Physics 501-21
Assignment 5

1) Show that to first order in ϵ, a phase fluctuation does not alter the pressure exerted by an electromagnetic file on a mirror, while an amplitude fluctuation does.

Assume that the background incoming field is

$$\mathfrak{A}\sin(\omega_0(t-x)) \tag{1}$$

The phase fluctuation is

$$\mathfrak{A}(\sin(\omega_0(t-x)+\epsilon cos(\hat\omega(t-x)) \tag{2}$$

Assume that$\hat\omega << \omega_0$). while an amplitude fluctuation is

$$\mathfrak{A}((1-\epsilon\cos(\hat\omega(t-x))sin(\omega_0(t-x)) \tag{3}$$

You can ignore terms in the pressure with frequencies of order ω_0 or larger.

2)A generalised squeezed state is the "vacuum" state for two modes where the annihilation operators with annihilation operators $A,\ B$ where the new annihilation operators are

$$\tilde A = e^{i\phi}\cosh(\theta)A + e^{-i\phi}\sinh(\theta)B^\dagger \tag{4}$$

$$\tilde B = e^{i\phi}\cosh(\theta)B + e^{-i\phi}\sinh(\theta)A^\dagger \tag{5}$$

Show that these operators obey the commutation relations for independent annihilation and creation operators if $A,\ B$ do. What is the vacuum state for $\tilde A,\ \tilde B$ in terms of the vacuum state for $A,\ B$?

What phase ϕ would you choose to minimize the expectation value of $(e^{i\delta}A+e^{-i\delta}B^\dagger)^2$ in the $\tilde A,\ \tilde B$ vacuum state.

3.) Show that the state for the light bouncing off a "free" mirror (equation of motion in absence of an electromagnetic field is $\partial_t^2 X=0$) converts vacuum state for the incoming field into a squeezed state for the outgoing field. The boundary condition for the field at the mirror is that $\phi(t,x)=0$. To simplify the system, assume that the incoming field is

$$e^{i\omega_0(t-x)}(\mathfrak{A} + Ae^{i\hat\omega(t-x)} + Be^{-i\hat\omega(t-x)} + \mathbf{HC} \tag{6}$$

(Ie, there are just two frequencies $\omega_0\pm\omega$ in the field with their annihilation operators.) for a single frequency $\hat\omega$. $\mathfrak{A}$ is a c-number, while A and B are annihilation operators. Show that for the beam leaving the mirror, the original vacuum state for $A,\ B$ is a squeezed state for the operators leaving the mirror. $\omega << \omega_0$.

You can assume that $X(t)$ is always very small, and that ω is the same order as the time variation of $X(t)$ (Ie, also that one can neglect any oscillations on X of order ω_0 or higher