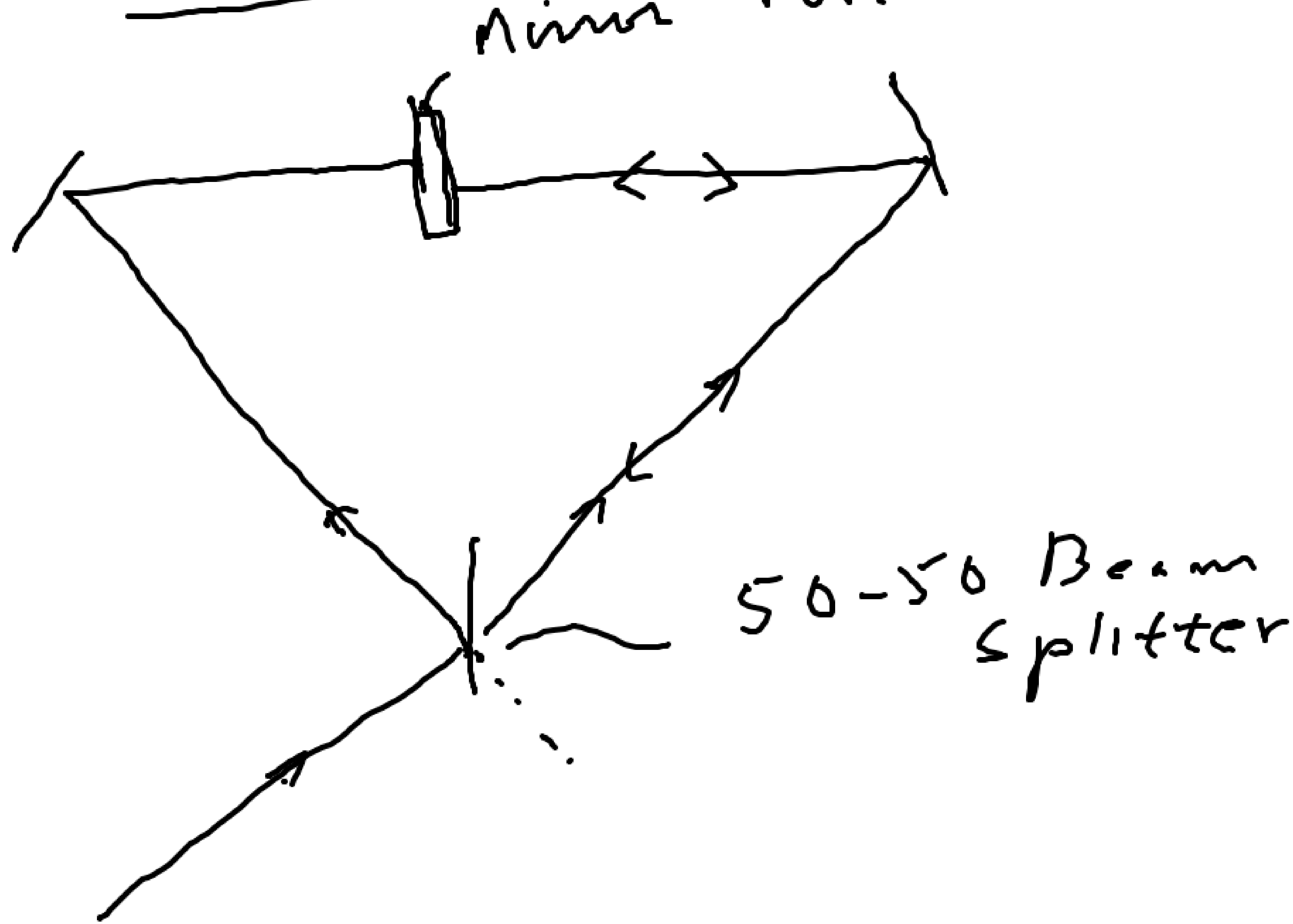
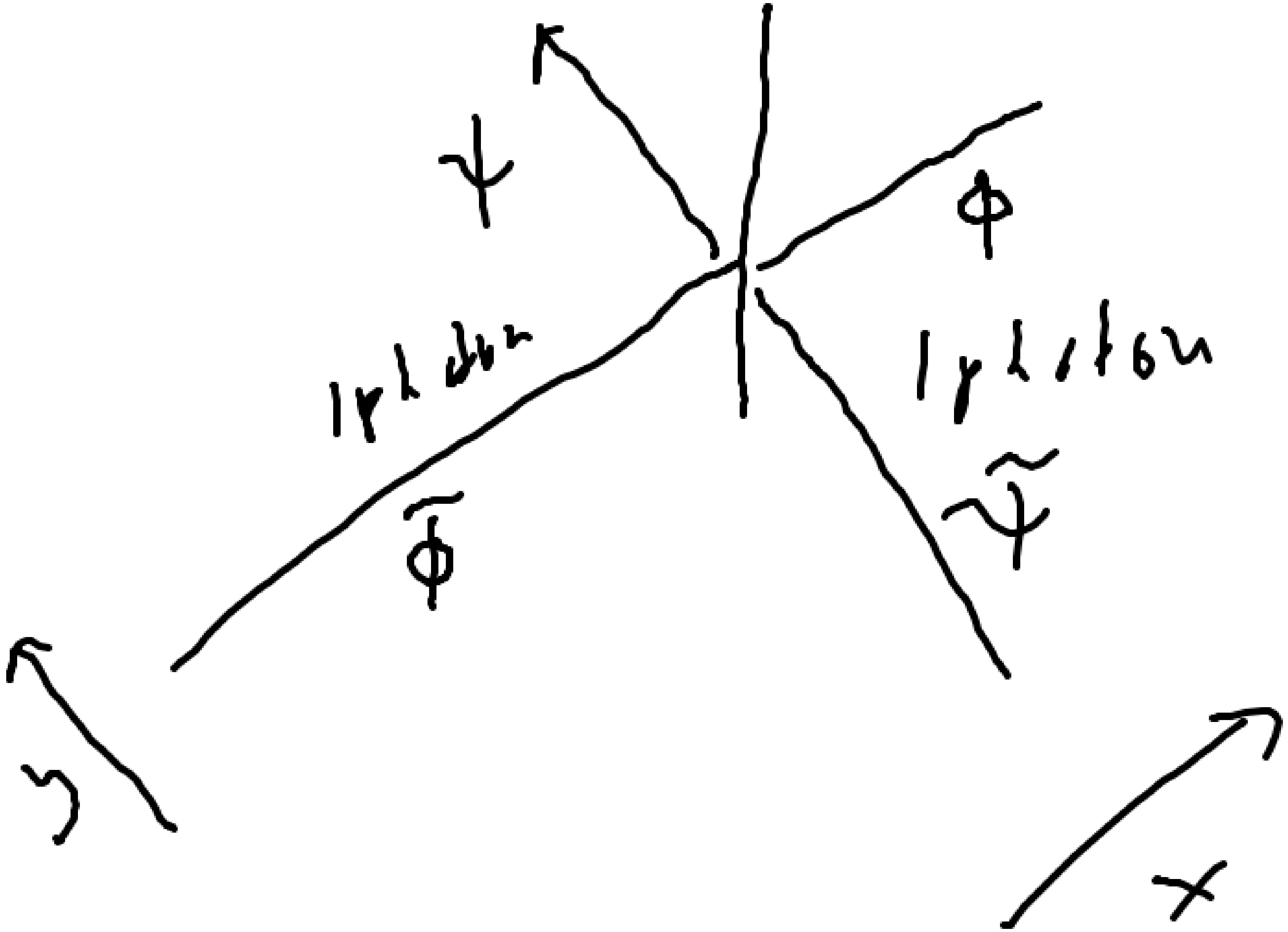
LIGO



$\tilde{\Phi}(t,x)$



$\alpha\ \tilde{\phi}_i(t-x)$

$\beta\ \tilde{\psi}(t-y)$

$$\phi(t-x) = \frac{1}{\sqrt{2}}\left(\tilde{\phi}(t-x) + \tilde{\psi}(t-y)\right)$$

$$\langle \phi, \phi \rangle = 1 \qquad i\int \phi^* \underbrace{\partial_t \phi}_{\pi} - \underbrace{\partial_t \phi^*}_{\pi^*} \phi$$

$$\tilde{\psi}(t) = \tilde{\phi}(t)$$

$$\Phi = A\,\Phi(t-x) + H.C. \quad \dots$$

$$\tilde{\Psi} = B\,\Psi(t-y) + H.C. \quad \dots$$

$$\Phi = C\,\phi(t-x) + H.C$$

$$\Psi = D\,\psi(t-x) + H.C.$$

$$C = \frac{A+B}{\sqrt{2}}$$

$$D = \frac{A-B}{\sqrt{2}}$$

$$|\psi\rangle = A^+ B^+ |0\rangle$$

$A^+|0\rangle \to$ state

1 particle state

$$C^+ = \frac{A^+}{\sqrt{2}}|0\rangle$$

$$D^+ = \frac{A^+}{\sqrt{2}}|0\rangle$$

$$A^{+} = \frac{C^{+} + D^{+}}{\sqrt{2}}$$

$$B^{+} = \frac{C^{+} - D^{+}}{\sqrt{2}}$$

$$A^{+} B^{+} = \frac{(C^{+} + D^{+})(C^{+} - D^{+})}{2} |0\rangle$$

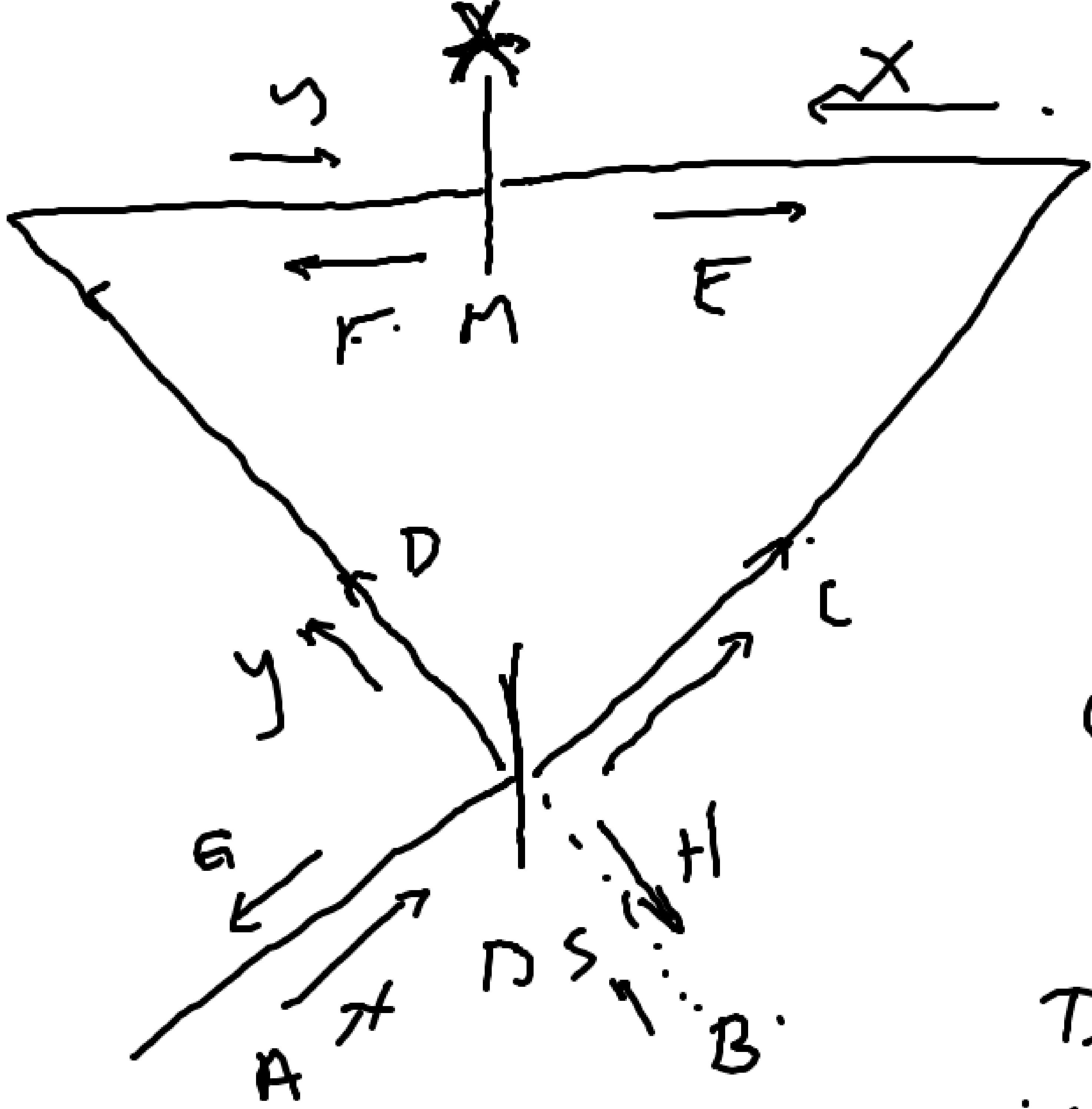
$$= \frac{(C^{+} C^{+} - D^{+} D^{+})}{2} |0\rangle$$

$$= \frac{|2_C\rangle + |2_D\rangle}{\sqrt{2}}$$

$\left(\frac{C^{+2}}{\sqrt{2}} |0\rangle \right.$ is norm 2 part state.)

Hung ou Mandel 1987

Two input photons on diff inputs
a: output has only two photons
on same arm.



Restposn is at $x = y = 0$

$\Phi(t-x)$

$+ \hat{\Phi}(t+x)$

Boundary cond is $\left.\frac{\partial\phi}{\partial x} = 0\right|_{x=X}$

$\phi(t-X) = \hat{\Phi}(t+X)$

$\phi(t-x)$, $\hat{\phi}(t+x)$ $\hookrightarrow$ $\hat{\Phi}(t) = \phi(t-2X)$

If $\phi(t) = e^{-i\omega t}$

$$\hat{\phi}(t) = e^{-i\omega(t - 2X(t))}$$

$$\hat{\phi}(t+x) = e^{-i\omega(t+x - 2X(t+x))}$$

(Assume X is small, and $\partial_t X$ is small.)

$$\hat{\psi}(t+y) = e^{-i\omega(t+y + 2X(t+y))}$$

Lets assume that the input state is a "coherent" state.

$$A|\alpha\rangle = \alpha|\alpha\rangle$$

$$|\alpha\rangle = e^{\alpha A^+}|0\rangle$$

$$[A, A^+] = 1 \Longrightarrow A = \frac{\partial}{\partial A^+}$$

$$\frac{\partial}{\partial A^+} e^{\alpha A^+}|0\rangle$$

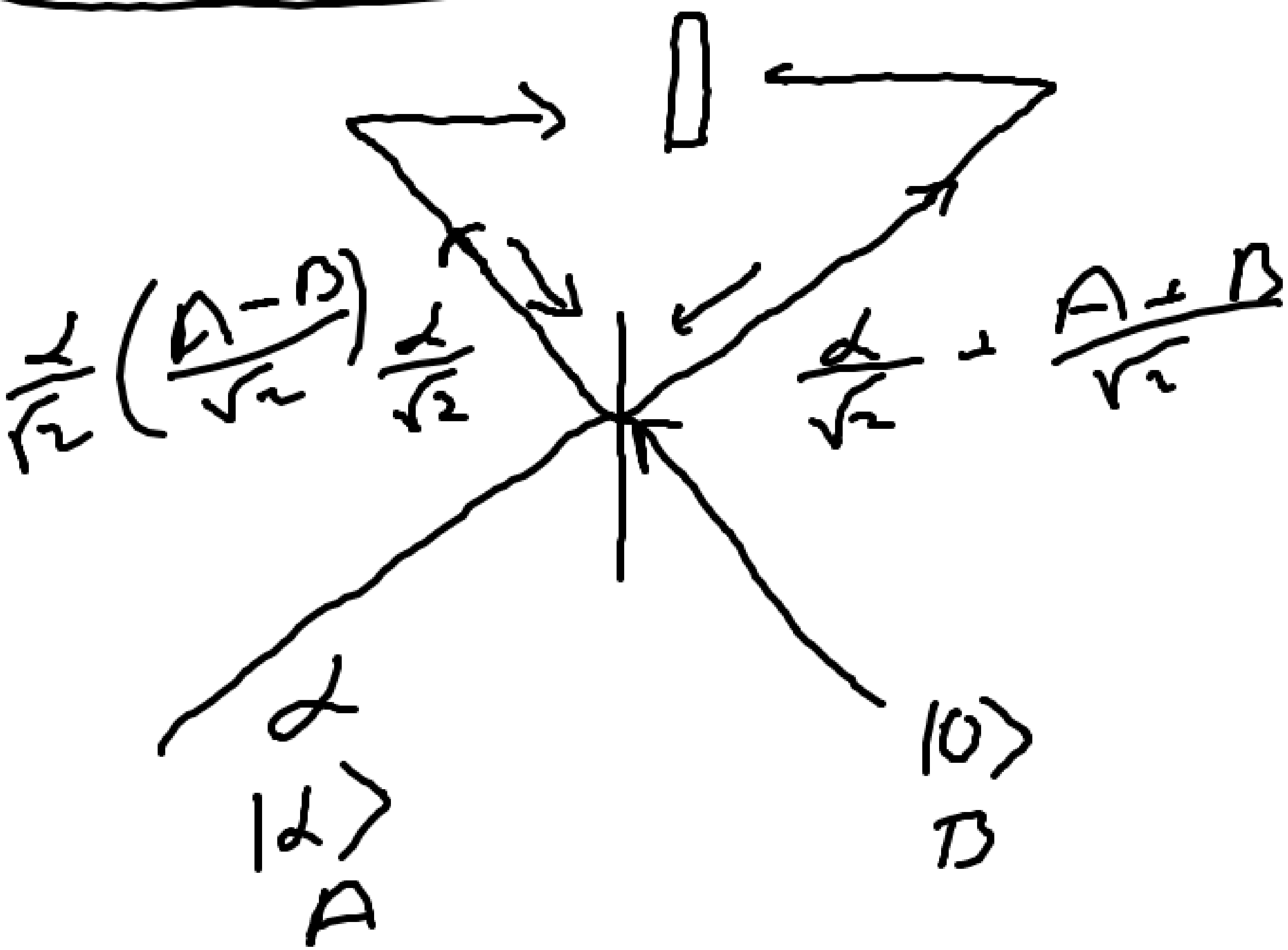
$$= \alpha \underbrace{e^{\alpha A^+}|0\rangle}_{|\alpha\rangle}$$

$$\langle 0|e^{\alpha^* A} e^{\alpha A^+}|0\rangle$$

$$= e^{|\alpha|^2}$$

The $A \to (A-\alpha)$

$$[(A-\alpha),(A^\dagger-\alpha^*)] = 1$$



phase shifted at mirror.

Each mode gets phase shifted by mirror.

If system exactly symmetric.

Then A.

Dark port

$e^{-i\omega(t+x-2X)} \approx e^{-i\omega(t+x)}(1+2i\omega X)$

& $e^{-i\omega(t+y+2X)} \approx e^{-i\omega(t+y)}(1-2i\omega X)$

e

$e^{-i\omega(t+y)} \, \underline{2X\omega}$ comes out Dark port.

Darkport has mirror displacement.
"Measure" outfield give meas of $\underline{X}$.

$2i\omega X(t) \rightarrow$ Amplifier of X.

$\Rightarrow$ 2 channels, <u>Noise</u>.

$$\tilde{\Phi} = \frac{\alpha\, e^{-i\omega_0(t-x)}}{\sqrt{2\pi|\omega|}} + \int A_\nu \frac{e^{-i(\omega_0+\nu)(t-x)}}{\sqrt{2\pi|(\omega\cancel{+\nu})|}}\, d\nu$$

$$+ \; H.c.$$

$$\tilde{\Psi} = \quad + \int B_\nu \frac{e^{-i(\omega+\nu)(t-y)}}{\sqrt{2\pi|\omega\cancel{+\nu}|}}\, d\nu$$

(we will only look at $|\nu| \ll \omega_0$)

$$\phi(t-x) = \frac{\alpha}{\sqrt{2}} e^{-i\omega_0(t-x)} + \int \frac{(A_\nu + B_\nu)}{\sqrt{2}} \frac{e^{-i(\omega+\nu)(t-x)}}{\sqrt{2\pi|\omega|}}\, d\nu$$

$$\psi(t-y) = \frac{\alpha}{\sqrt{2}} e^{-i\omega_0(t-x)} + \int \frac{A_\nu - B_\nu}{\sqrt{2}} \frac{e^{-i(\omega+\nu)(t-y)}}{\sqrt{2\pi|\omega|}}$$

Assume $\alpha \gg 1$

Reflection

$$\Phi(t+x) = \frac{\alpha}{\sqrt{2}} \frac{e^{-i\omega(t+x-2X)}}{\sqrt{2\pi\omega}} \quad (1+2i\omega X)$$

$$+ \int \frac{A_\nu B_\nu}{\sqrt{2}} \frac{e^{-i\omega(t+x-\cancel{2X})}}{\sqrt{2\pi\omega}} \, d\nu + HC$$

$$\chi = (1-2i\omega X) \int \frac{A_\nu - B_\nu}{\sqrt{2}} \frac{e^{-i\omega(t+y+\cancel{2X})}}{\sqrt{2\pi\omega}} \, d\nu + HC.$$

XA is very very small.

Lets assume there is a small extra phase shift back to mirror 2.

at ~~m~~ B.S.

$$\propto e^{i(\omega_0(t+x) - \delta)}$$

$$e^{i(\omega_0(t+y) + \delta)}$$

Then output to port.

$$\tilde{\psi}(t+x) = \sin\delta \, \alpha e^{i\omega(t+y)} + \cos\delta \cdot \alpha e^{-i\omega(t+y)} - 2iX + \sin\delta \int A_v e^{-i(\omega_1+v)(t+y)} dv + \cos\delta \int A_v e^{-i(\omega_1+v)(t+y)} dv$$

$\boxed{\delta \ll 1}$