$$\frac{e^{\pi\Omega/a}}{\sqrt{e^{2\pi\Omega/a}-1}} e^{i\Omega(\tau-\rho)} \tag{44}$$

Thus the first order correction to the state is

$$|\Psi_1(t)\rangle = \int e^{iE\tau}(\partial_\tau e^{i\Omega\tau})\mu_0 e^{-\frac{\tau^2}{2T^2}} A_{R\Omega}|0\rangle|\uparrow\rangle d\Omega d\tau d\Omega \tag{45}$$

$$= \frac{e^{\pi\Omega/a}}{\sqrt{e^{2\pi\Omega/a}-1}}(i\Omega)\sqrt{2\pi T}e^{-T^2(E+\Omega)^2} A_{R\Omega}|0\rangle|\uparrow\rangle d\Omega \tag{46}$$

$$\approx \frac{e^{-\pi E/a}}{\sqrt{4\pi E(1-e^{-2\pi E})}}\sqrt{2\pi T} A_{R(-E)}|0\rangle|\uparrow\rangle \tag{47}$$

where the last line assumes that T is very large, and that $e^{-T^2(E+\Omega)^2/2}A_{R\Omega}d\Omega \approx A_{R,(-E)}|0\rangle$. Ie, the state of the field is a one photon state with mode function $\hat{\phi}^*_{R(-E)}$. Note that this mode is larger in the acausal region ($\tilde{\rho}$) than it is in the region where the detector is by a factor of roughly $e^{\pi E/a}$ as pointed out by Unruh and Wald (PRD 1984)

One can ask if this is actually measureable. One can place another particle detector at $\tilde{\rho}=0$, and ask whether one can find that the second detector in the causally disconnected region excited due to the absorption of the quantum emitted by the first detector at $\rho=0$. leaving the state of the field in its vacuum state after both detectors have been excited. This was looked at by Svidzinsky, Asisi, Benjamin, Unruh and Scully in 2021, and they found that the probability of this was non-zero. However alternatively, the probability of detecting a particle by a second detector in the same region where the first detector was accelerated (eg at position $a\rho=1$ with the first detector at $\rho=0$). Here the probability of both detectors being excited and the state of the system being the vacuum was zero. Ie, that second detector cannot absorb the particle emitted by the first detector. The two detectors can both emit particles, and the first can stimulate the second to emit a particle. Ie, the probability of the two detectors both emitting the same mode is larger than the product of the probability that each detector emits that mode.

[1]