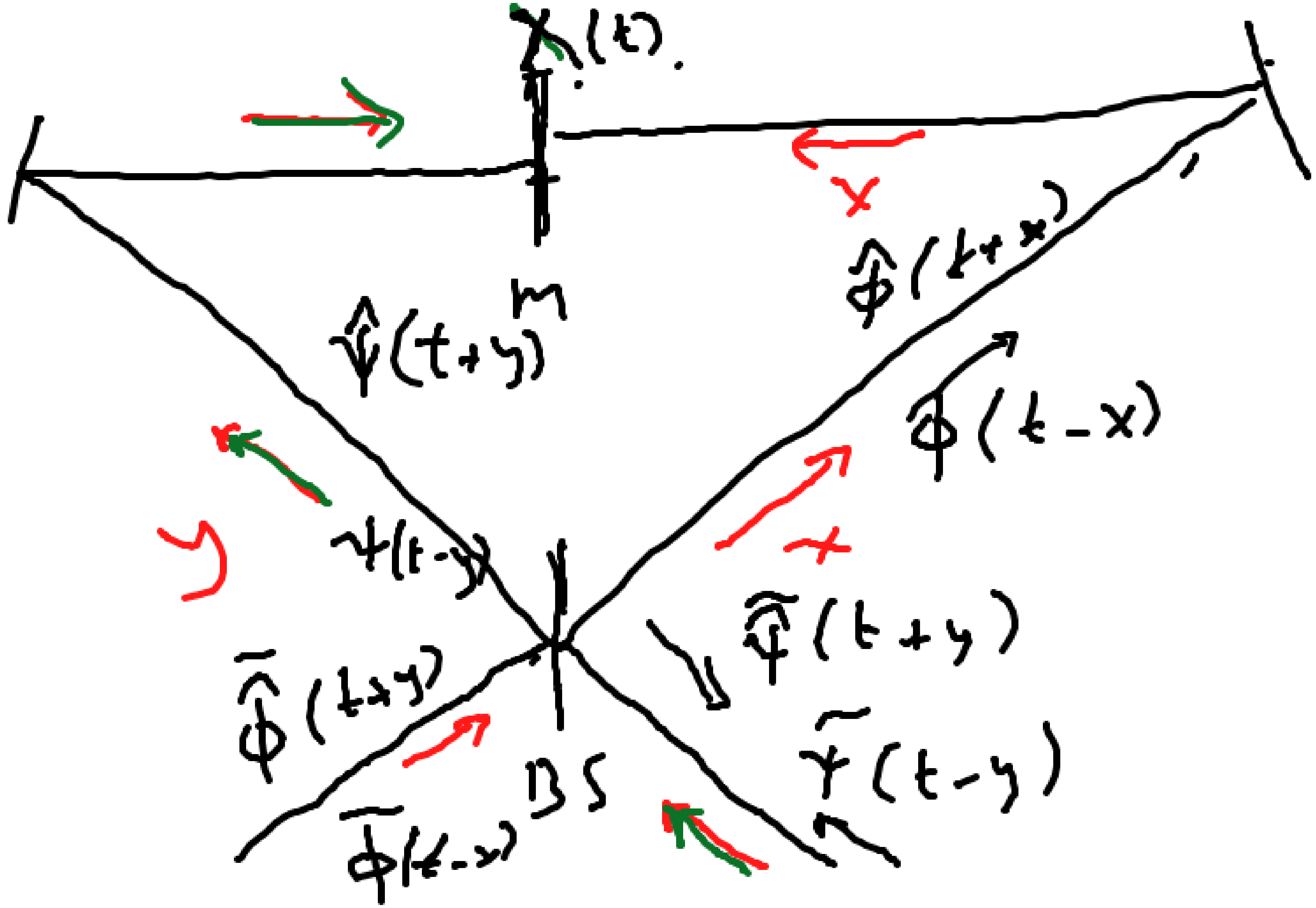


$$X(t) = X_0 + \hat{X}(t)$$

$$\phi(t-x) = \frac{1}{\sqrt{2}}\left(\tilde{\phi}(t-x) + \tilde{\psi}(t-x)\right)$$

$$\psi(t-y) = \frac{1}{\sqrt{2}}\left(\tilde{\phi}(t-y) - \tilde{\psi}(t-y)\right)$$

Mirror

$$\partial_x \left(\phi(t-x) + \hat{\phi}(t+x) \right) \Big|_{x = X(t)}$$

$$\hat{\phi}(t) = \phi(t - 2X(t))$$

$$X(t-x) \approx X(t) \cancel{-2\dot{X}(t)\dot{X}(t)} \dots$$

$$\hat{\psi}(t) = \psi(t + 2X(t))$$

$$\tilde{\hat{\phi}} = \frac{1}{\sqrt{2}} \left(\hat{\phi} + \hat{\psi} \right) = \tilde{\phi}$$

$$\tilde{\hat{\psi}} = \frac{1}{\sqrt{2}} \left(\hat{\phi} - \hat{\psi} \right) = \tilde{\psi} \quad \text{if } X(t) = 0$$

$$\tilde{\tilde{\Psi}}(t+y) = \frac{1}{2}\Big(\tilde{\Psi}_-\big(t+y-2X(t)\big) \leftarrow \text{rt branch}$$

$$+ \tilde{\Psi}\big(t+y+2X(t)\big) \rightarrow \text{left branch}$$

$$+ \tilde{\phi}\big(t+y-2X(t)\big) - \text{right}$$

$$- \tilde{\phi}\big(t+y+2X(t)\big) - \text{left}.$$

$$X(t) \to X(t+y)$$

$$X(t) = X_0 + \hat{X}(t)$$

<u>Mirror</u>

$$M\,\partial_t^2 X = F(t) + Pa$$

$$P = \frac{1}{4}\Big(\big(\partial_t \Phi_T(t,x)\big)^2 + \big(\partial_x \Phi_T(t,x)\big)^2$$

$$- \big(\partial_t \Psi_T(t,y)\big)^2 - \big(\partial_x \Psi_T(t,y)\big)^2 \Big)\Big|_{x=-y=X(t)}$$

Variation of Lagrangian

$$\frac{1}{2}\int \sqrt{\det g}\,\Big(g^{tt}\,(\partial_t \phi)^2 + g^{xx}\,(\partial_x \phi)^2 \Big)\,dx$$

$$g = -1/g^{tt} g^{xx}$$

Vary wrt g^{xx} $\Longrightarrow$ Pressure.

$(g^{tt} = -g^{xx} = 1)$

$$\Phi_T(t,x) = \phi(t-x) + \hat{\Phi}(t+x)$$

Boundary cond $\quad \partial_x \Phi_T(t,x)\Big|_{x=X} = 0$

$$P = \frac{1}{4}\left(\left(\frac{\partial \Phi_T}{\partial t}\right)^2 - \left(\frac{\partial \psi_T}{\partial t}\right)^2\right)$$

$$\hat{\phi} = \phi(t+x-2X(t))$$

$$\partial \phi_T = \dot{\phi}(t-X(t)) + \dot{\phi}(t-X(t))(1-2\dot{X}(t))$$

$$P = \left(\dot{\phi}(t-\cancel{X(t)})(2-2\dot{X})\right)^2 - \left(\dot{\psi}(t-\cancel{X(t)})(2+2\dot{X})\right)^2$$

$$P = -(\quad)^2 \dot{X}$$

$$m\partial_t^2 X = -(\quad)^2 \dot{X} + \cdots$$

$$m\partial_t^2 X + \underbrace{(\quad)^2 \dot{X}}_{\text{Damping term.}} = \cdots\cdots$$

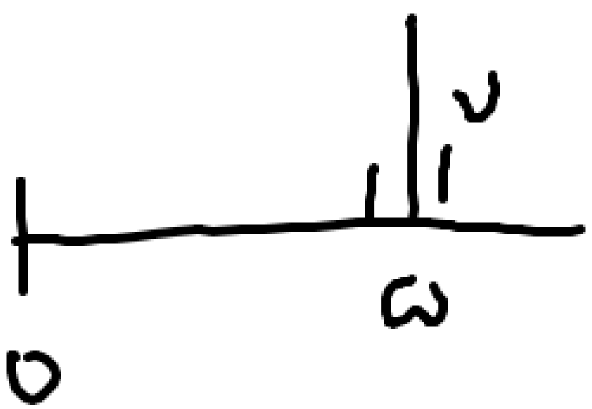
Input.

$$\tilde{\phi}(t-x) = \frac{\alpha e^{-i\omega(t-x)} + e^{-i\omega(t-x)} \int_{-\Delta}^{\Delta} A_\nu e^{-i\nu(t-x)}\, d\nu}{\sqrt{2\pi/\omega}}$$

$A(t-x)$

$+$ H.c.

$A_\nu \to$ annih. op for freq $\omega+\nu$



$\omega \gg \nu$

Assume α = Real.

$\alpha \ggg 1$

$$\tilde{\Psi} = \left(\int B_\nu \, e^{-i\nu(t-y)} \, d\nu \right) \frac{e^{-i\omega(t-y)}}{\sqrt{2\pi(\omega+\cancel{\nu})}} \, d\nu$$

$$+ \ H.C$$

$$\tilde{\hat{\psi}} = \frac{1}{2}\Big(\psi(t+\cancel{y}-2X) + \psi(t+y+2X) + \phi(t+y-2X) - \phi(t+y+2X) \Big)$$

$$= \frac{1}{2}\left(2B(t)\, e^{-i\omega(t+y)} + \frac{\alpha\left(e^{-i\omega(t+y)-2x)} - e^{-i\omega(t+y+2x)}\right)}{\sqrt{2\pi\omega}} + \cancel{A} - \cancel{A} \right)$$

$$\tilde{\hat{\psi}} = \frac{1}{\sqrt{2\pi w}} \left[\alpha e^{-iw(t+y)} (-iw)(4X(t)) + e^{-iw(t+y)} \left(B(t) \right) \right]$$

output has signal + Q. Noise.

$$X(t) = X_0 + \hat{X}(t)$$

$$(X_0)^2 \gg \langle X^2 \rangle$$

$\tilde{\hat{\Psi}} \longrightarrow$ Detector $\to$ counts photons coming out.

Photon flux :

$$\text{Photon\#} = \frac{i}{2}\int \left[\tilde{\hat{\Psi}}^{+} \partial_t \tilde{\hat{\Psi}} - (\partial_t \tilde{\hat{\Psi}}^{+})\tilde{\hat{\Psi}}\right] dx$$

$$\text{Photon flux} = \frac{i}{2}\left[\tilde{\hat{\Psi}}^{+} \partial_x \tilde{\hat{\Psi}} - \partial_x \tilde{\hat{\Psi}}^{+} \tilde{\hat{\Psi}}\right].$$

Linearize with $\hat{X}$, B, B^{+} (X_0 not lin.)

$$\hat{N}F = \frac{1}{2\pi}\left[\alpha^2 X_0^2 \omega^2 + \alpha\omega X_0\, i(B - B^{+}) + 2\alpha^2 \omega^2 X_0 \underbrace{\hat{X}(t)}_{B+B^{+}}\right]$$

$$\langle NF \rangle = \frac{1}{2\pi}\left(|\alpha|^2 X_i^2 \omega^2 + 2\alpha^2 \omega^2 X_0 \int \frac{F_v(e^{-ivt})}{-v^2 m} dv \right)$$

$$\left\langle \left(NF - \langle NF \rangle \right)^2 \right\rangle$$

$$= \left\langle \int \left(\alpha \omega X_0 \, i(B_v - B_v^{+}) + 2\alpha^2 \omega^2 X_0 \, a \frac{(B + B_{-v}^{+})_v}{M v^2} \right)^2 dv \right\rangle$$

$$B_v |0\rangle = 0$$

$$\left((\quad) \left(e^{i\delta} B + e^{-i\delta} B^{+} \right) \right)^2$$

Squeezed State.

By making $B_\nu e^{i\delta\nu} + B^\dagger_{-\nu} e^{-i\delta\nu}$ small, we can

Reduce the Quantum noise by a lot.

Grav Wave Detector.
