Addm notes on Adiabatic Thm.

$$i\partial_t|\psi,t\rangle = H(t)|\psi,t\rangle$$

Define

$$H(t)\,|n,t\rangle = E_n(t)\,|n,t\rangle$$

Let $|\psi,t\rangle = \sum_n c_n(t)\,e^{-i\int^t E_n(t')dt'}\,|n,t\rangle$

Then $i\partial_t|\psi,t\rangle = \sum_n\Big[(i\partial_t c_n(t) + E_n(t)c_n)|n,t\rangle$
$$+\, i\,c_n(t)\,\partial_t|n,t\rangle\Big]e^{-i\int^t E_n dt'}$$

So $0 = i\partial_t|\psi,t\rangle - H(t)|\psi,t\rangle$
$$= \sum_n\left(i\,\partial_t c_n|n\rangle + i\,c_n\,\partial_t|n\rangle\right)e^{-i\int^t E_n dt'}$$

Take inner product wrt $|m\rangle$

$$0 = \partial_t c_m|n\rangle + c_m\langle m|\partial_t|m\rangle + \sum_{n\neq m}\langle m|\partial_t|n\rangle\,e^{i\int^t(E_m-E_n)dt'}$$

$\langle m|\partial_t|m\rangle$ is related to Berry phase

$$\gamma(t) = \int^t\langle m,t'|\partial_{t'}|m,t'\rangle\,dt'.$$

Since $\langle m,t'|m,t'\rangle = 1$, $(\partial_t\langle m,t'|)|m,t'\rangle$
$$= -\langle m,t'|\partial_{t'}|m,t'\rangle$$

But $(\partial_t\langle m,t'|)|m,t'\rangle = [\langle m,t'|\partial_{t'}|m,t'\rangle]^*$

so γ is pure imaginary. (ie a phase)

$H(t)|n,t\rangle = E_n(t)\,|n,t\rangle$ so $\partial_t H\,|n\rangle + H\,\partial_t|n\rangle$
$$= \partial_t E_n|n\rangle + E_n\partial_t|n\rangle$$

$m\neq n \Rightarrow \langle m|\partial_t H|n\rangle + E_m\langle m|\partial_t|n\rangle = E_n\langle m|\partial_t|n\rangle$

$$\text{or } \langle m|\partial_t|n\rangle = \frac{\langle m|\partial_t H|n\rangle}{(E_n - E_m)}$$

Berry phase: If $H(t) = H(\vec{\theta}(t))$

(ie H changes in time because parameters change in time) $\vec{\theta} \to$ vector $(\theta_1, \theta_2, \ldots, \theta_N)$

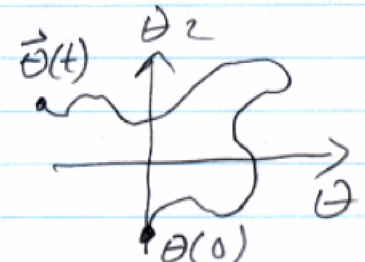
Then $|n, t\rangle = |n, \vec{\theta}(t)\rangle$

$$\partial_t |n,t\rangle = \sum_i \partial_{\theta_i} |n, \theta\rangle \frac{d\theta_i}{dt}$$

$$= \nabla_{\vec{\theta}} |n, \theta\rangle \cdot \frac{d\vec{\theta}}{dt}$$

$$\int^t \langle n, \vec{\theta} | \nabla_{\vec{\theta}} | n, \vec{\theta}\rangle \cdot \frac{d\theta}{dt}\, dt$$

$$= \int_{\text{Path}} \langle n, \vec{\theta} | \vec{\nabla}_{\vec{\theta}} | n, \vec{\theta}\rangle \cdot d\vec{\theta}$$



Depends only on path in $\vec{\theta}$ space, not on function of time.

If $|n, \vec{\theta}\rangle \to |n, \vec{\theta}\rangle = e^{i\phi(\vec{\theta})} \widetilde{|n, \vec{\theta}\rangle}$

$$\langle n, \vec{\theta} | \vec{\nabla}_{\theta} | n, \vec{\theta}\rangle = \widetilde{\langle n, \theta|} \vec{\nabla}_{\theta} \widetilde{|n, \vec{\theta}\rangle} + i \vec{\nabla}_{\theta} \phi$$

Like a gauge transf.

If path is closed path $(\vec{\theta}(T) = \vec{\theta}(0))$

then $\int_0^T i \vec{\nabla}_{\theta} \phi(\vec{\theta}(t)) \cdot \frac{d\vec{\theta}}{dt}\, dt = 0$

So $\int_0^T \langle n, \vec{\theta} | \partial_t | n, \vec{\theta}\rangle\, dt = \int \widetilde{\langle n, \vec{\theta}|} \partial_t \widetilde{|n, \vec{\theta}\rangle}\, dt$

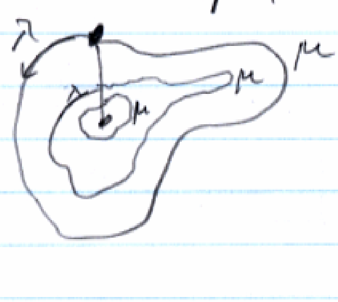
The integral over closed curve in independent of the extra phases. express basis vectors $|n, \vec{\theta}\rangle$ in terms of. This extra phase is also called Berry's phase. It depends on path in parameter space but not on extra phases of basis functions.

Stokes Thm.

$$\int_{\substack{\text{closed}\\ \text{curve}}} \vec{V} \cdot \frac{d\vec{\theta}}{d\lambda}\, d\lambda = \iint \nabla_i V_j$$

$$= \int \left(\nabla_i V_j - \nabla_j V_i\right) \frac{d\theta_i}{d\lambda} \frac{d\theta_j}{d\mu}\, d\lambda\, d\mu$$

where λ, μ are coords covering the interior surface.



"Coordinates λ, μ are arbitrary.