QM5 solutions.

1) Ah well. I am ~~just~~ wrong in this question. It is much smaller not zero.

Incoming $A\sin(\omega(t-x)+\epsilon\cos\hat\omega(t-x))$

Boundary cond at $x=0$

$$\frac{\partial\phi}{\partial x}=0$$

Full soln. (solves $\frac{\partial\phi}{\partial x}(t,0)=0$)

$$A\left(\sin(\omega(t-x)+\epsilon\cos\hat\omega(t-x))\right.$$
$$\left.+\sin(\omega(t+x)+\epsilon\cos\hat\omega(t+x))\right)$$

To lowest order in ϵ this is

$$A\left(\sin(\omega(t-x))+\epsilon\cos(\omega(t-x))\cos\hat\omega(t-x)\right.$$
$$\left.+\sin(\omega(t+x))+\epsilon\cos\omega(t+x)\cos\hat\omega(t+x)\right)$$

the pressure at $x=0$ is prop to

$$P\propto\left.\left(\frac{\partial\phi}{\partial t}\right)^2\right|_{x=0}\quad\left(\text{since } \frac{\partial\phi}{\partial x}=0\right)$$

so

$$P\propto A^2\left(2\omega\cos\omega(t)+\epsilon\left(-\omega\sin\omega t\cos\hat\omega(t)-\hat\omega\cos\omega t\sin\hat\omega(t)\right)\right)^2$$

$$\approx 4A^2\left(\omega^2\cos^2\omega(t)-2\epsilon\omega^2\cos\omega t\sin\omega t\cos\hat\omega t-2\epsilon\omega\hat\omega\cos^2\omega t\sin\hat\omega(t)\right)$$

$$\approx 4A^2\left[\frac{\omega^2}{2}(1+\cos 2\omega t)-\epsilon\omega^2\sin 2\omega t\cos\hat\omega t-\epsilon\omega\hat\omega(1+\cos 2\omega t)\sin\hat\omega t\right]$$

In the limit where $\omega\gg\hat\omega$ and we assume the system is insensitive to high freq.

$$P\propto A^2\left(\frac{\omega^2}{2}-\epsilon\omega\hat\omega\sin\hat\omega t\right)$$

Ie it does depend on ϵ.

For amplitude modulation

$$\phi = A\Big(\sin\omega(t-x)\,(1+\epsilon\cos\hat{\omega}(t-x))$$
$$+\sin\omega(t+x)\,(1+\epsilon\cos\hat{\omega}(t+x))$$
$$+\sin\omega(t+x)\,(1+\epsilon\cos\hat{\omega}(t+x))\Big)$$

$$P \propto A^2\Big[2\omega\cos\omega(t)\,\cancel{x}\,(1+\epsilon\cos\hat{\omega}(t))\Big]^2$$
$$+\,2\epsilon\ \sin\omega(t+x)\ \epsilon\hat{\omega}\,(-\sin\hat{\omega}(t))$$

$$= 4A^2\Big[\omega^2\cos^2\omega(t)$$
$$+\,2\epsilon\big[\cos\omega t^2\cos\hat{\omega}(t)\,\omega^2$$
$$-\omega\cos\omega t\sin\omega t\ \hat{\omega}\sin\hat{\omega}t\big]$$

$$\approx 4A^2\Big[\frac{\omega^2}{2} + \frac{2\epsilon\omega^2}{2}\cos\hat{\omega}t\Big]$$

Thus, although the ~~Am~~ Phase modulation force is much smaller than the amp modulation force (by a factor of $\frac{\hat{\omega}}{\omega}$) the phase modulation pressure is not zero.

QM 2-1

2). $[\tilde{A}, \tilde{A}^+] = \cosh^2(\theta)[A, A^+] + \sinh^2\theta\,([B^+, B])$

$= \cosh^2(\theta) - \sinh^2\theta = 1$

and sim for $[\tilde{B}, B^+]$

Also $[\tilde{A}, \tilde{B}^+] = 0$ (since both are linear combinations of A and B^+ which commute)

$[\tilde{A}, \tilde{B}] = \cosh\theta\sinh\theta\,[A, A^+]$

$+ \cosh\theta\sinh\theta\,[B^+, B]$

$= 0$

as required

$\tilde{A}|\tilde{0}\rangle = \tilde{B}|\tilde{0}\rangle = 0$

so $e^{i\phi}\cosh\theta\, A + e^{-i\phi}\sinh\theta\, B^+ |\tilde{0}\rangle = 0$

and $e^{i\theta}\cosh\theta\, B + e^{-i\phi}\sinh\theta\, A^+ |\tilde{0}\rangle = 0$

So formally. $A = \frac{\partial}{\partial A^+}$ $\quad B = \frac{\partial}{\partial B^+}$

so $\frac{\partial}{\partial A^+} + e^{-2i\phi}\tanh\theta\, B^+ |\tilde{0}\rangle = 0$

$\frac{\partial}{\partial B^+} + e^{-2i\phi}\tanh\theta\, A^+ |\tilde{0}\rangle = 0$

$|\tilde{0}\rangle = f(A^+, B^+)|0\rangle$

$f = e^{-e^{-2i\phi}\tanh\theta\, A^+B^+}$ ~~pos~~

$$A = e^{-i\phi}(\cosh\theta\,\tilde{A} - \sinh\theta\,\tilde{B}^+)$$
$$B^+ = e^{i\phi}(\cosh\theta\,B^+ - \sinh\theta\,A)$$

$$Q = e^{i\delta}A + e^{-i\delta}B^+$$
$$= e^{i(\delta-\phi)}(\cosh\theta\,\tilde{A} - \sinh\theta\,\tilde{B}^+)$$
$$+ e^{-i(\delta-\phi)}(\cosh\theta\,\tilde{B}^+ - \sinh\theta\,\tilde{A})$$
$$= ((\cos(\delta-\phi)\,e^{-\theta} + i\sin(\delta-\theta)\,e^{\theta})\,\tilde{A}$$
$$+ \cos(\delta-\phi)\,e^{-\theta} - i\sin(\delta-\theta)\,e^{\theta})\,\tilde{B}^+)$$

clearly $\delta = \phi$ the eqn is smallest for as it goes as $e^{-\theta}$.

However $\tilde{A}|\tilde{0}\rangle = \langle\tilde{0}|\tilde{B}^+ = 0$

so the square expectation value is zero for all values of δ.

If instead I had asked for $\langle 0|Q^+Q|\rangle$ I would have had its expectation value to be proportional do $e^{-2\theta}$

The larger the squeezing the smaller the ~~no~~ "noise"

3) This question is a disaster. - ie it is hard to figure out, even for me, if it makes any sense. I e, do not include in max mark but it could give bonus points.

One interpretation might be to simply regard it as going through the in class derivation but with the boundary condition at the mirror being $\Phi = 0$ rather than $\frac{\partial \Phi}{\partial x} = 0$ and $\frac{\partial \psi}{\partial y} = 0$ at the mirror.

Then almost everything goes through as before except

$$\phi_{in}(t-x) + \phi_{out}(t+x) = 0 \text{ at}$$

$x = X(t)$ ie

$$\phi_{in}(t - X(t)) + \phi_{out}(t + X(t)) = 0$$

$$\text{or } \phi_{out}(t) \approx -\phi_{in}(t - 2X(t) + x)$$

(note we have neglected extra terms like $\phi_{in}(t - 2X(t-\cancel{X(t)}) + x)$ by assuming $X(t)$ small and slowly varying w.r.t. Φ.

Ie, ϕ_{out} is $\cancel{\phi_{in}}$ ϕ_{in} rather than
$\phi_{out} = \phi_{in}$ in case of $\frac{\partial \phi}{\partial \cancel{t}} = 0$.
~~Note~~ Then the pressure is
$P = (\partial_t \phi)^2 + (\partial_x \phi)^2 \Big|_{x=X}$, since $\phi(t, X(t)) = 0$
then $\partial_t \phi(t, X) = 0$. and the pressure
comes from the $(\partial_x \phi)^2$ term.
But $\partial_x \phi_{out} = \partial_t \phi_{out}$ and
$\partial_x \phi_{in} = -\partial_t \phi_{in}$ (function of $t-x$)
this says the pressure is again
the sum of $(\partial_t \phi_{in} - \partial_t \phi_{out})^2$
$\simeq$ ~~∂_x~~ which is the pressure if
ϕ obeyed the $\partial_x \phi = 0$ boundary
cond. Ie the result for this
alternative bndy cond. is the same
for both possible bndry cond.