Notes on error correction.

Classical : Majority rule :

Only error is "spinflip" $1 \leftrightarrow 0$ for a bit

encode 1 as 3 physical bits 111

0 " " " 000

Error flips one bit : $111 \to 101$ or $000 \to 010$

Measure the three bits. flip the minority bit to correct error.

Quantum - cannot measure - destroys entanglement. Also cannot duplicate (No cloning Thm.

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $\quad |\psi\rangle|\psi\rangle = \alpha^2|0\rangle|0\rangle + \alpha\beta(|1\rangle|0\rangle + |0\rangle|1\rangle) + \beta^2|0\rangle|0\rangle$

Thus $|\psi\rangle|\psi\rangle$ is non linearly related to $|\psi\rangle$.

Only transformation is Unitary transf.

Can "clone" basis vectors.

$|0\rangle|0\rangle \to |0\rangle|0\rangle$

$|1\rangle|0\rangle \to |1\rangle|1\rangle$

Complete to unitary

eg.

$|0\rangle|1\rangle \to |0\rangle|1\rangle$

$|1\rangle|1\rangle \to |1\rangle|0\rangle$

unique inputs give unique outputs.

Start with state $|\psi\rangle$ plus annicilla bits ($|0\rangle$)

Peres (~1985)

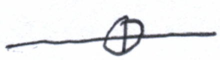
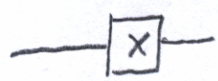
Bitflip correction for quantum.

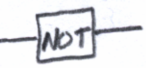
Graphical notation.

Horizontal lines — single physical bits

horizontal represents time (or rather order of operations.)).

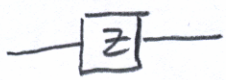
operation on single bit → □ with label saying which operation

 or 

or 

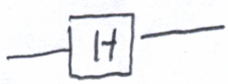
Bitflip
$|1\rangle \to |0\rangle, |0\rangle \to |1\rangle$

From $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$



(from $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$)

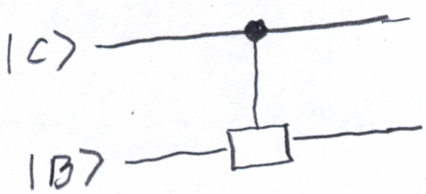
signflip $|0\rangle \to |0\rangle$
$|1\rangle \to -|1\rangle$



Hadamard
(change from ~~σ_z~~ basis to σ_x basis)

$\frac{1}{\sqrt{2}}(\sigma_x + \sigma_z) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$



Carry out the 1 bit operation on selected bit only if bit with dark dot is $|1\rangle$

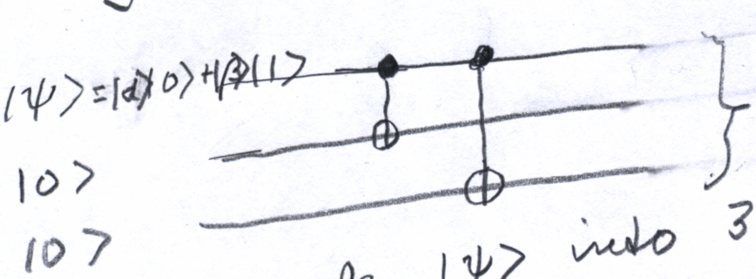
unitary

eg. 

$\Rightarrow$

$|B\rangle|C\rangle$	
$|0\rangle|0\rangle$	$\to |0\rangle|0\rangle$
$|0\rangle|1\rangle$	$\to |0\rangle|1\rangle$
$|1\rangle|0\rangle$	$\to |1\rangle|1\rangle$
$|1\rangle|1\rangle$	$\to |1\rangle|0\rangle$

If vertical line crosses a bit line without dot, then nothing happens to that bit eg



$|\alpha\rangle|000\rangle + |\beta\rangle|111\rangle$

Encode $|\psi\rangle$ into 3 bits "cat" state.

($|111\rangle$ or $|000\rangle$ called a cat state because it could sort of represent a Schroedinger's cat state.

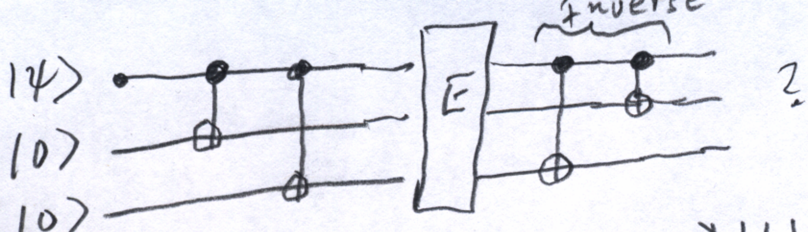
$|000\rangle \rightarrow$ atom not decayed and cyanide vial not broken and cat alive

$|111\rangle \rightarrow$ atom decayed and cyanide vial broken and cat dead.)

Now lets say that someone goes in and applies a one bit bit flip. you dont know which bit has flipped. you want to correct the flip but keeping the entangleded state

$|\alpha\rangle|000\rangle + |\beta\rangle|111\rangle \rightarrow (|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle)$(any state of ancilla)

you want the final state to again be a superposition on 1st bit producted with some (you don't care what) state of ancilla. No correlation between the bit and ancilla.



$E\ (|\alpha\rangle|000\rangle + |\beta\rangle|111\rangle)$

gives one of 4 poss. after E

No bitflipped $|\alpha\rangle|000\rangle + |\beta\rangle|111\rangle$

2nd bit flipped $|\alpha\rangle|010\rangle + |\beta\rangle|101\rangle$

3rd bit flipped $|\alpha\rangle|001\rangle + |\beta\rangle|110\rangle$

1st bit flipped $|\alpha\rangle|100\rangle + |\beta\rangle|011\rangle$

After inverse

No bit flipped $|\alpha\rangle|000\rangle + |\beta\rangle|100\rangle \Rightarrow (|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle)\underbrace{}_{|0\rangle|0\rangle}$

2nd bit flipped $|\alpha\rangle|010\rangle + |\beta\rangle|110\rangle \Rightarrow (|\alpha\rangle|0\rangle + |\beta\rangle|1\rangle)(|10\rangle)$

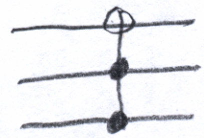
3rd bit flipped $|\alpha\rangle|001\rangle + |\beta\rangle|101\rangle = (|\alpha\rangle|0\rangle + \beta|1\rangle)(|01\rangle)$

1st bit flipped $|\alpha\rangle|111\rangle + |\beta\rangle|011\rangle = (|\alpha\rangle|1\rangle + |\beta\rangle|0\rangle)|11\rangle$

The output on the 1st three cases on the first bit is the original state $|\psi\rangle$ the 4th case it is the state with spgain flipped on $|\psi\rangle$ (ie $\sigma_x|\psi\rangle$ or NOT $|\psi\rangle$)

Thus, the output on the 1st bit is the same as the input unless the 1st bit was the one flipped. Then the output has the bit flipped. But the two ancilla also have the value $|11\rangle$

Thus if we put in a Toffoli gate (Control Control NOT)

1 

$\tilde{\Psi} = |\alpha\rangle|1\rangle + \beta\rangle|0\rangle$

Then $|\psi\rangle|00\rangle$ or $|\psi\rangle|01\rangle$ or $|\psi\rangle|10\rangle$ are the same on output as on input.

But $|\tilde{\Psi}\rangle|11\rangle \rightarrow |\psi\rangle|11\rangle$ by the Toffoli gate. At the end, the output on the 1st bit is always $|\psi\rangle$. We can now throw away the ancilla. The ancilla are not entangled with anything else in the system.

There are two other kinds of one bit changes : sign flip and both spin flip and sign flip.

Sign flip.

$|0\rangle \to |0\rangle \quad |1\rangle \to -|1\rangle \quad (\sigma_z)$

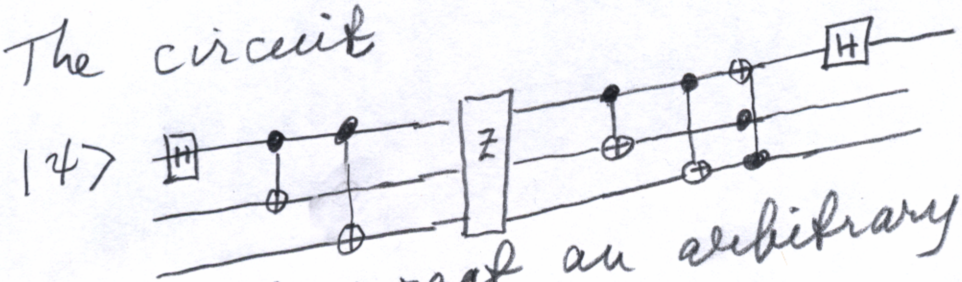
A sign flip in Z basis is a spin flip in the x basis.

The Hadamard changes from z basis to x basis.

$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |-\rangle$

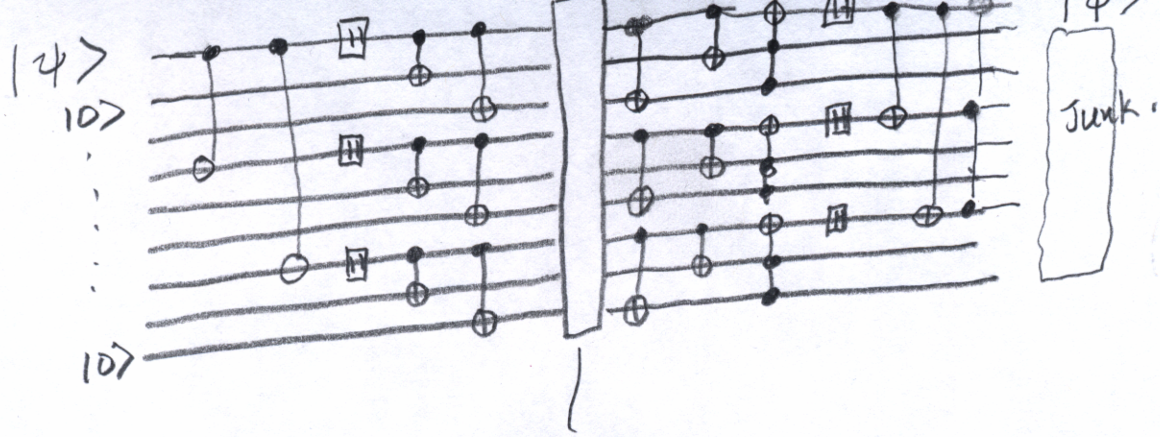
$|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |+\rangle$

The circuit



will correct an arbitrary sign flip.

To correct both, Shor used a concatenation. He used 9 bits to represent a logical bit.



X, Z
or XZ
onebit.

This circuit will correct a single Z on any line and a single X on any line.
It will thus correct also an X and a Z on any a single line.

Now if any error has a prob p of occuring on any bit, it will have a prob of p^2 of happening twice.
these processes cannot correct double errors. but their prob. is much lower.
But one can now concatenate these.

Treat each line above as a logical bit line instead of a physical bit
Thus with 81 physical lines one should be able to correct 2 bit errors. as well but at huge cost.
(81 bits).

Note: This cannot correct "Decoherence" errors.

$|e\rangle, |e'\rangle$ are environ.

Eg $|0\rangle|e\rangle \to |0\rangle|e\rangle$

$|1\rangle|e\rangle \to |0\rangle|e'\rangle$

where e' is some enviroment and $\langle e'|e\rangle = 0$. or the error is of form of a projection operator $\frac{1}{2}(\mathbb{1} + \sigma_z)$

Peres.

$$|\alpha\rangle|000\rangle + |\beta\rangle|000\rangle \to |\alpha\rangle|e\rangle|000\rangle + |\beta\rangle|e'\rangle|110\rangle$$

$$\to |\alpha\rangle|e\rangle|000\rangle + |\beta\rangle|e'\rangle|101\rangle$$

$$|\alpha\rangle|e\rangle + |\beta\rangle|e\rangle$$

$\to |\alpha\rangle|e\rangle, \beta|e'\rangle$ decohere $|\alpha\rangle, |\beta\rangle$

This will destroy the entanglement.

One cannot get at $|e\rangle$ to work on it.