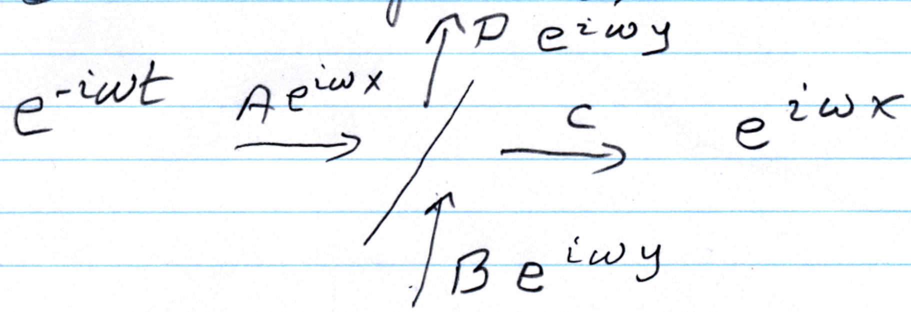
Parts of Interferometer:

① Beam splitter, – 50-50.



Assume $\begin{pmatrix} C \\ D \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$

② EM as scalar field.

$$\partial_t \vec{D} = +\nabla \times \vec{H} \qquad n^2 E = D$$

$$\partial_t \vec{B} = -\nabla \times \vec{E} \qquad H = B$$

Prop in $x-y$ plane. E_z = polsm.

$$n^2 \partial_t^2 E_z = -\nabla \times \frac{\partial B}{\partial t} = -\nabla \times (\nabla \times D)$$

$$= -\partial_z \underbrace{(\nabla \cdot E)}_{=0} + \nabla^2 E_z$$

$$\xi = \frac{x+y}{\sqrt{2}} \qquad \eta = \frac{x-y}{\sqrt{2}}$$

Beam splt. surface at $\eta = 0$.

{ What is actual Beam splitter transfer function? - I can not find this. It is important.

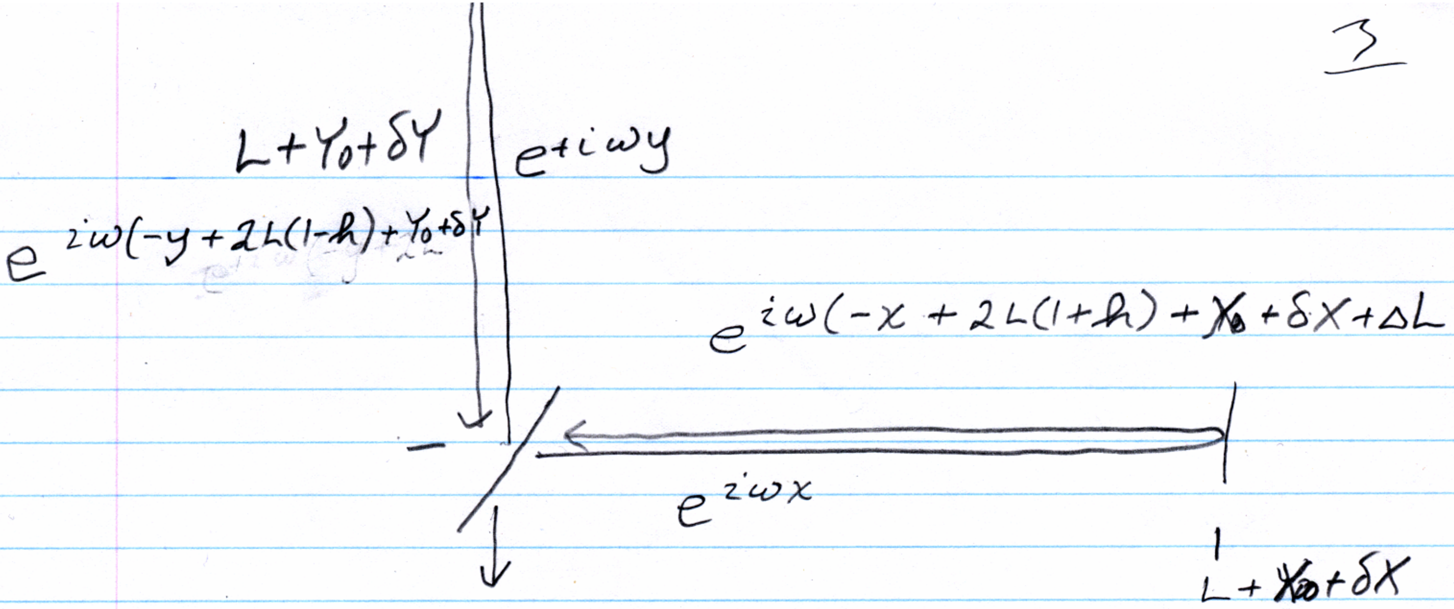
$$A: \quad \frac{a}{\sqrt{4\pi\omega_0}} e^{-i\omega_0(t-x)} + \int \frac{A_{\hat{\omega}} e^{-i(\omega_0+\hat{\omega})(t+x)}}{\sqrt{4\pi\,\omega_0+\hat{\omega}}} + H.C.$$

$$B = \int \frac{B_{\hat{\omega}} e^{-i\omega(t-y)}}{\sqrt{4\pi(\omega_0+\hat{\omega})}} + H.C.$$

$$C = \frac{a}{\sqrt{2}} e^{-i\omega_0(t-x)} + \int \frac{A_{\hat{\omega}} e^{-i(\omega+\hat{\omega})(t-x)}}{\sqrt{2}\sqrt{4\pi(\omega_0+\hat{\omega})}} + \int \frac{B_{\hat{\omega}} e^{-i(\omega+\hat{\omega})(t-x)}}{\sqrt{2}\sqrt{4\pi\,\omega_0+\hat{\omega}}} + H.C$$

We will assume $|\hat{\omega}| \lll \omega_0$ (10^{-12})

$$D = \frac{a}{\sqrt{2}\sqrt{4\pi\omega_0}} e^{-i\omega_0(t-y)} + \int A_{\hat{\omega}} \frac{e^{-i(\omega+\hat{\omega})(t-y)}}{\sqrt{4\pi\,\omega+\hat{\omega}}} - \int B_{\hat{\omega}} \frac{e^{-i(\omega+\hat{\omega})(t-y)}}{\sqrt{4\pi(\omega+\hat{\omega})}} + H.C.$$



δY, h, ΔL all small

$$\frac{1}{\omega_0} \gg \Delta L \gg \delta Y,\ hL$$

Return $\quad Ce^{i\mu} \quad De^{i\nu}$

$$\begin{pmatrix} A_R \\ B_R \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} Ce^{i\mu} \\ De^{i\nu} \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{i\mu} & 0 \\ 0 & e^{i\nu} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} e^{i\mu}+e^{i\nu} & e^{i\mu}-e^{i\nu} \\ e^{i\mu}-e^{i\nu} & e^{i\mu}+e^{i\nu} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

Assume $\mu, \nu \ll \pi$

$$B_R \approx \frac{1}{2}\, i(\mu-\nu)A + B.$$

$$\mu = \omega_0 2(L + X_0) + 2\omega_0 \Delta L + 2\omega_0 \delta X - 2\pi N + \omega_0 L h$$

$$\nu = \omega_0(2L+Y_0) + 2\omega_0 \delta Y - 2\pi N - \omega_0 L h$$

Signal

$$\mu - \nu = 2\omega_0 \Delta L + 2\omega_0(\delta X - \delta Y) + 4\omega_0 L h$$

$$B_R \approx \overbrace{\left(2\omega_0 \Delta L\right)}^{10^{-6}} \frac{a}{\sqrt{4\pi\omega_0}} e^{-i\omega_0(t-y)} + H.C. + \underbrace{4\omega_0 L h}_{\text{signal}}$$

$$+ 2\omega_0(\delta X - \delta Y) \qquad \approx 10^{-22} \times 10^{10}$$

$$+ \int \frac{B_{\hat\omega}\, e^{-i(\omega_0+\hat\omega)(t+y)}}{\sqrt{4\pi\omega_0}} + H.C.$$

Neglect $\cancel{X}$ $+ (2\omega_0 \Delta L) \int A_{\hat\omega}\, e^{-i(\omega_0+\omega)(t+y)} + H.C.$

Effect of $B_{\hat\omega}$ and of $(\delta Y - \delta X)$
about same. (depends on freq.)

Pressure on mirror δY.

Pressure = energy density $(c=1)$

$$\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial t}\right)^2 + \underbrace{\left(\frac{\partial\phi}{\partial x}\right)^2}_{=0 \text{ by bndry cond. At } X}\right]$$

$$\phi = \frac{a}{\sqrt{2}\sqrt{4\pi\omega_0}} e^{-i\omega_0(t-L)} +$$

$$+ e^{-i\omega_0(t-L)} \int \left(\frac{A_{\hat\omega}}{\sqrt{2}} + \frac{B_{\hat\omega}}{\sqrt{2}}\right) e^{-i\hat\omega(t-L)}\, d\omega.$$

$$+ H.C.$$

Frequencies only near 0 will affect the motion of mirror.

when I square ϕ get terms of $e^{2i\omega_0 t}$ and $e^{-2i\omega_0 t}$. They are have negl. effect on mirrors.

(10^{15} Hz pushes have no effect on mirror.

Thus $\frac{a\omega_0^2}{8\pi\omega_0}\int (A^+_{\hat\omega} + B^+_{\hat\omega})\, e^{+i\hat\omega(t-L)}$

$+ \frac{a^*\omega_0^2}{8\pi\omega_0}\int (A_{\hat\omega} + B_{\hat\omega})\, e^{-i\hat\omega(t-L)}$

At freq $\hat\omega$, the force is

$$\frac{a\omega_0}{8\pi}(A_{\hat\omega} + B_{\hat\omega}) + \frac{a^*\omega_0}{8\pi}(A^+_{-\hat\omega} + B^+_{-\hat\omega})$$

Assume a real

If $\hat\omega \gg \Omega$

$$M\delta\ddot{X} = P$$

$$\text{or } \delta X_{\hat\omega} = \frac{a\omega_0}{M 8\pi\hat\omega^2}\left[A_{\hat\omega} + A^+_{-\hat\omega} + B_{\hat\omega} + B^+_{-\hat\omega}\right]$$

(~~the~~

Similarly for δY.

$$\delta Y_{\hat\omega} = \frac{a\omega_0}{M 8\pi \hat\omega^2}\left[A_{\hat\omega} + A^{+}_{-\hat\omega} - B_{\hat\omega} - B^{+}_{\hat\omega}\right]$$

$$\delta X_{\hat\omega} - \delta Y_{\hat\omega} = \frac{2\, a\omega_0}{M\, 8\pi \hat\omega^2}\left(B_{\omega} + B^{+}_{-\hat\omega}\right)$$

Ie Radn pressure noise is prop to $(B_{\hat\omega} + B^{+}_{-\omega})\dfrac{a\omega_0}{4\pi M \hat\omega^2}$

Similarly the "shot nouse is also equal to $B_{\hat\omega} + B^{+}_{-\hat\omega}$

$$\langle B_{\omega} + B^{+}_{\omega} \rangle = 0$$

$$\langle (B_{\hat\omega} + B^{+}_{-\hat\omega})^{+} (B_{\hat\omega} + B^{+}_{-\omega}) \rangle \neq 0$$

If state is vacuum state

$$\langle 0 | (B^{+}_{\hat\omega} + B_{-\hat\omega})(B_{\omega} + B^{+}_{-\omega}) | 0\rangle$$

$$= \langle 0 | B_{-\hat\omega} \cdot B^{+}_{-\hat\omega} | 0 \rangle = 1$$

Create Squeezed State.

$$|S\rangle = \frac{Ne^{r B^+_{\hat\omega} B^+_{-\hat\omega}}}{N} |0\rangle$$

$$\left(\langle 0| e^{r^* B_{\hat\omega} B_{-\hat\omega}} e^{r B^+_{\hat\omega} B^+_{-\hat\omega}} |0\rangle \right.$$

$$= \langle 0| \sum_{nm} r^{*n} \frac{(B_{\hat\omega} B_{-\hat\omega})^n}{n!} r^m \frac{B^+_{\hat\omega} B^+_{-\hat\omega}}{m!} |0\rangle$$

$$= \langle 0| \sum_n |r|^{2n} \frac{B_{\hat\omega}^n B^{+n}_{\hat\omega} B_{-\hat\omega}^n B_{-\hat\omega}^n}{(n!)^2} |0\rangle$$

$$= \langle 0| \sum_{n=0}^\infty |r|^{2n} |0\rangle = \frac{1}{1-|r|^2} \qquad (|r|<1)$$

$$N = \sqrt{1-|r|^2}$$

$$\langle s| (B_\omega + B^+_{-\omega})(B^+_\omega + B_\omega) |s\rangle$$

$r = \tanh\theta$ $\quad (\cosh\theta\, B_{\hat\omega} - \sinh\theta\, B^+_{-\hat\omega}) |s\rangle = 0$

$$(\cos\theta\, B_{-\hat\omega} + \sinh\theta\, B^+_{\hat\omega}) |s\rangle = 0$$

$$\tilde B_1 = \cosh\theta\, B_{\hat\omega} - \sinh\theta\, B^+_{-\hat\omega}$$

$$\tilde B_2 = \cos\theta\, B_{-\hat\omega} - \sinh\theta\, B^+_{-\hat\omega}$$

$$[B_1, B_1^+] = 1, \quad [B_2, B_2^+] = 1$$

$$[B_1, B_2] = [B_1, B_2^+] = 0$$

$B_1 |s\rangle = B_2 s.$

$$B_{\hat\omega} + B^+_{-\hat\omega} = (B_1 + B_2^+)e^{-\theta}$$

$$(B_\omega + B^+_{-\omega})(B^+_{\omega} + B_{-\hat\omega}) = e^{-2\theta}(B^+_1 + B_2)(B_1 + B_2^+)$$

$$\langle s | (B^+_{\omega} + B_{-\hat\omega})(B_\omega + B^+_{-\omega}) |s\rangle$$

$$= e^{-2\theta}$$ If $r = \tanh\theta$

then squeezed state can reduce the noise by almost arbitrary amount. This reduces both the ~~rad~~ shot noise and the pressure noise.

It depends on the Beam splitter transfer function being real $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Can also have $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$

Then Radn pressure noise is turns out to be $\frac{ai}{M\hat\omega^2}(B_{\hat\omega} - B^+_{-\hat\omega})$

while shot noise is still $\frac{c}{M\omega^2}(B_\omega + B^+_{-\omega})$

$$\langle s| \left(B_{\omega} - B^+_{-\omega}\right)^+ (B_\omega - B_{-\omega}) |s\rangle$$

$$= e^{+r} \quad \text{Ie bigger.}$$

thus if beam splitter is imag parts
Need squeezing to vary with freq
Need freq dep squeezing to
squeezing which kills shot noise
at high freq + Radn pressure at
low freq.