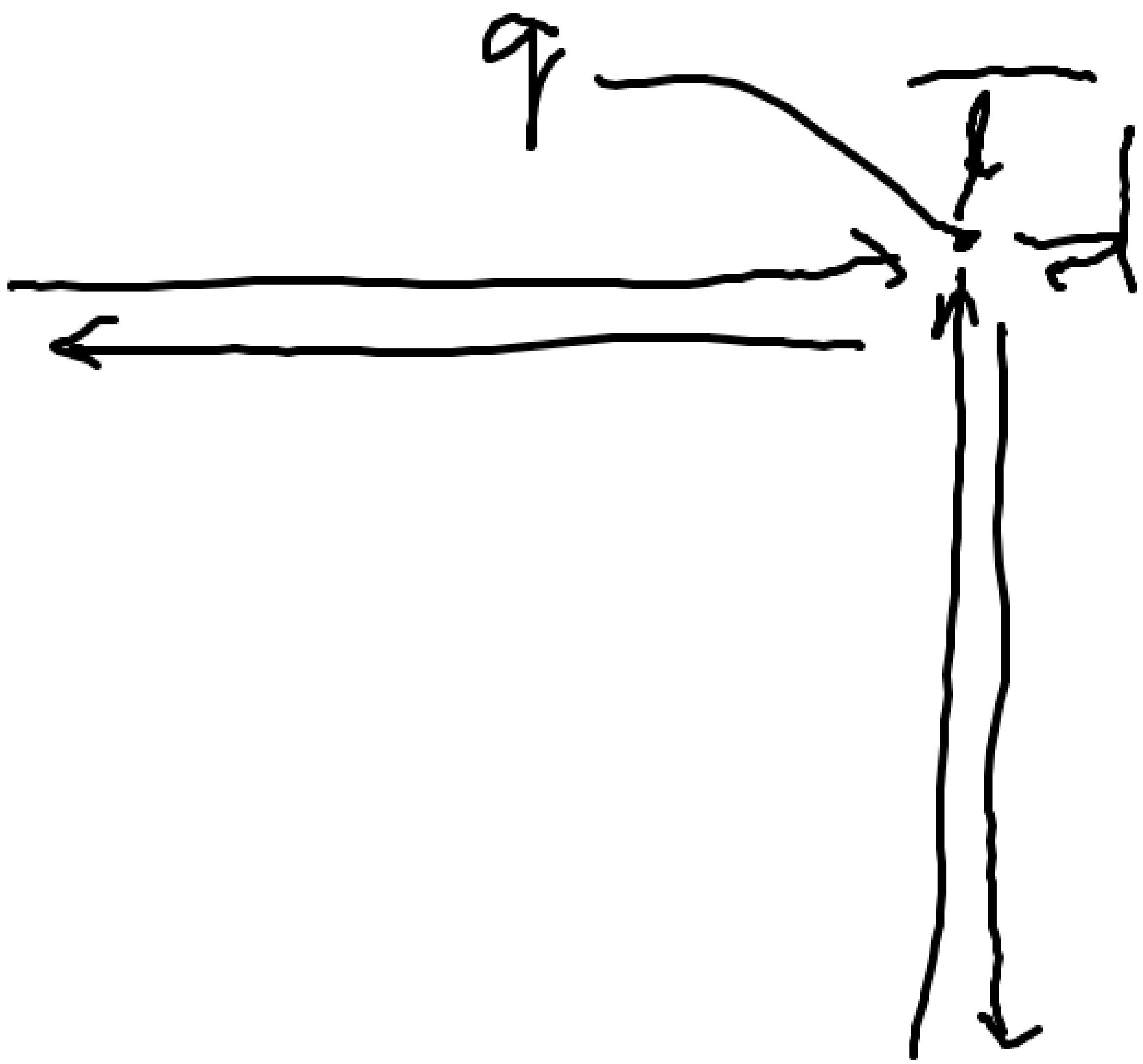
Amplifier

$x = y = -\epsilon$



E

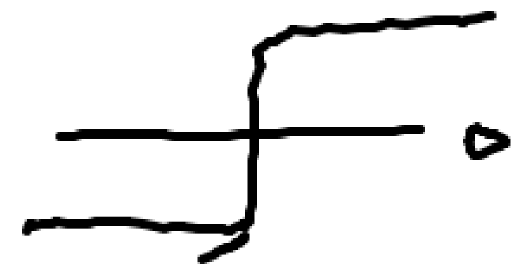
Equation:

$$-\partial_t^2 \phi + \partial_x^2 \phi + \lambda \partial_t q \, \delta(x+\epsilon) = 0$$

$$\partial_t^2 \psi - \partial_y^2 \psi + \mu \partial_t q \, \delta(y+\epsilon) = 0$$

$$-\partial_t^2 q - \lambda \partial_t \phi(t,-\epsilon) - \mu \psi(t,-\epsilon) = 0$$

$$-\partial_t^2 \phi \quad \partial_x^2 \phi + \lambda \partial_t q \, \delta(x+\epsilon) = 0$$



$$\frac{\lambda}{2} q(t - |x+\epsilon|)$$

$$\partial_x\left(\frac{\lambda}{2} q(t - |x+\epsilon|)\right) = -\frac{\lambda}{2} \partial_t q \, \sigma(x+\epsilon)$$

$\sigma(\xi) = +1$ if $\xi > 0$, -1 if $\xi < 0$

$$\partial^2 q = \partial x \left(\qquad \right) - = \partial t^2 q \quad \sigma^2 \qquad \sigma^2 = 1$$

$$- \frac{\lambda}{2} \partial t\, q\, 2\, \delta(x+\epsilon)$$

$$- \partial t^2 \phi + \partial x^2 \phi + \lambda\, \partial t\, q\, \delta(x+\epsilon) = 0$$

$$\phi = \phi_0(t-x) + \phi_0(t+x)$$

$$+ \frac{\lambda}{2} q\left(t - |x+\epsilon|\right) + \frac{\lambda}{2} q(t+x-\epsilon)$$

$$\psi = \psi_0(t-x) + \psi_0(t+x) - \frac{\mu}{2} q(t-|y+\epsilon|)$$

$$- \frac{\mu}{2} q(t+y-\epsilon)$$

$$\lim_{\epsilon \to 0} \quad \psi = \psi_0(t-y) + \psi_1(t+y) - \mu\, q(t+y)$$

$$\phi = \phi_0(t-x) + \phi_0(t+x) + \frac{\lambda}{2} q(t+x)$$

except at $x = -\epsilon$ $\qquad \underbrace{q(t) + q(t-2\epsilon)}_{2q(t)}$

$$\partial_t^2 q + (\lambda^2 - \mu^2)\partial_t q$$

$$= \partial_t \left[\lambda\, 2\phi_0(t) + \mu\,(2\psi_0(t))\right]$$

Now mode $e^{-i\omega t}$

$$-\omega^2 q - i\omega(\lambda^2 - \mu^2) q = -i\omega(2\lambda\phi_\omega + 2\mu\psi_\omega)$$

$$q_\omega = \frac{2\lambda\phi_\omega + 2\mu\psi_\omega}{-i\omega + \underbrace{(\lambda^2 - \mu^2)}}$$

ϕ : $e^{-i\omega(t-x)} \longrightarrow$ incoming

ψ $e^{-i\omega(t-y)} \longrightarrow$ incoming

$e^{-i\omega(t+x)} \longrightarrow$ outgoing.

$\phi_{in} = \phi_0 \, in \qquad \psi_{in} = \psi_0 \, in$

$\phi_{out} = \phi_0 \, out. + \lambda q_\omega$

$\psi_{out} = \psi_0 \, out - \mu q_\omega$

standard soln.

$$\phi_{out} = \int \phi_{0\omega} + \frac{2\lambda(\lambda\phi_0 - \mu\psi_0)}{-i\omega + (\lambda^2 + \mu^2)} e^{-i\omega(t+x)} \, d\omega$$

$$\psi_{\text{out}}(t+x) = \int \left(\psi_{\omega} - 2\mu \frac{(\lambda\,\phi_{0\omega} - \mu\,\psi_{0\omega})}{-i\omega + (\lambda^2 - \mu^2)} \right) e^{-i\omega(t+x)}\, d\omega$$

quantize. $\omega > 0$

$$\phi_{0\omega} \longrightarrow \frac{A_\omega}{\sqrt{2\pi|\omega|}}$$

$$\psi_{0\omega} \longrightarrow \frac{B^{+}_{\omega}}{\sqrt{2\pi|\omega|}}$$

$\bar{A}$ is out. annihil of $\cdot$ $\left.\right\}$ $e^{-i\omega(t+x)}$

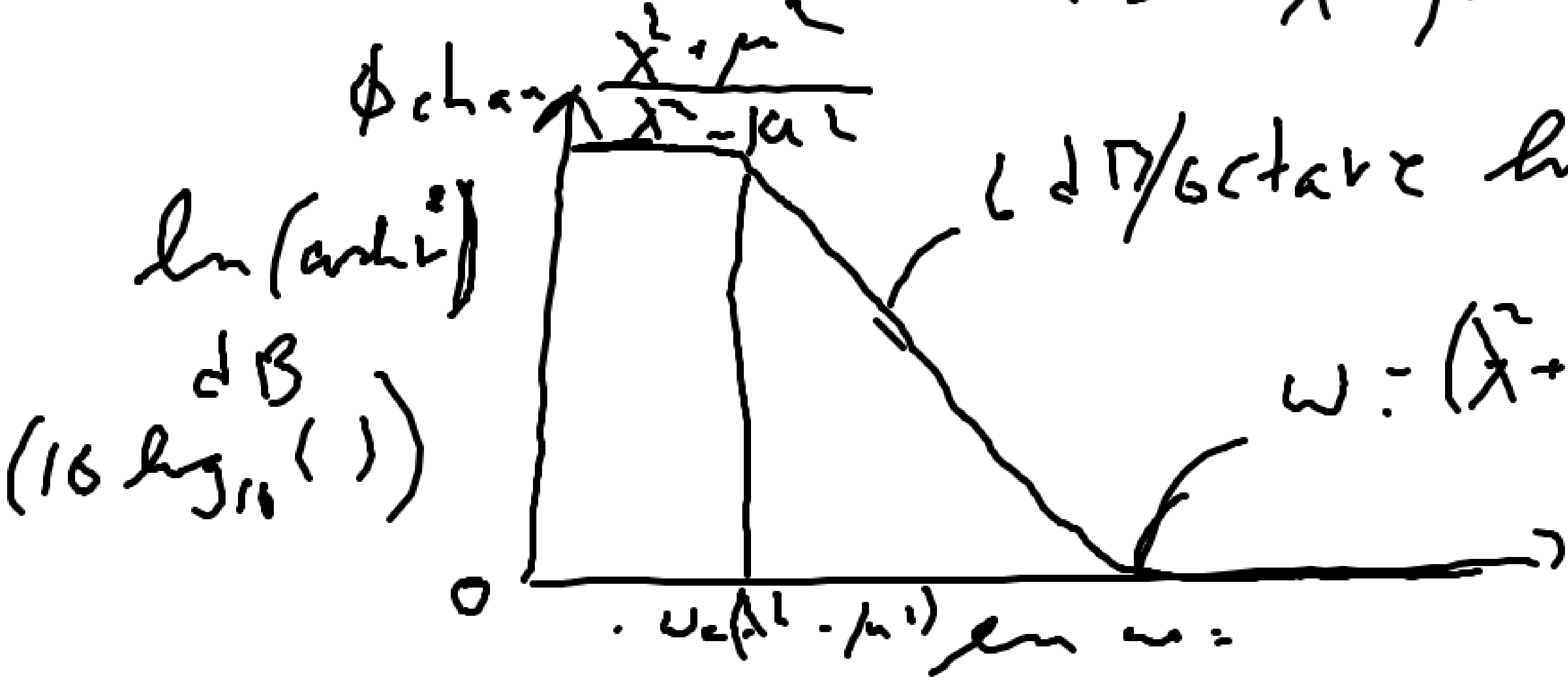
$\bar{B}^{+}$ " " creation $\left.\right\}$ $e^{+i\omega(t-x)}$

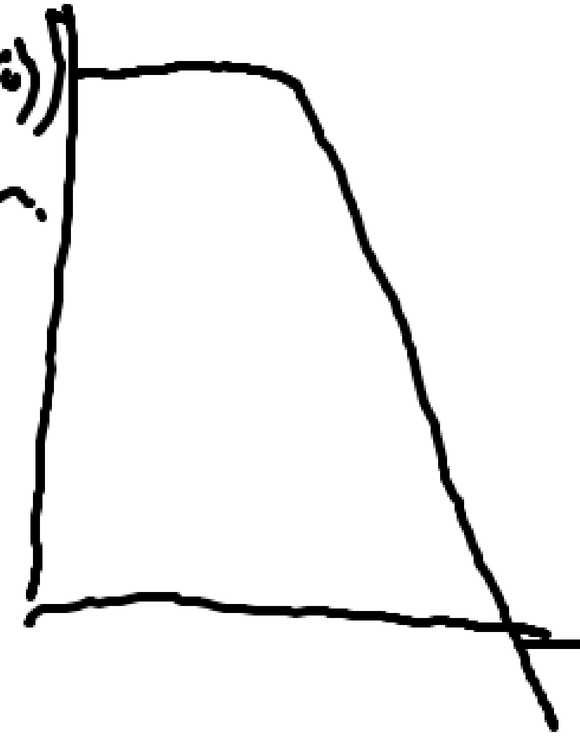
$$\tilde{A}_\omega = A_\omega \underbrace{\left(\frac{-i\omega - \lambda^2 - \mu^2}{-i\omega + \lambda^2 - \mu^2}\right)}_{\cosh r_\omega e^{i\varphi_\omega}} + \frac{2\lambda\mu}{-i\omega + \lambda^2 + \mu^2} B^+_\omega$$

$+ \sinh r_\omega e^{i\hat{\varphi}_\omega}$

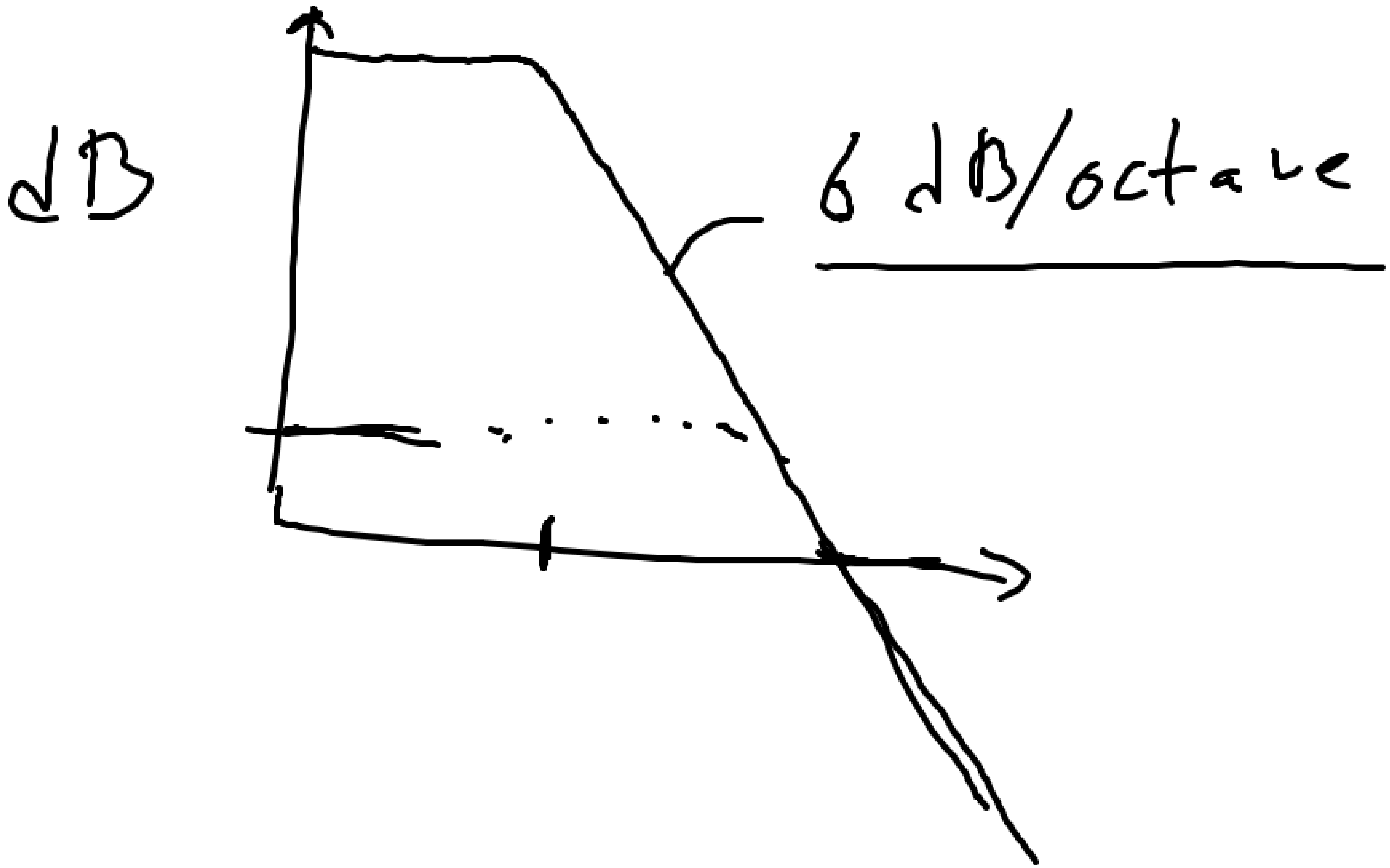
$$\lambda^2 + \mu^2 > \lambda^2 - \mu^2$$

$$\bar{B}^+_\omega = \frac{-i\omega + \lambda^2 + \mu^2}{-i\omega + \lambda^2 - \mu^2} B^+_\omega + \frac{2\lambda\mu}{-i\omega + \lambda^2 - \mu^2} A_\omega$$





Data book for linear amplifiers chips.



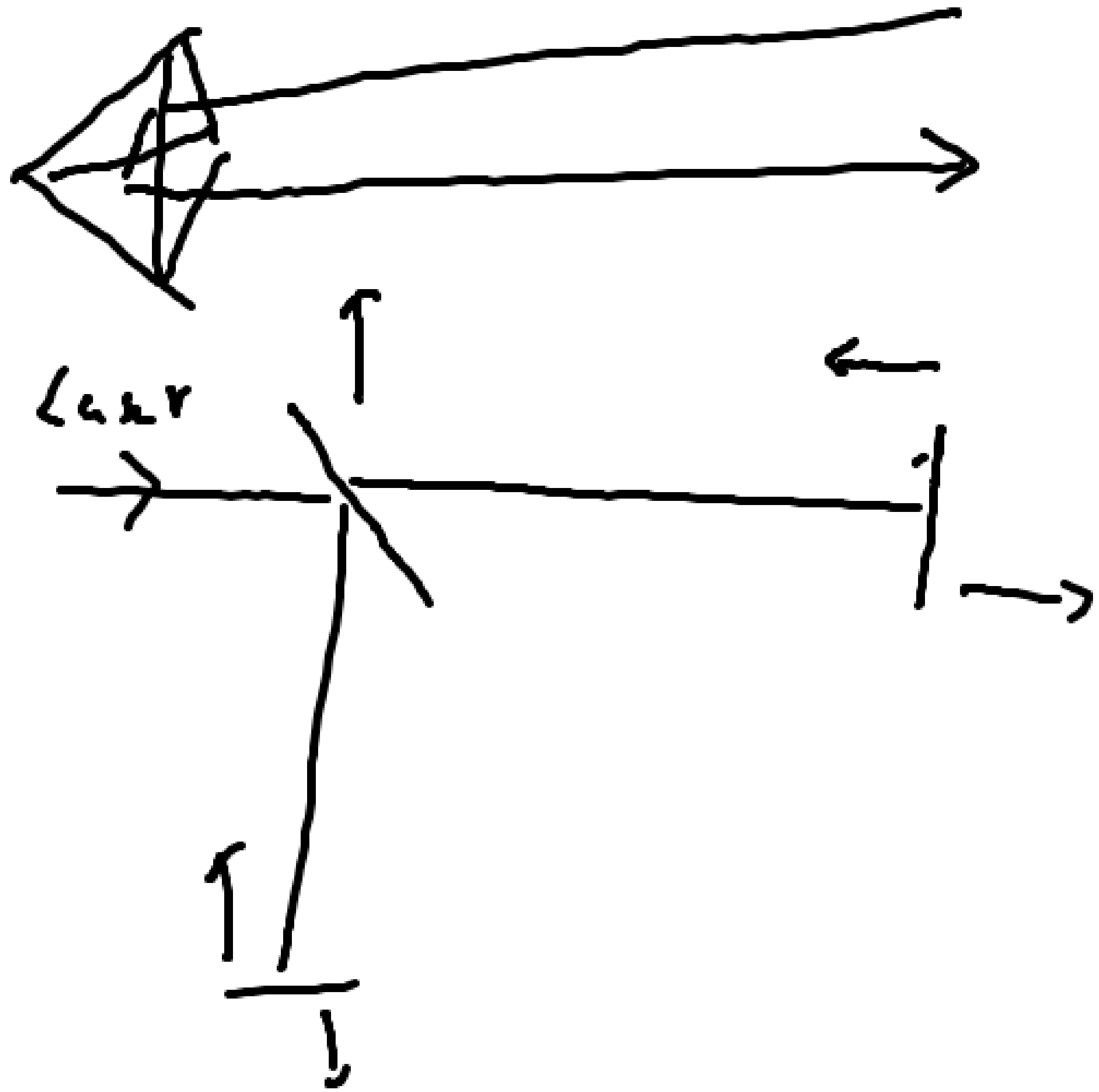
Model with negative Ham. field theory is a good mode for looking at the Q.M. of a generic amp. (Hifi, maser laser, ...)

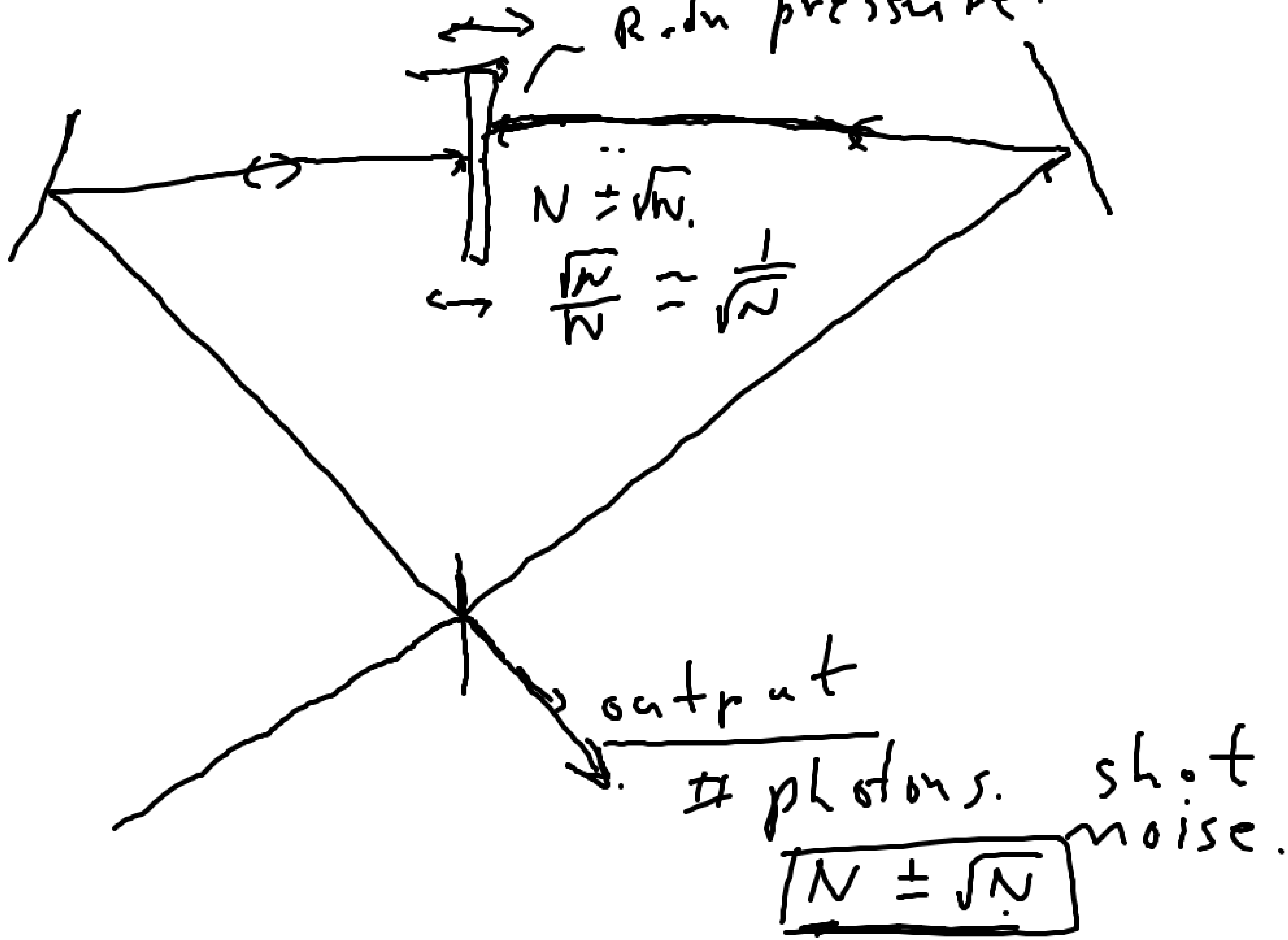
$\longrightarrow$

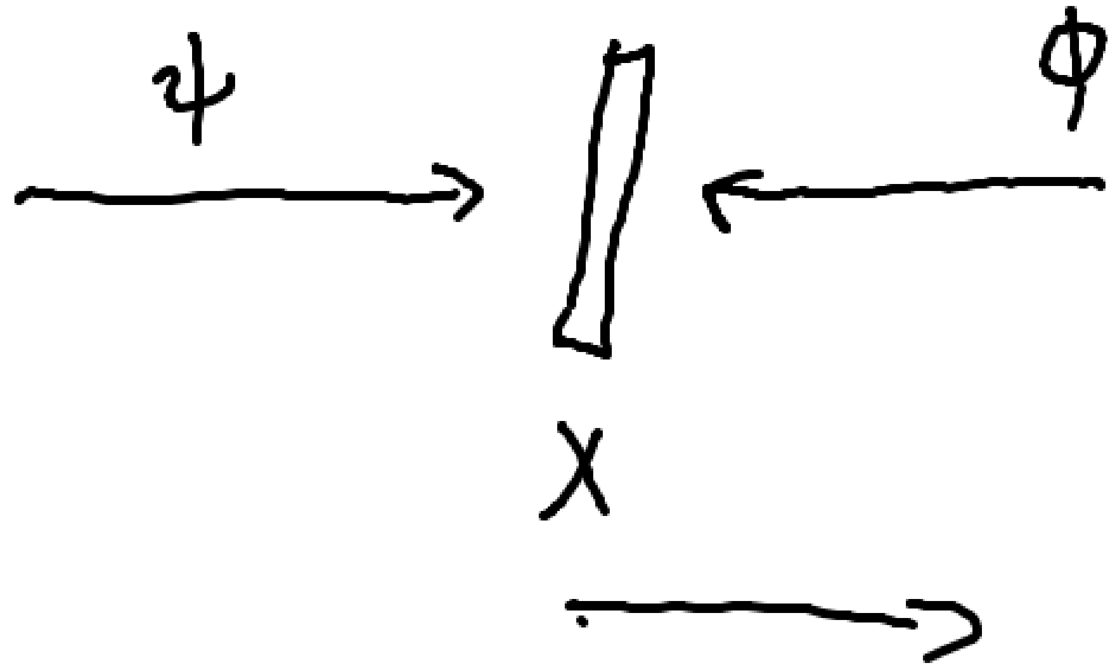
(Model of generic amplifier)

Detectors.

Q.M. of LIGO (Laser Int. detector for Grav. radn)




Radn pressure.
$N \pm \sqrt{N}$
$\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$
output
photons.
shot noise.
$\boxed{N \pm \sqrt{N}}$



In Ligo
Intensity
is ~ 1 MW.
99.999999 %

Next time

(LIGO Q.M.)