Physics 501-20
Assignment 1

1.) Consider the Hamiltonian

$$H = \frac{1}{2}(P^2 + (1 + \epsilon\delta(t))Q^2) \quad (1)$$

Assume that at $t = 0^-$ (ie just before $t = 0$ the operators P, Q are given by $P = P_0, \quad Q = Q_0$.

Assume that the intial state of this quantum system is the lowest energy state at time $t = 0^-$ (the ground state).

a) Find the Heisenberg equations of motion, and solve them for arbitrary time in terms of $P_0 = P(0^-)$ and $Q_0 = Q(0^-)$

b)Define the Annihilation operators $A_0 = \frac{1}{\sqrt{2}}(Q_0 + iP_0)$. Show that $[A_0, A_0^\dagger] = 1$ and that $H(t < 0) = \frac{1}{2}A_0A_0^\dagger + A_0^\dagger A_0$. The minimum energy state $|0\rangle$ will therefor be given by

$$A_0 |0\rangle = 0 \quad (2)$$

Given the solution of the Heisenberg equations of motion, write the solutions for $P(t), Q(t)$ in terms of $A_0, \quad A_0^\dagger$. at all times.

c) Find the expectation value of the energy H in the state $|0\rangle$ as a funtion of time.

d)Solve the Schroedinger equation for this problem with the same initial conditions, and explicitly find the expectation value of the energy as a function of time. (If necessary, solve this to lowest non-trivial order in ϵ. Note that the Heisenberg equations can be solved exactly to all orders in ϵ).

Which is easier?

Note the state of the system after the t=1 is called a squeezed state, which has become a very powerful tool in quantum optics in increasing the sensitivity of certain detectors beyond what one might naively call the quantum limit. We will look at this later in the course.

2)Consider the Hamiltonian

$$H = \frac{1}{2}(P^2 - Q^2) \quad (3)$$

Find the solution to the Heisenberg equations of motion for this system, assuming that at $t = 0$ the operators are $P_0, \; Q_0$.

Consider the state of the system to be

$$|\psi\rangle = \int e^{-q^2/2} |q\rangle \, dq \quad (4)$$

In terms of the operators P_0, Q_0, what equation does this initial state satisfy? Write this equation in terms of $P(t)$ and $Q(t)$. In terms of the eigenvalues of $Q(t)$ what is the state of the system? (This is the Schroedinger equation solution.)

3)Consider the Hamiltonian with dynamic variables $X,\ P$

$$H = \delta(t)X \tag{5}$$

What are the Hamiltonian representation solutions for this and what is the Schroedinger equation?

Assume that just before t=0, the wave function is $\alpha e^{\beta x^2/2}$ where $\alpha,\ \beta$ are complex constants. In the Schroedinger representation what will the state be just after t=0? (Can ψ be continous? If not what does a delta funtion times a non-continous function mean?)

What is the solution for the Schroedinger equation just after t=0 using the Heisenberg solution to find it? What does this suggest about the solution to the above problem of continuity?