$Cor = \langle A(C+D) + B(C-D) \rangle$

Both classical (with counterfactual reality) and Quantum.

A, B, C, D all ± 1 e-values.

classical : $|C+D|$ and $|C-D|$ anti correlated. $\to$ If $|C+D| = 2$ then $|C-D| = 0$ $\Longrightarrow$ $-2 \leq Cor \leq 2$

Quantum $C+D$ and $C-D$ have e-values $\pm\sqrt{2}$

choose $A = \sigma_z^1$ $B = \sigma_x^1$ $C+D = \sqrt{2}\,\sigma_z^2$ and $C-D = \sigma_x^2$

Quantum

$Cor = \sqrt{2} \langle \sigma_z^1 \sigma_z^2 + \sigma_x^1 \sigma_x^2 \rangle$

Consider $\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ e-genstate of both $\sigma_z^1 \sigma_z^2$ and of $\sigma_1^x \sigma_2^x$ with e-value $+1$ in each case Thus.

$Cor = \sqrt{2}\langle 1 + 1 \rangle = 2\sqrt{2}$

Greenberger-Horne-Zeilinger.

3 particles all 2 level systems.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right)$$ Cat state

$$\sigma_x|\uparrow\rangle = |\downarrow\rangle \quad \sigma_x|\downarrow\rangle = |\uparrow\rangle$$

$$\sigma_x^1\sigma_x^2\sigma_x^3|\psi\rangle = |\psi\rangle$$

$$\sigma_y|\uparrow\rangle = i|\downarrow\rangle \quad \sigma_y|\downarrow\rangle = -i|\uparrow\rangle$$

$$\sigma_x^1\sigma_y^2\sigma_3^y|\psi\rangle = i^2\left(|\downarrow\downarrow\downarrow\rangle + (-1)^2(|\uparrow\uparrow\uparrow\rangle)\right)$$

$$= -|\psi\rangle$$

$$\sigma_y^1\sigma_x^2\sigma_y^3|\psi\rangle = \sigma_y^1\sigma_y^2\sigma_y^3|\psi\rangle = -|\psi\rangle$$

If counterfactual values for all of σ_x^i, σ_y^i Then with values $s_y^i = \pm 1$

$$\sigma_x^1\sigma_y^2\sigma_z^3 \rightarrow s_x^1 s_y^2 s_z^3 = -1$$

$$\underbrace{(s_x^1 s_y^2 s_y^3)(s_y^1 s_x^2 s_y^3)(s_y^1 s_y^2 s_x^3)} = -1$$

But s $\quad s_x^1 s_x^2 s_x^3 = -1$

But $\sigma_x^1\sigma_x^2\sigma_x^3|\psi\rangle = +1|\psi\rangle$

$-1 \neq +1$

Counterfactual values cannot exist.

Hardy Chain.

2 systems A, B ; X, Y 2 level.

state : $\sin\theta|\uparrow\uparrow\rangle + \cos\theta|\downarrow\rangle(\sin\phi|\uparrow\rangle + \cos\phi|\downarrow\rangle)$

4 statements.

a → If ~~a~~ A measured and has value +1 then X measured to have value +1

x – If X meas to have value +1 then B measured to have value +1

ß → If B measured to have have value +1 then D meas to have value +1

y → If A meas to have value +1 then D measured to have value +1

a true always., X true always

ß true always.

y true almost never.

$A = |\uparrow_1\rangle\langle\uparrow_1|$ e-values 1, 0

$\frac{1}{2}(\sigma_z^1 + 1)$

$X = |\uparrow_2\rangle\langle\uparrow_2|$

$$B = \frac{(\sin\theta|\uparrow_1\rangle + \sin\phi\cos\theta|\downarrow_1\rangle)(\sin\theta\langle\uparrow_1| + \sin\phi\cos\theta\langle\downarrow_1|)}{\sin^2\theta + \sin^2\phi\cos^2\theta}$$

$$Y = \frac{(\sin^2\theta|\uparrow_2\rangle + \sin^2\phi\cos^2\theta|\uparrow_2\rangle + \cos^2\theta\sin\phi\cos\phi|\downarrow_2\rangle)(\sin^2\theta + \cos^2\theta\sin^2\phi)\langle\uparrow_2| + \cos^2\theta\sin\phi\cos\phi\langle\downarrow_2|)}{N}$$

If θ small, ϕ small, $\cos\theta = \cos\phi \simeq 1$, $\sin\theta \simeq \theta$, $\sin\phi \simeq \phi$

$$Y = \frac{((\theta^2+\phi^2)|\uparrow_2\rangle + \phi|\downarrow_2\rangle)((\theta^2+\phi^2)\langle\uparrow_2| + \phi\langle\downarrow_2|)}{(\theta^2+\phi^2)^2 + \phi^2}$$

choose $\theta = \phi$

$$Y \approx \frac{(2\phi^2|\uparrow_2\rangle + \phi|\downarrow_2\rangle)((\theta^2+\phi^2)\langle\uparrow_2| + \phi\langle\downarrow_2|)}{\phi^2}$$

$$\approx (2\phi|\uparrow_2\rangle + |\downarrow\rangle)(2\phi|\uparrow_2\rangle + |\downarrow\rangle)$$

If $A=1$ the Prob $Y=1$ is

$\text{Prob} = |(2\phi\langle\uparrow_2| + \langle\downarrow_2|)(|\uparrow_2\rangle)|^2 \approx 4\phi^2$

small ϕ small prob.

But if small ϕ

$$|\psi\rangle = \phi|\uparrow\uparrow\rangle + (\phi|\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$= |\downarrow\downarrow\rangle + \phi(|\uparrow\uparrow\rangle + |\downarrow\uparrow\rangle)$$

small entanglement. If $\phi = 0$

$|\psi\rangle = |\downarrow\downarrow\rangle$ No entanglement.

In limit $\phi \to 0$ no entang.

In limit $\phi \to 0$

Prob ( ) $\to 4\phi^2 \to 0$.

$$a \Rightarrow X \Rightarrow B \Rightarrow Y$$

Quantum Mech differs from classical Mech more the less entangled the state is

As $\theta = 0$ Prob $A=1$ goes to 0. (as θ^2)

As no entanglement $A \Rightarrow 1$ goes to 0

Prob $Y \to 0$.

But $\lim_{\theta, \phi \to 0}$ Prob Y

Violation of Classical Logic is greatest when state is not entangled.

(Violation of Bell's ineq. or GHZ max when state max entangled).

It is not entanglement that makes Q.M. $\neq$ Classical.