QUANTUM ERROR Correction.

Logical bit ⟶ Many physical bits.

$1 \Rightarrow 111$ $\quad 0 \rightarrow 000$ Majority rules

Measure bits, findout which value is in majority + flip back the minority bit.

5bits 11111. Measurement destroys superposition in Q.Case.

$|\psi\rangle \Rightarrow |\psi\rangle|\psi\rangle|\psi\rangle$

$|\psi\rangle \Rightarrow |\psi\rangle|\psi\rangle$

$\alpha|1\rangle + \beta|0\rangle \Rightarrow \alpha^2|1\rangle|1\rangle + \alpha\beta(|1\rangle|0\rangle + |0\rangle|1\rangle)$
$+ \beta^2(|0\rangle|0\rangle)$.

Q.M is linear. (Unitary ops.)

$U(|\psi\rangle + |\phi\rangle) = U|\psi\rangle + U|\phi\rangle$

No cloning Thm.

$\left.\begin{array}{l} |1\rangle|0\rangle \Rightarrow |1\rangle|1\rangle \\ |0\rangle|0\rangle \Rightarrow |0\rangle|0\rangle \end{array}\right\}$ Unitary.

CNOT $\rightarrow$ Bit flip second if 1st bit is $|1\rangle$

$$|00\rangle \Rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

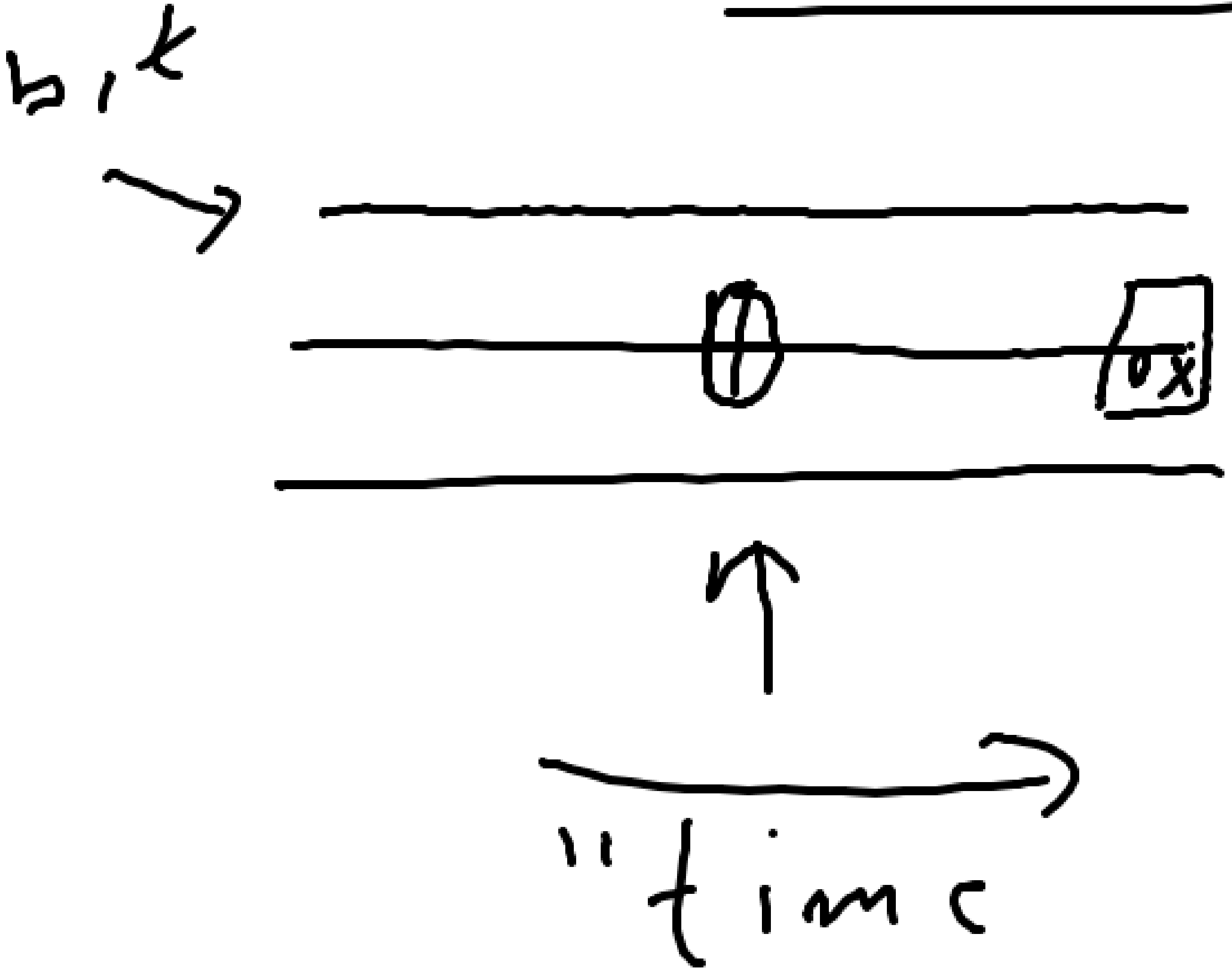
CNOT or XOR.

Own Inverses.

$$|\psi\rangle|0\rangle = (\alpha|0\rangle + \beta|1\rangle)|0\rangle$$
$$= \alpha\underbrace{|00\rangle} + \beta\underbrace{|11\rangle}$$

$\underline{|nnnnnn\rangle}$ — CAT STATES.

NOTATION



NOT.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_x \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \to \begin{matrix} \beta \\ \alpha \end{matrix}$$

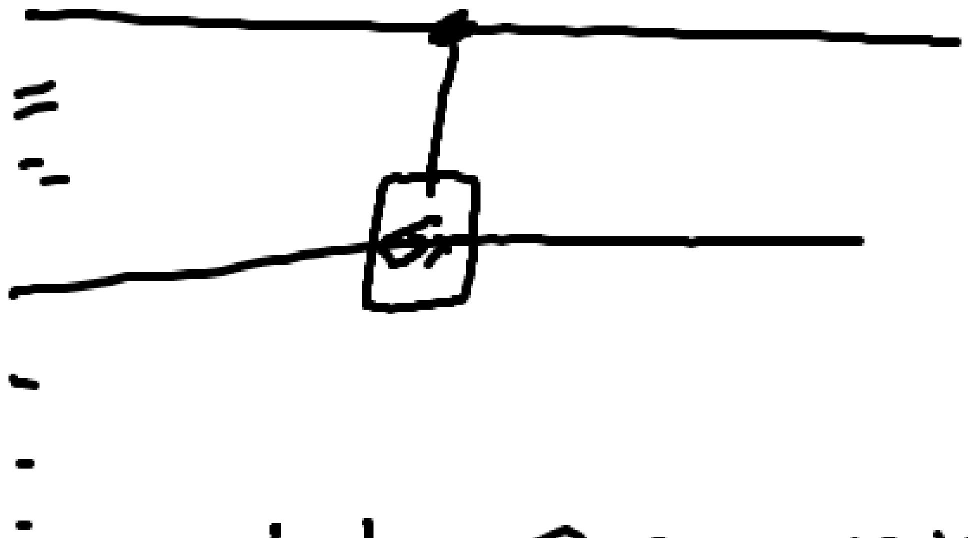
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \to \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$$

$$\sigma_z \sigma_x = i\sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$|0\rangle \to |0\rangle$

$|1\rangle \to -|1\rangle$

2 bit ops. → control opn.

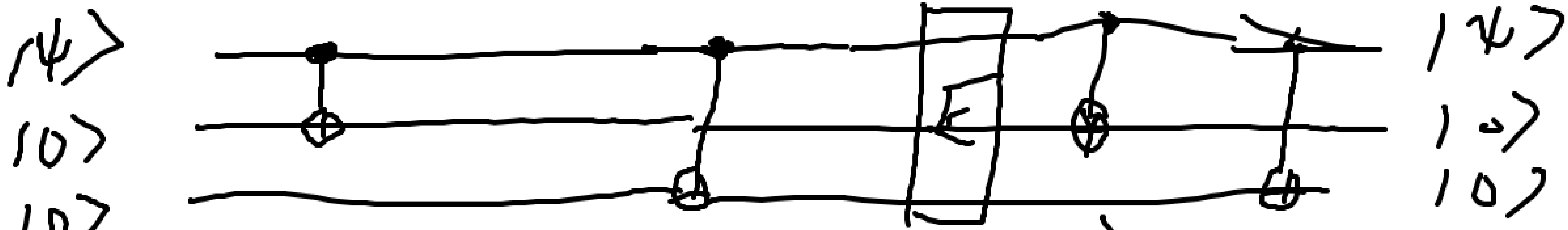


$C\sigma_x$, (CX) $(CNOT)$

(XOR)

HADAMARD $\frac{1}{\sqrt{2}}(\sigma_z + \sigma_x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

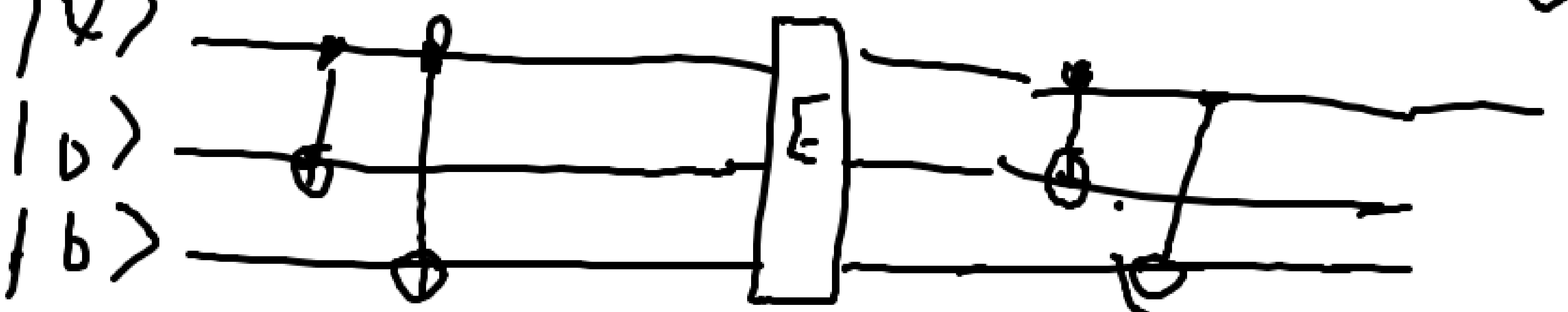
"Copying"



$\alpha|000\rangle+\beta|110\rangle$ $\alpha|000\rangle+\beta|111\rangle$

E is a 1 bit unitary op

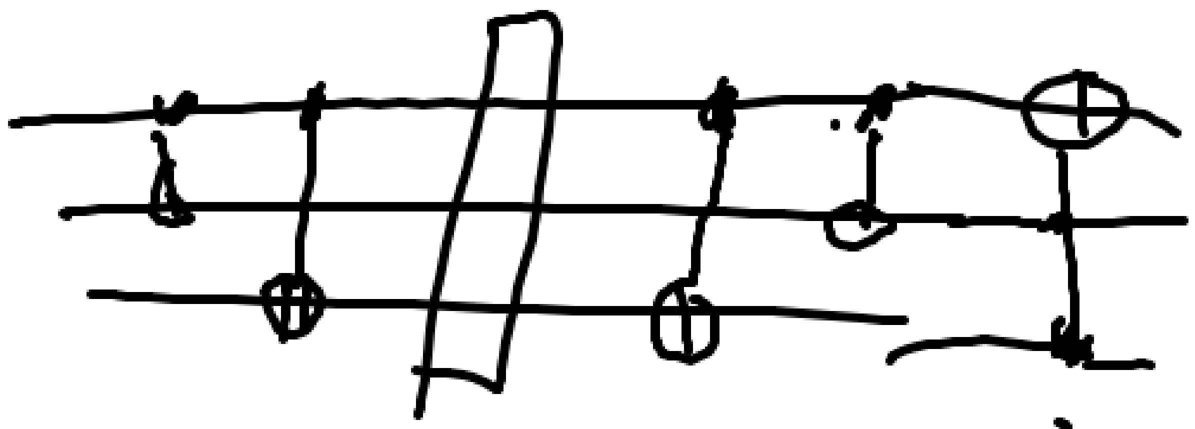
bit flip.



$\alpha|0,10\rangle$ $\alpha|010\rangle$

$\beta|11,01\rangle$ $\beta|111\rangle$

0 flip	2 flip	3 flip	1 flip
$\lvert\psi\rangle$	$\lvert\psi\rangle$	$\lvert\psi\rangle$	$\lvert\bar{\psi}\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$



$|\psi\rangle$ $|0$

$|0\rangle\ |1\rangle\ |0\ \ |1\rangle$

$|0\rangle\ |0\rangle\ |1\rangle\ |1\rangle$

Toffoli gate.

2 ancilla bits uncorrelated with 1st bit $\longrightarrow$

α, β $\quad \underbrace{\alpha = |\phi_1\rangle \quad \beta = |\phi_2\rangle}_{\text{states of other bits in system.}}$

Corrects single bit bit flips.

Sign flip.

$|0\rangle \to |0\rangle$

$|1\rangle \to -|1\rangle$

$\alpha|0\rangle + \beta|1\rangle$

$\to \alpha|0\rangle - \beta|1\rangle$

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Sign flip $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|+\rangle$

$= |-\rangle$

Sign flip in z basis $\Longrightarrow$ spin flip in x basis

$$\alpha|0\rangle + \beta|1\rangle \Longrightarrow \alpha|-\rangle + \beta|+\rangle$$

Hadamard.

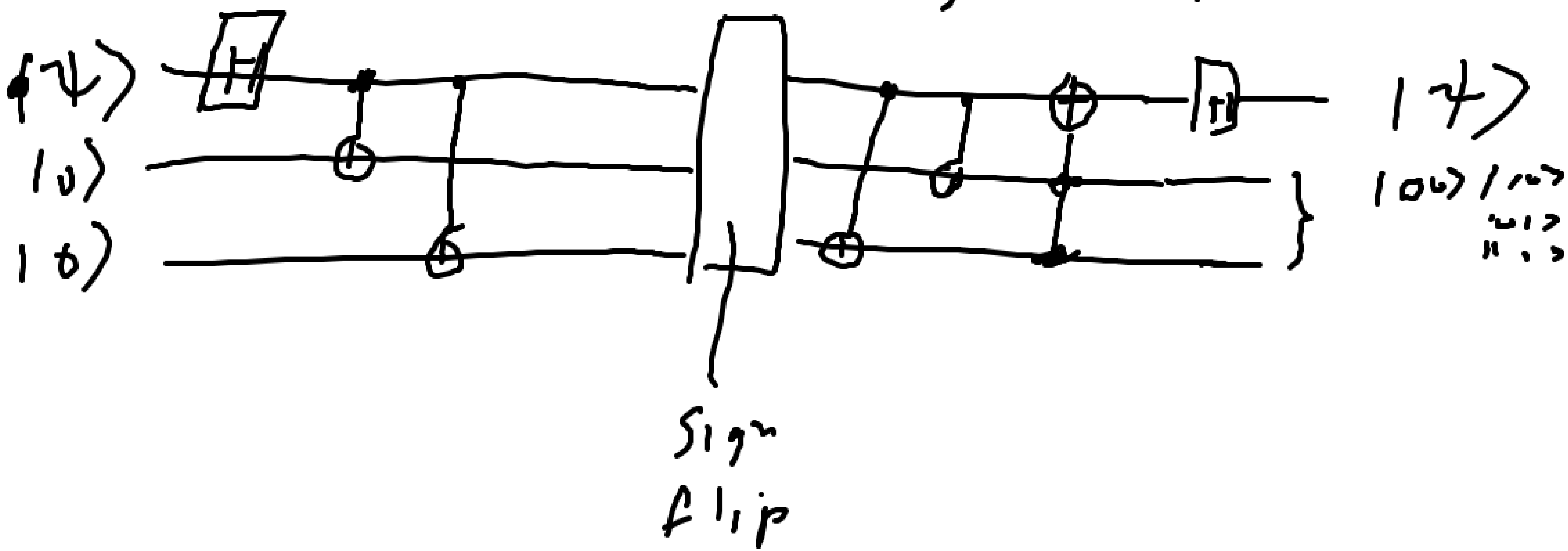
$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

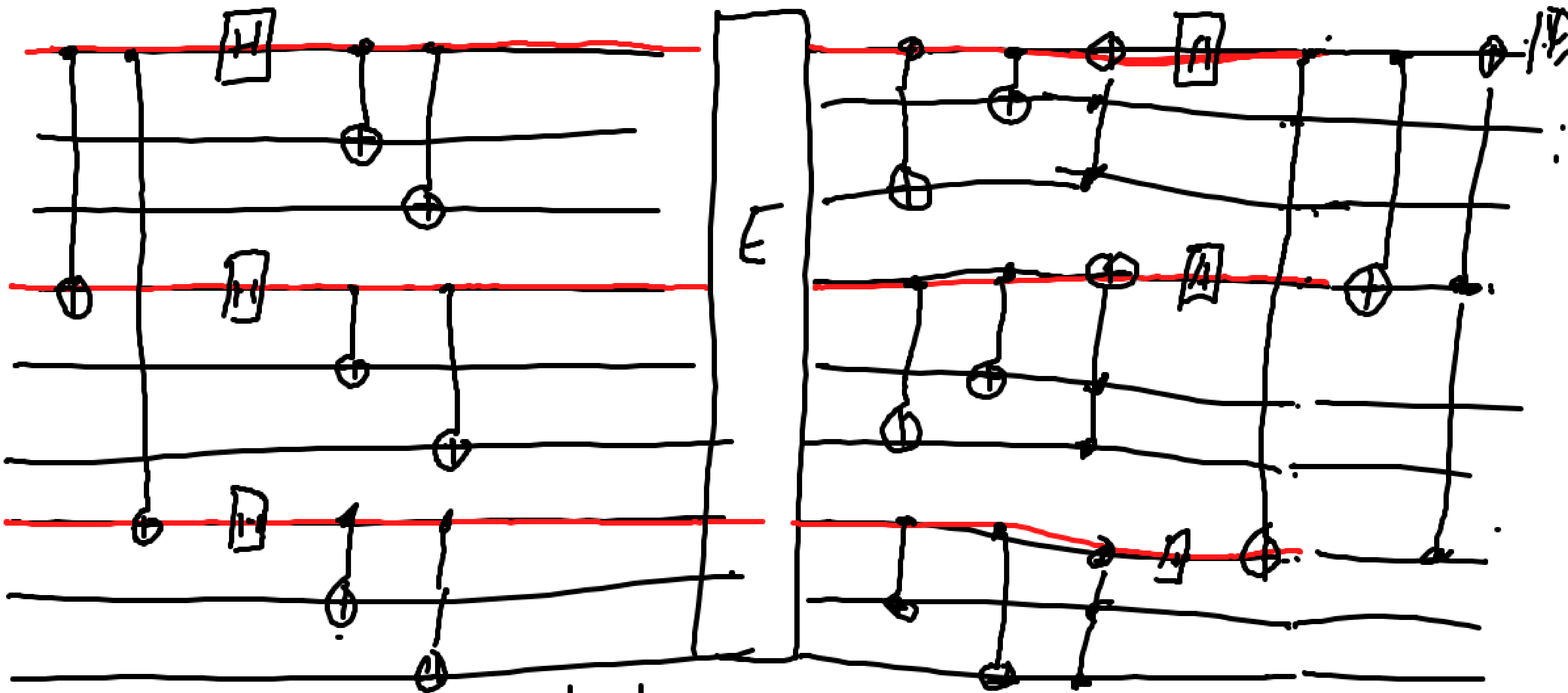
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|-\rangle + \beta|+\rangle$$

sign flip in + basis.

Corrects any single sign flip error.



Shor Error Corr.



Corrects arbitrary sign + spin flips on single bits.

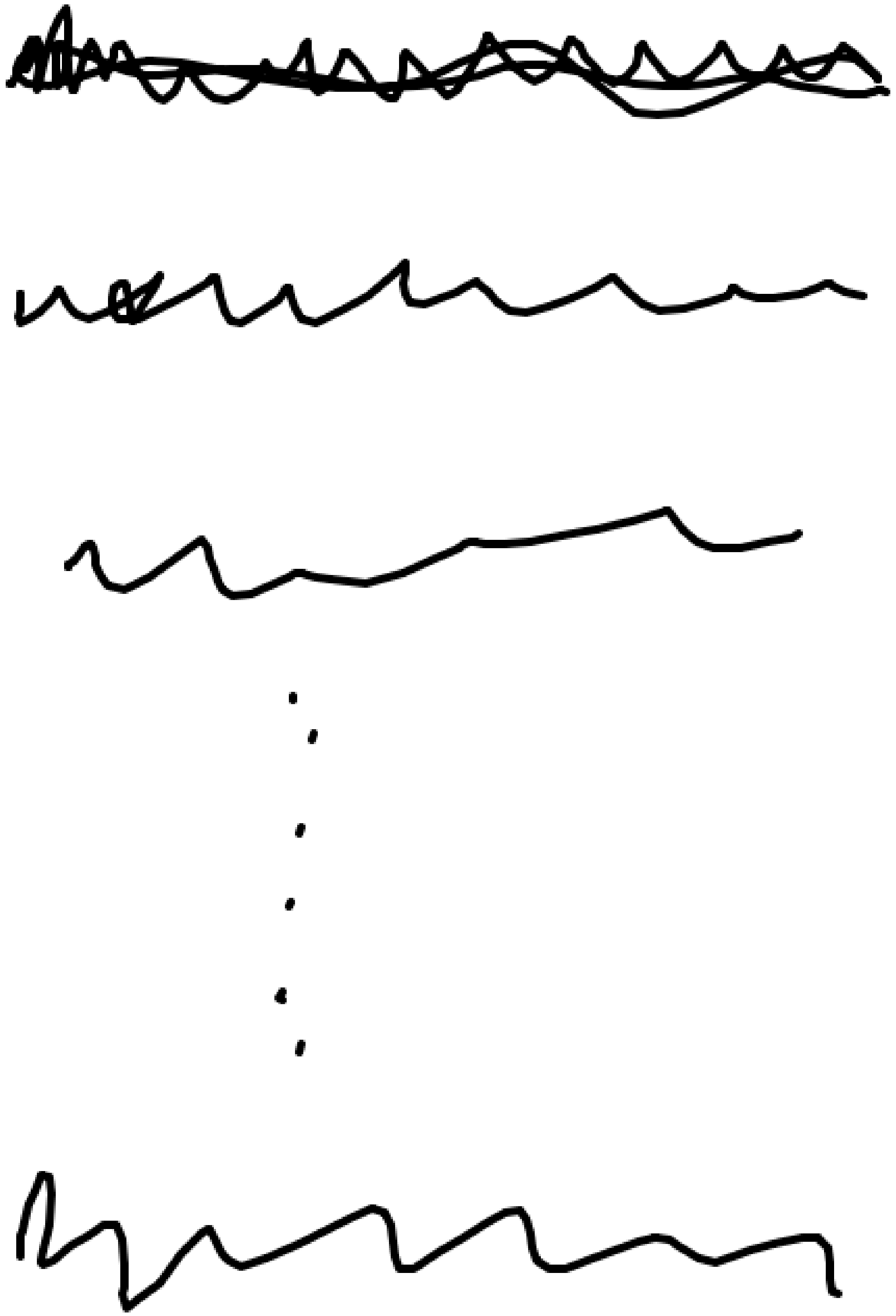
sign + spin-flip on single bit

This process corrects arbitrary unitary 1bit errors.

$$|1\rangle \to |0\rangle \quad \text{Non Unitary error.}$$
$$|0\rangle \to |0\rangle$$

Expensive – Need 9 bits (physical) for each logical bit.

9 phys. bits.



8 phys bits per logical bit

1996 Laflamme : Showed that
5 physical bit is smallest
needed to error correct
1 bit errors. with example

showed that one could make
the calc. error tolerant.

$|0\rangle \to .995|1\rangle + .1|0\rangle$

Again corrected if prob small