Physics 501
Midterm Exam
Feb 26 2020

This exam is 1hr 15 min (75 min) in length. It is closed book.

On your answer sheet, please include a statement that you have not and will not receive help in answering any of the question from any other person, and that you have not and will not read any material, whether in books or online to help you answer these questions, and sign that declaration.

This exam has 5 questions, all of equal value.

When the exam has been marked, and handed back to you, you will have the option of redoing any of the questions on which you did not get full marks and handing the result in for remarking. As your midterm mark you will get the average of the mark you received on the midterm writing itself and the mark you get on the redo of the question, but in no case will you get less than that the mark you received on the midterm marking itself. This "re-do" will be due 1 week after the results of the marking of the original midterm are sent back to you, with no extention of that time. You may discuss this redo with others, and use any notes, text-books, etc. in doing so, except you may not simply copy someone else's solution(s).

1) Consider the sigma matrix

$$\sigma_\theta = cos(theta)\sigma_z + sin(\theta)\sigma_x \tag{1}$$

and the Hamiltonian for this system of

$$H = 0 \tag{2}$$

At 9AM σ_z was measured and found to have a value of +1. At 11AM σ_x was measured and found to have a value of +1. At 10AM σ_θ was measured . What is the probability that the value of +1 was found this measurement,as a function of θ given the above conditions.

2. Consider the free particle Hamiltonian

$$H = \frac{1}{2}p^2 \tag{3}$$

Consider the complex mode solution

$$x = 1 - it \tag{4}$$

a)What is p for this mode?

b)What is the norm for this mode?

c)What is the annihilation operator corresponding to this solution in terms of the momentum and position operators at time $t = 0$ and what is the annihilation operator in terms of the momentum and position operators at time t?

3) Give the argument from Hardy's chain argument, that classical Mechanics cannot mimic the results from a quantum system.a Go into some detail.

4)Show that the unitary matrix

$$U = e^{i(\alpha P + \beta X)} \tag{5}$$

is a translation operator on both X and P, where P and X are the usual momentum and position operators. What linear combination of the operators X and P does this operator not change at all?

5)Consider the Lagrangian for a time dependent oscillator of

$$L = \frac{1}{2} t^2 \left((\partial_t X)^2 - X^2 \right) \tag{6}$$

a) What is the (time dependent) Hamiltonian for this problem.

b) Doing one round of the assymptotic adiabatic transformation, what are $\hat{X}$, $\hat{P}$, τ in terms of $X,\ P,\ t$? What is $\hat{H}$ in terms of $\hat{X}$, $\hat{P}$, τ?

c) Using the annihilation operator expressed in terms of $X(t),\ P(t)$, and the representation of $X(t)\psi(t,x) = x\psi(t,x)$, $P(t)\psi(t,x) = -i\partial_x \pi(t,x)$, find the function $\psi(t,x)$ which is the $||\rangle\, 0 >$ state for the above annihilation operator in part b). [This is the Schroedinger representation of that state, which is, or course, time dependent.)