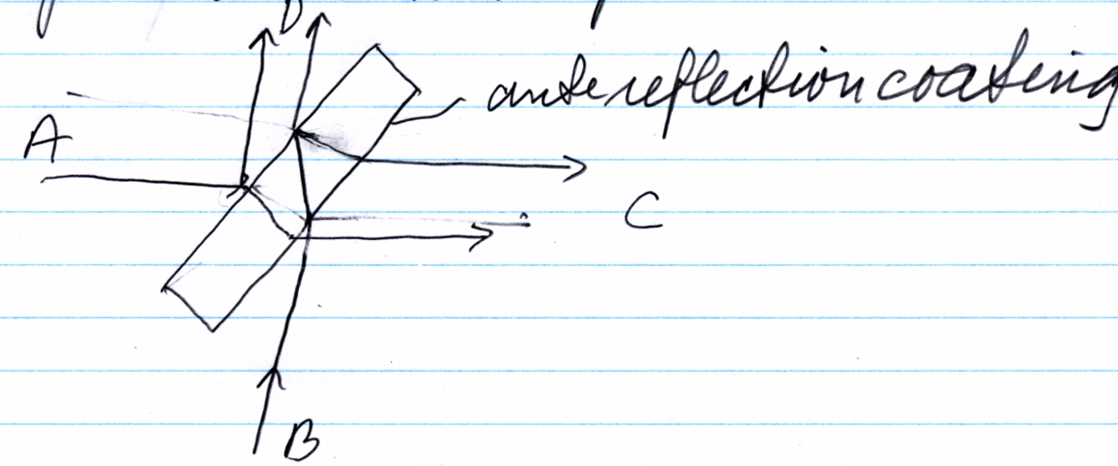
(a) Beam Splitter:

The form I used in my slides is possible to set up.



$$n^2\omega^2 - k_\xi^2 - k_\eta^2 = 0$$

where $n=1$, $|k_\xi| = |k_\eta| \Rightarrow |k_\xi|^2 = \frac{\omega^2}{2}$

Inside $k_\eta'^2 = n^2\omega^2 - \frac{\omega^2}{2} = (n^2 - 1/2)\frac{\omega^2}{}$

At boundary

$$\left(A e^{ik\eta} + D e^{-ik\eta}\right)\Big|_{\eta=0} = \left(B e^{-ik'\eta} + C e^{ik'\eta}\right)$$

Also 1st deriv is cont. (integrate the equ of motion from $\eta = 0^-$ to $\eta = 0^+$)

$$k(A - D) = k'(C - B)$$

$\sqrt{2}$

$$C = \frac{2k}{k+k'} A - \frac{(k-k')}{k+k'} B$$

$$D = \frac{(k-k')A}{k+k'} + \frac{2k'}{k+k'} B.$$

The number flux goes as

kA^2, $k'B^2$, $k'C^2$, kD

So we want for A that

$D = \frac{1}{\sqrt{2}} A$ (D on same side as A)

$(k'|C|^2 = k|A|^2)$

Thus $\quad \frac{k'-k}{k+k'} = \frac{1}{\sqrt{2}}$,

$$k' = \frac{\sqrt{2}+1}{\sqrt{2}-1} k =$$

$$2n^2 - 1 = \frac{\sqrt{2}+1}{\sqrt{2}-1} \Rightarrow n = 1.84$$

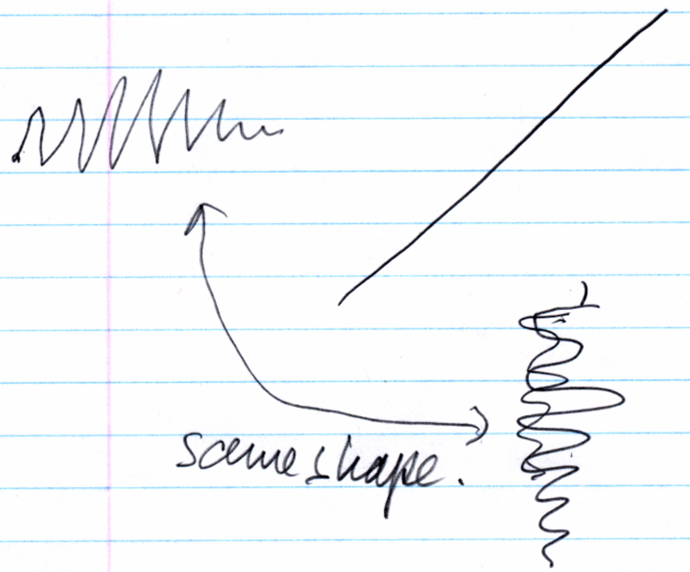
(Fused Silica 1.45)
Ta_2O_5 $n \sim 2.076$

For other side $C = -\frac{1}{\sqrt{2}} B$ gives same value for n

So both sides reflect 50-50

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Hong Ou Mandel Effect.



$$A_\phi = \langle \Phi(t-x), \Phi(x) \rangle \qquad \phi = \int \alpha_\omega e^{-i\omega(t-x)} d\omega$$

$$B_\phi = \langle \phi(t-y), \Phi(y) \rangle \qquad \alpha\omega \sim$$

After beam splitter.

$$C_\phi = 1/\sqrt{2}(A_\phi + B_\phi)\ \phi(x) \qquad A_\phi = \frac{C_\phi - D_\phi}{\sqrt{2}}$$

$$D_\phi = \frac{1}{\sqrt{2}}(-A_\phi + B_\phi)\ \phi(y) \qquad B_\phi = \frac{C_\phi + D_\phi}{\sqrt{2}}$$

state: $A^+_\phi B^+_\phi |0\rangle = |\psi\rangle$

$$(C^+_\phi C_\phi)\ |\psi\rangle$$

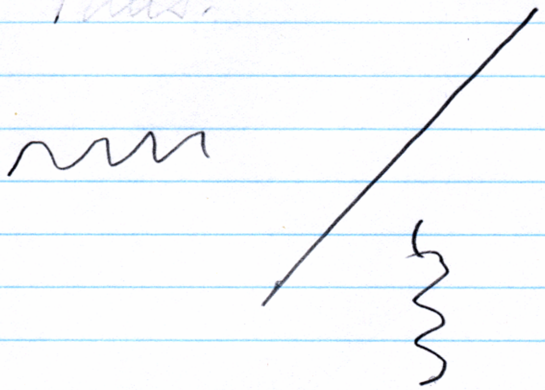
$$A^+_\phi B^+_\phi = \frac{1}{2}(C^+_\phi - D^+_\phi)(C^+_\phi + D^+_\phi)$$

$$= \frac{1}{2}(C^+_\phi C^+_\phi - D^+_\phi D^+_\phi)|0\rangle$$

N particle state: $\frac{(C^+)^n}{\sqrt{n!}}|0\rangle$

$$A^+_\phi B^+_\phi |0\rangle = \frac{1}{\sqrt{2}}(|2_c, 0\rangle - |0_c, 2_D\rangle$$

Classical.



$$\phi(t-x) + \phi(t-y)$$

$$\underset{A}{\phi(t-x)} \longrightarrow \frac{1}{\sqrt{2}}\underset{C}{\phi(t-x)} + \frac{1}{\sqrt{2}}\underset{D}{\phi(t-y)}$$

$$\underset{B}{\phi(t-y)} \longrightarrow \frac{1}{\sqrt{2}} \cdot -\phi(t-x) + \frac{1}{\sqrt{2}}\phi(t-y)$$

$$\phi(t-x) + \phi(t-y) \longrightarrow \frac{2}{\sqrt{2}}\phi(t-y)$$

$$\phi(t-x) + e^{i\phi}\phi(t-y) \longrightarrow \frac{1}{\sqrt{2}}(1-e^{i\phi})\phi(t-x) + \frac{1}{\sqrt{2}}(1+e^{i\phi})\phi(t-y)$$

- ie as phase shifts, classical field goes from all out of D → 50 50 C and D → all out of C → 50 50 C+D → ...