Physics 501-22
Assignment 3

1.) Hardy system: Given the state

$$|\Psi\rangle = \alpha\,|\uparrow\rangle\,|1\rangle + \beta\,|\downarrow\rangle\,(S\,|1\rangle + C\,|0\rangle) \quad (1)$$

where this is a unit vector with $|\alpha|^2 + |\beta|^2 = 1$ and $|C|^2 + |S|^2 = 1$

i)Argue that we can always choose the coefficients as real and positive by adjusting the phases of the basis vectors..

ii) Find the value of S that minimizes the probability of having the final value of "D" be equal to +1. (Recall from the lectures that one has two systems, with A,B being attributes of the first system, and C,D of the second. They are such that $A \to 1 \Rightarrow C \to 1, C \to 2 \Rightarrow B \to 1, B \to 2 \Rightarrow D \to 1$, but $A \to 1$ does not imply that $D \to 1$ (in fact the probability that when A has value 1 it is highly improbably that D also has the value 1. $A \to 1$ here means A is found to have value 1. $\Rightarrow$ means "implies that"– ie if one makes measurements on the system, then it is always true that if A and X are measured, then whenever A is found to value 1, X always also has value 1.)

iii) Given that value of S, what is the largest value of the the ratio of the eigenvalues $\lambda_1,\ \lambda_2$ where the two λ are the two eigenvalues of the reduced density matrix of particle 1 with $\lambda 1$ being the smallest of the eigenvalues.

(recall that the density matrix for the second particle associated with a pure entangled state on the whole system is

$$|\Psi\rangle = \sum_i \lambda_i\,|\phi_i\rangle\,|\psi_i\rangle \quad (2)$$

is

$$\rho = \sum_{i,j} \lambda_i^* \lambda_j\,\rangle\phi_i|\,|\phi_j\rangle\,|\psi_j\rangle\,\rangle\psi_i| \quad (3)$$

where $|\phi\rangle$ is a state for the first particle/system, while $|\psi\rangle$ is a state for the second particle/system. For the Hardy system, use the two component vector to find the matrix representing the reduced density matrix for the second particle.

2) Assume that we have a Hamiltonian

$$H = \frac{1}{2}\left(\frac{p_1^2}{m_1^2} + \frac{p_2^2}{m_2} + k_1 x_1^2 + k_2 x_2^2 + 2\epsilon x_1 x_2\right) \quad (4)$$

a)What are the 4 eigenvalues $\pm i\omega_1,\ \pm\omega 2$ of the Hamiltonian equations for this Hamiltonian in terms of the constants $m_1, m_2, k_1, k_2, \epsilon$.

b) Is there any condition on $k_i, m_i, epsilon$ such that $\omega_1 = \omega 2$?

c) If $m_1 = m_2$, $k_1 = k_2$, is there any condition on ϵ such that the eigenvalues are not purely imaginary?

d) What are the normalised (using the symplectic norm) eigenvectors if $m_1 = m_2$, $k_1 = k_2$ and $\epsilon \neq 0$?

3). Consider the Hamiltonian $H = \frac{1}{2}(p^2 - x^2)$.

a)What are the eigenvalues of the Hamiltonian (The "diagonalization of the Hamiltonian" values for omega? Show that there are no purely real eigenvalues.

b)Find a positive norm, normalised mode. (Recall that if you have two independent classical solution, the sum of the first plus i times the second is a complex mode solution.) What is the time dependence of this mode. Show that its norm is independent of time explicitly.

d)Find the Annihilation and Creation operators corresponding to this mode, and show explicitly that they are independent of time.

e) What is the quantum Hamiltonian in terms of these annihilation and creation operators?