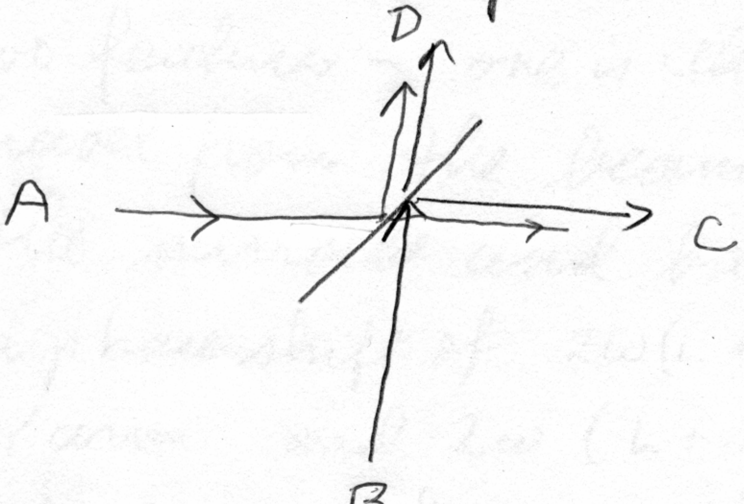
Beam splitter



$$\begin{pmatrix} C \\ D \end{pmatrix} = \frac{1}{\sqrt{2}} \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \begin{matrix} A \\ B \end{matrix}$$

Q Any unitary matrix with
with "equal amplitudes"
& Absolute values of all
co-efficients $= \frac{1}{\sqrt{2}}$

Also opposite direction

$$\begin{pmatrix} A_R \\ B_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{matrix} C_R \\ D_R \end{matrix}$$

If we have small phase shift.

$$C_R = C_0 e^{i\mu} \qquad D_R = D_0 e^{i\nu}$$

$$\begin{pmatrix} A_R \\ B_R \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^{i\mu} & 0 \\ 0 & e^{i\nu} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\begin{pmatrix} A_R \\ B_R \end{pmatrix} = \begin{pmatrix} e^{i\mu}+e^{i\nu} & e^{i\mu}-e^{i\nu} \\ e^{i\mu}-e^{i\nu} & e^{i\mu}+e^{i\nu} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

The phase shifts μ and ν will arise from two features - one is that the beams travel from the beam splitter to the mirrors and back - this gives a phase shift of $2\omega(L+Y)$ along the Y arm and $2\omega(L+X)$ along the X arm. choose such that

$2\omega_0(L+X_0) = 2n\pi$ for the x arm and $2n\pi+\epsilon$ for the y arm where $\epsilon \ll 1$. The motion of the distances to the mirrors $\delta X, \delta Y$ add to the phase shift by $e^{-i\omega\delta X}$ and $e^{i\omega\delta Y}$

Part of this is due to the motion of the mirrors, and part due to the grav wave changing the distance directly ($L \to L(1+h(t))$ for x arm and $L \to L(1-h(t))$ for Y in optimal case). [This assumes that the grav wave freq is much less than the time of transit of the light through the interferometer $2L/\omega_0 \ll h/(dh/dt)$

or $B_R = (e^{i\mu} - e^{i\nu})A + e^{i\mu + i\nu} B$

If $e^{i\mu}$ and $e^{i\nu}$ very close to 1.

$$B_R = i(\mu - \nu) A + B$$

ie there is a small admixture of the A stream, and the B stream comes back completely.

In particular, we choose the A stream to be $a \frac{e^{-i\omega_0(t-x)}}{\sqrt{4\pi\omega_0}}$. For $\mu-\nu$ very small, the quantum attribute of the Incoming A stream will be negligible when multiplied by $\mu-\nu$. The B stream will be quantum noise

$$\delta\Phi = \int \left\{ A_{\omega_0+\hat{\omega}} e^{-i(\omega_0+\hat{\omega})(t-y)} + A^{+}_{\omega_0+\hat{\omega}} e^{i(\omega_0+\hat{\omega})(t-y)} \right\} d\hat{\omega}$$

Both our measuring of the output and our interaction with the masses will be sensitive only to $\hat{\omega}$ small.

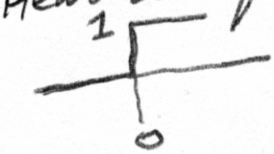
Radiation on mirrors.

$\frac{a}{\sqrt{4\pi\omega}} e^{-i\omega_0(t-x)}$ $\longrightarrow$

$\longleftarrow$ $\tilde{a}\, e^{-i\omega_0(t+x)}$

$L + \delta X$

Heaviside functn



$$\mathcal{L} = \int_{-\infty}^{\infty} \frac{1}{2}\left((\partial_t \phi)^2 - (\partial_x \phi)^2 \right) \Theta(2L + \delta X(t) - x)\, dx$$
$$+ \frac{1}{2} M\left(\frac{dX}{dt}^2 - \Omega^2 X \right)$$

Vary wrt ϕ

$$\int \delta\phi \left(-\partial_t \Theta(\ )\frac{\partial\phi}{\partial t} + \partial_x \Theta(\ )\partial_x\phi \right) dx$$

$$\Longrightarrow \left(\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} \right) = 0 \quad \text{for } x < L + \delta X$$

$$\left(\frac{\partial\phi}{\partial x} - \frac{\partial X}{\partial t}\frac{\partial\phi}{\partial t} \right) = 0 \text{ at } x = (L + \delta x)$$

(Direchlet boundary cond.).

Bkgnd soln $e^{-i\omega_0(t-x)} + e^{-i\omega_0(t+x-2L-2\delta X)}$ will solve these eqns and boundary condition.

The eqn of motion for the mirror is

$$-\partial_t^2 \delta X - \Omega^2 \delta X + \frac{1}{2}\left((\partial_t \phi)^2 - (\partial_x \phi)^2 \right)\Big|_{x = L + \delta}$$

$\partial_x \phi = 0$ at bndry so

~~and~~ $\cancel{\phi = \left[\frac{a}{\sqrt{4\pi\omega_0}} e^{-i\omega_0(t - L - \delta X)} - + \frac{a}{2\pi\omega_0} e^{-i\omega(t - L - \delta X)} \right] + H.C.}$

~~$\frac{1}{2}(\partial_t \phi)^2$ has terms which go as $e^{\pm 2i\omega_0 t}$. These are such high freq they do not contrib. to mirror motion. Only cross terms apply.~~

$$\cancel{\frac{|a|^2}{4\pi\omega} e^{-i\omega_0(\delta X^* - \delta X)}}$$

$$\phi = \frac{a}{\sqrt{4\pi\omega_0}} e^{-i\omega_0(t - x)} + \frac{a^*}{\sqrt{4\pi\omega}} e^{-i\omega_0(t + x - 2L - 2\delta)}$$

$$+ \frac{1}{\sqrt{2}} \int \left(B_{\hat\omega} \frac{e^{i(\omega_0 + \hat\omega)(t - x)}}{4\pi\omega_0} + B^\dagger_\omega e^{+i(\omega_0 + \hat\omega)(t + x - 2L - 2\delta)} \right) d\omega$$

where B is assumed small

The pressure is then, neglecting terms of order $e^{\pm 2i\omega_0 t}$

$$\text{Pressure} = \frac{\omega_0 a^2}{\sqrt{4\pi}} + \frac{2\omega_0 a}{\sqrt{2}} \int B_{\hat{\omega}}\, e^{-i\hat{\omega}(t-L)} + B^+_{-\hat{\omega}}\, e^{i\hat{\omega}(t-L)}\, d\hat{\omega}$$

Fourier transforming the Y eqn and assuming $\hat{\omega} \gg \Omega$ and damping

$$\delta Y(\hat{\omega}) = \sqrt{2}\,\omega_0 a \left(B_{\hat{\omega}} + B^+_{-\hat{\omega}}\right)$$

(assume $e^{2i\hat{\omega}_0 L} \approx 1$)

$$M(-\hat{\omega}\,\delta Y) = \frac{\sqrt{2}\,\omega_0 a}{\hat{\omega} M}\left(+B_{\hat{\omega}} + B^+_{-\hat{\omega}}\right)$$

Thus the phase shift at the end will be $\mu = 2Lh + i\frac{\sqrt{2}\,\omega_0 a}{\hat{\omega} M}\left(B_{\hat{\omega}} + B^+_{-\hat{\omega}}\right)$

$$\nu = -2Lh - i\frac{\sqrt{2}\,\omega_0}{\hat{\omega} M} a \left(B_{\hat{\omega}} + B^+_{-\hat{\omega}}\right)$$

$$(\mu - \nu) = 4Lh - i\frac{\sqrt{2}\,\omega_0}{\hat{\omega} M} a \left(B_{\hat{\omega}} - B^+_{-\hat{\omega}}\right)$$

Thus we get 2 noise sources.

The noise which came into the B port and reflected directly off the mirror, and the noise source which pushed the mirror and that produced a phase shift which came back out the port.

These two have opposite phases.