Physics 501-20
Assignment 1

1.) Consider the Hamiltonian

$$H = \frac{1}{2}(\frac{1}{m}P^2 + m(\Omega^2 + \epsilon\delta(t-1))Q^2) \tag{1}$$

Assume that at $t = 0$ the operators P, Q are given by $P = P_0, \quad Q = Q_0$.

Assume that the intial state of this quantum system is the lowest energy state at time t=0 (the ground state).

a)Solve the Heisenberg representation equations for the operators $P(t),\ Q(t)$ for this system at arbitrary times t.

b)Define the Annihilation operators $A_0 = \frac{1}{\sqrt{2}}(\sqrt{m\Omega}Q_0 + \frac{i}{\sqrt{m\Omega}}P_0)$. Show that $[A_0, A_0^\dagger] = 1$ and that $H(0) = \frac{\Omega}{2}A_0A_0^\dagger + A_0^\dagger A_0$. The minimum energy state $|0\rangle$ will therefor be given by

$$A_0 |0\rangle = 0 \tag{2}$$

Given the solution of the Heisenberg equations of motion, write the solutions for $P(t), Q(t)$ in terms of $A_0, \quad A_0^\dagger$.

c) Find the expectation value of the energy H in the state $|0\rangle$ as a funtion of time except at $t = 1$.

d)Solve the Schroedinger equation for this problem with the same initial conditions, and explicitly find the expectation value of the energy as a function of time. (If necessary, solve this to lowest non-trivial order in ϵ. Note that the Heisenberg equations can be solved exactly to all orders in ϵ).

Which is easier?

Note the state of the system after the t=1 is called a squeezed state, which has become a very powerful took in quantum optics in increasing the sensitivity of certain detectors beyond what one might naively call the quantum limit. We will look at this later in the course.