# Error Correction.

What happens if the environment suddenly affects the system?

Eg: error is a bit flip ($\sigma_x$)

This will mess up the computation.

$|01100010\rangle \rightarrow |01000010\rangle$ ?

$50 \longleftrightarrow 34$

Classical – repeat the bit by 3

$$\begin{matrix} 1 & & 1 \\ 0 & \Rightarrow & 1 \\ 0 & & 1 \end{matrix}$$

$$\begin{matrix} 0 & & 0 \\ 0 & \rightarrow & 0 \\ 0 & & 0 \end{matrix}$$

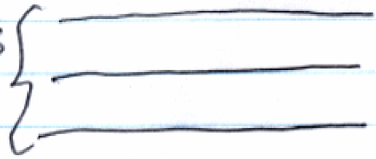
$$\alpha|\psi_1\rangle|1\rangle + \beta|\psi_2\rangle|0\rangle \Rightarrow \alpha|\psi_1\rangle|111\rangle + \beta|\psi_2\rangle|000\rangle$$

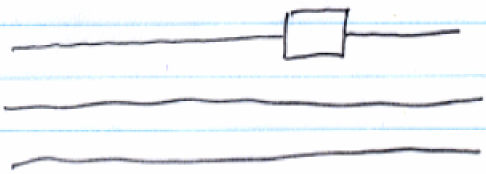
Classically – test if 3 bits are same. If not Majority rules. (Replace the mis behaving bit by the majority.)

Q. Cannot measure – Destroy superposn

# Diagramatic repn of quantum opns.

bits





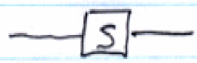
Single bit opn.
Label with what kind of operation

Hadamard  $\frac{1}{\sqrt{2}}(\sigma_z + \sigma_x) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$

—$\boxed{X}$— is also called NOT

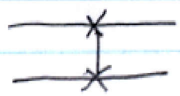
 (or Y or Z) $\sigma_x$ ($\sigma_y$, $\sigma_z$)

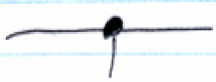
 Bit flip = $\sigma_x$

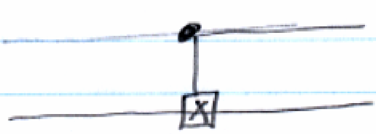
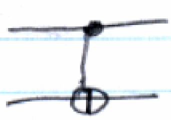
Phase  $\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$ (Square root of $\sigma_z$)

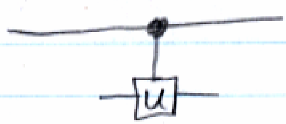
 $\begin{pmatrix}1 & 0\\ 0 & \sqrt{i}\end{pmatrix}$ (Square root of $\boxed{S}$

2 Bit gates.

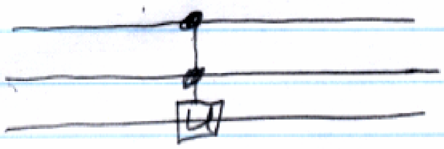
Swap  $|a, b\rangle \rightarrow |b, a\rangle$
a, b are 0, 1 or ...

 Control – The other end of operator carried out only if control bit is $|1\rangle$

 =  control not
XOR

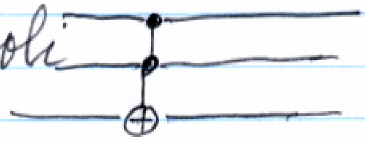
 control arbitrary Unitary.

3 bit

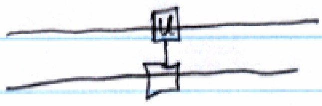



Control control Unitary.

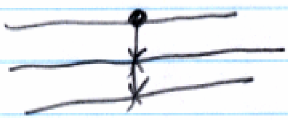
Eg Toffoli



control control NOT = CCNOT
control (XOR)




2 bit Unitary.
eg
control swap.



Encode.

$\alpha|\psi_1\rangle|0\rangle|0\rangle|0\rangle + \beta|\psi_2\rangle|1\rangle|0\rangle|0\rangle$

Apply $C\sigma_x$ (controled bit flip) from 1 ~~to~~ 2 + 1 & 3.

$$\frac{1}{2}\left[(1+\sigma_{1z}) + (1-\sigma_{1z})\,\sigma_{2x}\right.$$

$$= \frac{1}{2}\left[(1_1+\sigma_{1z})\,\mathbb{1}_2 + (\sigma_{2x} - \sigma_{z1}\sigma_{x2})\right]$$

$$= \frac{1}{4}\Big(2(1_1+\sigma_{1z})\,\mathbb{1} + 2\overbrace{(1_1+\sigma_{z1})(1-\sigma_{z1})}^{=0}\sigma_{2x}$$

$$+ \underbrace{\sigma_{2x}}_{1}\underbrace{(4\,1-\sigma_{z1})^2}_{2(1-\sigma_{z1})}\Big)$$

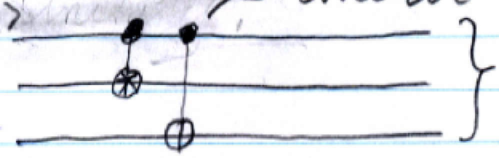
$$= \frac{1}{4}\,4\,\mathbb{1} \pm \mathbb{1}$$

control $\sigma_x$

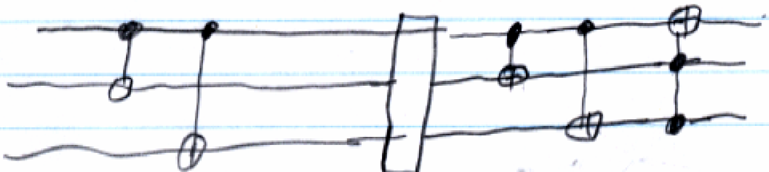
$\psi = \alpha|0\rangle + \beta|1\rangle$

0

0



$\alpha|000\rangle + \beta|111\rangle$



Bit flip correction.

8 poss.

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 ← |
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 |
| | 0 | 0 | | | |
| | 0 | 0 | | | |
| | 1 | 1 | | | |
| | 0 | 0 | | | |
| | 1 | 1 | | | |
| | 0 | 0 | | | |

In $\begin{matrix}1\\1\\1\end{matrix}$ out $\begin{matrix}1\\1\\0\end{matrix} \rightarrow \begin{matrix}1\\0\\1\end{matrix} \Bigg| \begin{matrix}1\\0\\1\end{matrix} \rightarrow \begin{matrix}1\\1\\0\end{matrix} \Bigg| \begin{matrix}0\\1\\1\end{matrix} \rightarrow \begin{matrix}1\\1\\1\end{matrix}$

Thus error correction. in 1st bit.

Same as classical. (Majority vote).

Hadamard. $H = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z) \rightarrow$ Unitary

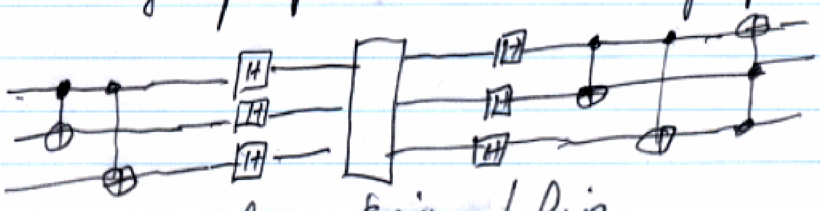
$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$H\sigma_z H = \frac{1}{2}(\sigma_x + \sigma_z)\,\sigma_z\,(\sigma_x + \sigma_z)$$

$$= \frac{1}{2}(-i\sigma_y + 1)(\sigma_x + \sigma_z) = \frac{1}{2}(\sigma_z + \sigma_x) + (\sigma_x + \sigma_z)$$

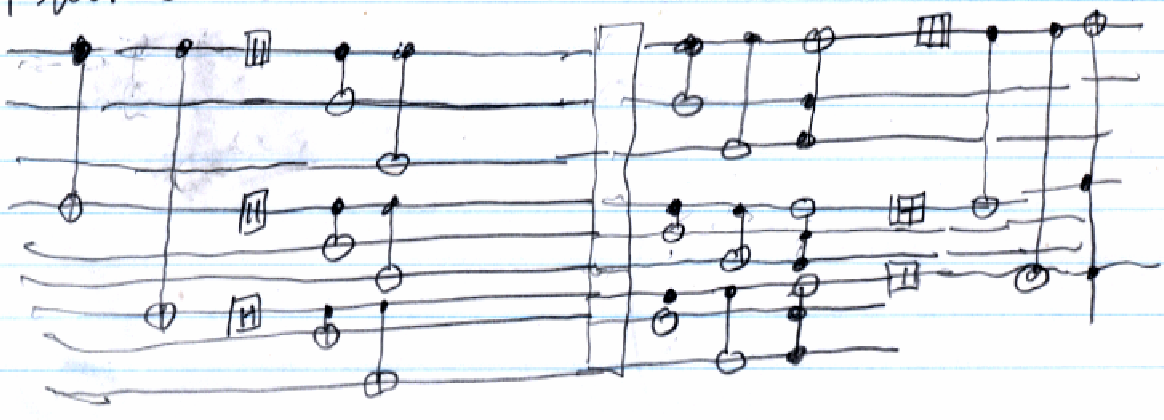
$$= 2\sigma_x$$

Sign flip becomes bit flip.



corrects p sign flip.

9 bit code Shore & Steane.



copy 1st bit to 4 + 7 Hadamard

copy 1st to 2 & 3 and 4 to 5 + 6
7 to 8, 9.

If $\sigma_x$ error. – corrected in
first sets on (1, 2, 3) (4, 5, 6)
(789)

but if there is a phase error it
will get copied to 1st bit.
the Hadamards & correction will
fix that.

This means that this code will fix
both single bit flip and single bit
pha sign errors.

Very expensive. Need 8 ancilla
/ which get thrown away after
each error correction.

Another layer $\Rightarrow$ 81 ~~fet~~ bits.
(80 thrown away).

(Error correcting codes with 1000 physical
bits per logical bit instead of 9.

No one has implimented an error
correction code.