Physics 501-22
Assignment 4

1) Consider the field theory with Lagrangian

$$\mathcal{L} = \int (\partial_t\phi - v\partial_x\phi)^2 - (\partial_x\phi)^2 - (\partial_x^2\phi)^2 dx \tag{1}$$

where x represents the one dimensional coordinate.

i)Find the Hamiltonian.

ii) Assume $v > 1$. Find the modes which diagonalize this Hamiltonian. (Hint use the spatial Fourier transform), and assume that the temporal solution goes as $e^- i\omega t$. What are the solutions for ω as a function of k the wavenumber? What is the condition on ω that the solutions have positive norm?

This is the field theory which represents sound waves in a BEC, where the spatial coordinates are expressed in terms of the "healing length" and the time is chosen so that the velocity of sound at long wavelengths is unity. This is looking at the BEC in a frame moving with velocity v with respect to the rest frame of the BEC.

2) Consider the harmonic oscillator with the Hamiltonian

$$H = \frac{1}{2}(p^2 + \mu(t)(px + xp)) \tag{2}$$

with .

a) What are the equations of motion for $p,\ x$.

b) Carry out the first stage of the adiabatic transformation. What equation must mu satisfy in order that the transformed equation are those of a free particle?

3. Consider the the field theory given by the one spatial dimensional Hamiltonian in a one dimensional cosmology writen in terms of the modes

$$\mathcal{L} = \frac{1}{2}\int \left(a(t)\left((\partial_t\phi_k(t))^2 - \frac{k^2}{a(t)^2}\phi_k(t)^2\right)\right) dk \tag{3}$$

a) What is the Hamiltonian for this system?

b)What is the "particle creation" at time $t + \delta$ if we use the Hamiltonan diagonalisation to define particles?a Is it finite?

c) What is the first order adiabatic transformation of this system?