Amplifiers

$$\phi(t) \longrightarrow \alpha\,\phi(t) \qquad \alpha > 1$$

$$Q \longrightarrow \alpha Q$$

$$P = \partial_t Q \longrightarrow \alpha P$$

$$[\alpha Q, \alpha P] = \alpha^2 [Q, P] = \alpha^2 i$$

$$\neq [Q, P]$$

$$Q \rightarrow \tilde{Q} = \alpha Q \quad , \quad \tilde{P} = \frac{1}{\alpha} P \qquad [\tilde{Q}, \tilde{P}] = [Q, P] = i$$

Phase sensitive Amp.

$$Q(t) = \cos\Omega t \, Q_0 + \frac{\sin\Omega t}{\Omega} P_0$$

$$P(t) = \cos\Omega t \, P_0 - \frac{\sin\Omega t}{\Omega} Q_0$$

$$Q_0 \to \alpha Q_0$$

$$P_0 \to \frac{1}{\alpha} P_0$$

$$[Q(t), P(t)] = \alpha \frac{1}{\alpha} \left([Q_0, P_0] = i \right.$$

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Phase insensitive Amp.?

2 chanels $A \to$ annihil op 1^{st}

$B \to$ " " 2^{nd}

~ = output

$$\tilde{A} = \cosh r\, A + \sinh r\, B^\dagger$$

$$[\tilde{A}, \tilde{A}^\dagger] = \cosh^2 \underbrace{[A, A^\dagger]}_{1} + \sinh^2 \underbrace{[B^\dagger, B]}_{-1}$$

$$= \cosh^2 r - \sinh^2 r = 1$$

$A|\psi\rangle = a|\psi\rangle$ coherent state

$B|\psi\rangle = 0$

$$\langle\psi|\tilde{A}|\psi\rangle = \langle\psi|\cosh r\,\overbrace{A|\psi\rangle}^{a|\psi\rangle} + \underbrace{\langle\psi|\sinh r\,B^{\dagger}|\psi\rangle}_{\sinh r\,(B|\psi\rangle)^{\dagger} = 0}$$

$$\langle\psi|\tilde{A}|\psi\rangle = a\cosh r$$

$$\langle\psi|\tilde{A}^{\dagger}\tilde{A}|\psi\rangle = |a|^2\cosh^2 r + \langle\psi B B^{\dagger}\psi\rangle \sinh^2 r$$

$$= |a|^2\cosh^2 r + \underbrace{\sinh^2 r}_{\text{Noise}}$$

$$\tilde{Q} = \langle\psi|\frac{\hat{A}+\bar{A}^+}{\sqrt{2}}|\psi\rangle = \left(\frac{a+a^*}{\sqrt{2}}\right)\cosh r$$

$$\langle\psi|P|\psi\rangle = \langle\psi|i\left(\frac{\bar{A}-\tilde{A}^+}{\sqrt{2}}\right)|\psi\rangle = i\left(\frac{a-a^*}{\sqrt{2}}\right)\cosh r$$

$$\langle\psi|\tilde{Q}^2|\psi\rangle = \langle\psi|\bar{Q}|\psi\rangle^2 + \underbrace{\text{Noise}}_{\cosh 2r}$$

① Phase insens. amp.

Put in time dep get

same amp for A + B

Model amplifier.

Mix $A, B^\dagger$

ϕ 1+1 dim field

$$\partial_t^2 \phi - \partial_x^2 \phi = 0$$

$$\Pi = \partial_t \phi \longrightarrow e^{-i\omega(t-x)}$$

inc norm state

Assoc with A_ω

Chanel ψ

$$\partial_t^2 \psi - \partial_y^2 \psi = 0$$

$e^{-i\omega t}$

$$\phi \Rightarrow \Pi = \partial_t \phi$$

$$\frac{i}{2}\left(\phi^* \pi - \pi^* \phi\right) > 0 \text{ for } e^{-i\omega t}$$

$\psi \quad \pi \rightarrow -\partial_t \psi$

$e^{-i\omega t} \rightarrow$ neg norm

$$\mathcal{L}_\phi = \frac{1}{2}\int (\partial_t \phi)^2 - (\partial_x \phi)^2 \, dx$$

$$\Pi_\phi(t,x) = \frac{\partial \phi(t,x)}{\partial t}$$

$$\mathcal{L}_\psi = -\frac{1}{2}\int (\partial_t \psi)^2 - (\partial_y \psi)^2 \, dx$$

$$\Pi = -\partial_t \psi$$

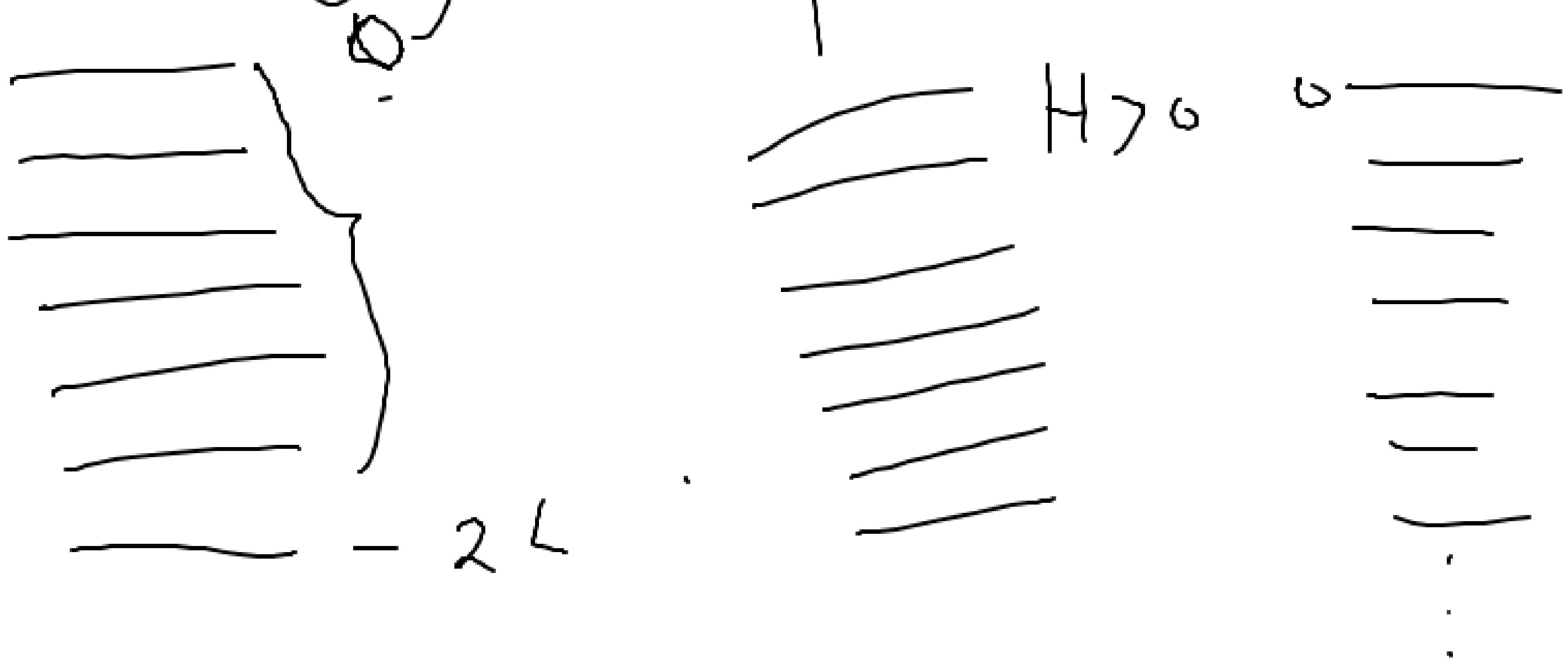
$$H = -\frac{1}{2}\int \pi^2(t,x) + (\partial_x \psi)^2 \, dx$$

Nothing exists with arb large neg energy.

- Non linear → for large ψ → energy becomes interesting

$$E = L_x$$

$$H = (L_x - L)$$



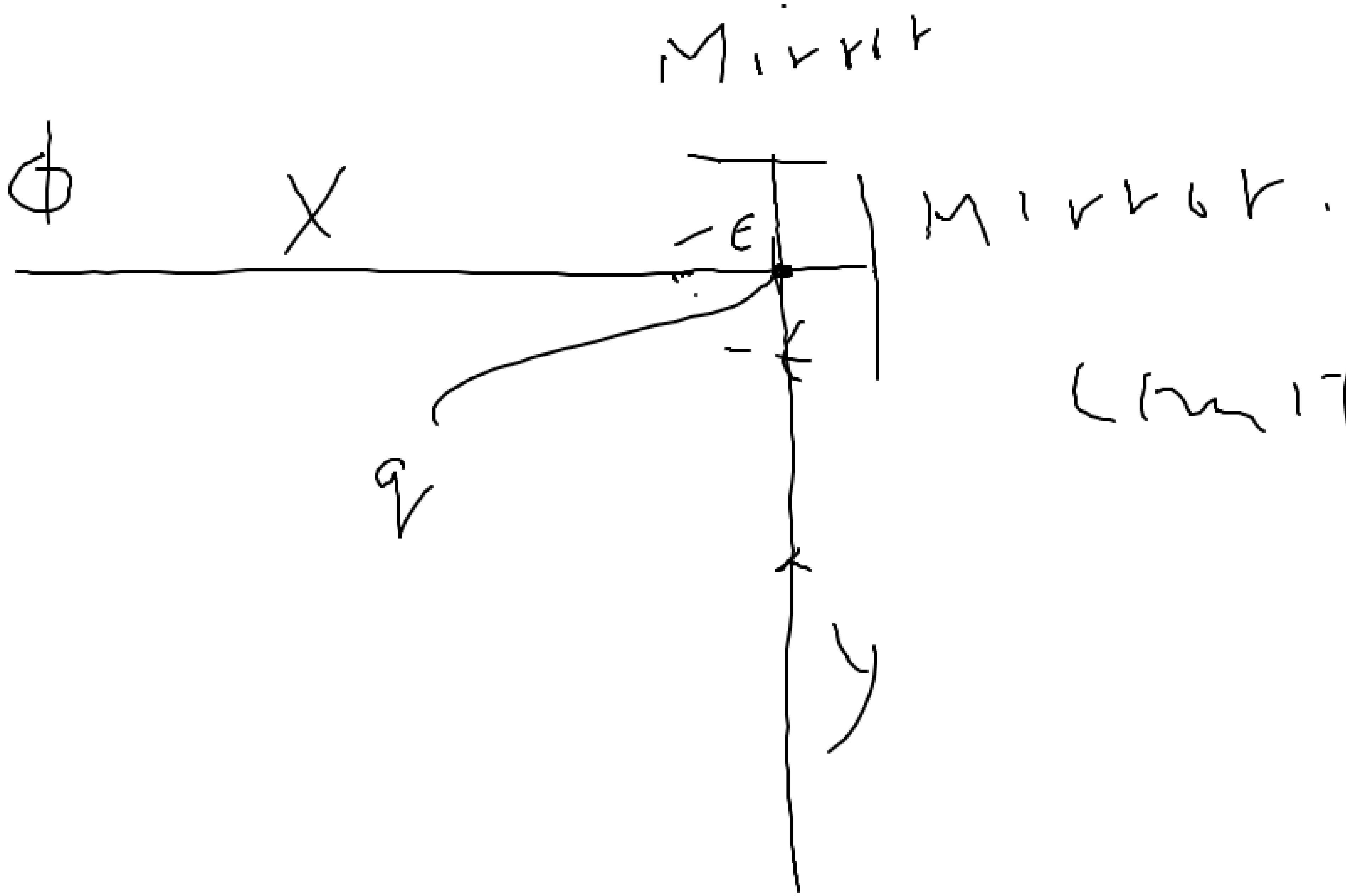
System. $\frac{1}{2}\left[\int (\partial_t \phi)^2 - (\partial_x \phi)^2 \, dx\right.$

$$- \int (\partial_t \psi)^2 - (\partial_y \cancel{\phi})^2 \, dy$$

Couple $x = y = -\epsilon$ via a single "free" particle

q $\quad \mathcal{L} = \frac{1}{2}(\partial_t q)^2$

Couple $\lambda \phi(t, -\epsilon) \underbrace{\partial_t q}_{|\mu| < |\lambda|} + \mu \psi(t, -\epsilon) \partial_t q$



Limit $\epsilon \to 0$.

Eqn of motion

$$-\Box\phi(t,x) + \lambda\partial_t q(t)\,\delta(x+\epsilon) = 0 \qquad \phi(t,-\epsilon)$$

$$\Box\phi = (\partial_t^2 - \partial_x^2)\,\phi$$

$$\Box\psi + \mu\partial_t q(t)\,\delta(y+\epsilon) = 0$$

$$-\partial_t^2 q - \lambda\phi(t,-\epsilon) - \mu\psi(t,-\epsilon) = 0$$

$$\& \quad \left.\frac{\partial\psi}{\partial y}\right|_{y=0} = \left.\frac{\partial\phi}{\partial x}\right|_{x=0} = 0$$

$$\phi(t,x) = \phi_0(t-x) + \phi_0(t+x)$$
$$+ \frac{\lambda}{2}\Big(q(t+x+\epsilon)\,\Theta(-(x+\epsilon))$$
$$+ q(t-x-\epsilon)\,\Theta(x+\epsilon)$$
$$+ q(t+x-\epsilon)\Big)$$

$$\Theta(\zeta) = \begin{cases} +1 & \text{if } \zeta > 0 \\ 0 & \text{if } \zeta < 0 \end{cases}$$

$$-\Box\phi(t,x) \Rightarrow \nabla_x^{1}\,\phi(t,x)$$

$$\psi(t,y) = \psi_0(t-y) + \psi_0(t+y) - \frac{\mu}{2}\Big(q(t+y+\epsilon)$$
$$+ q(t-y-\epsilon)\,\Theta$$
$$+ q(t+x-\epsilon)\Big)$$

$$\partial_t^2 q = \partial_t \left(\lambda \underbrace{\phi(t,-\epsilon)}_{} + \mu \Psi(t,-\epsilon) \right)$$

$$\partial_t^2 q + \frac{(\lambda^2-\mu^2)}{2} \partial_t \left(q(t) + q(t-2\epsilon) \right)$$

$$= \partial_t \Big(\lambda \phi_0(t+\epsilon) + \mu \Psi_0(t+\epsilon)$$

$$+ \lambda \phi_0(t-\epsilon) + \mu \Psi_0(t-\epsilon) \Big)$$

$\mathrm{Lim}\ \epsilon \to 0$

$$\underbrace{\partial^2 q + (\lambda^2-\mu^2)\partial_t q}_{} = 2\lambda \partial_t \phi_0(t)$$

$$+ 2\mu \partial_t \Psi_0(t)$$

$\lambda^2 > \mu^2 \longrightarrow$ damping of q.

- ϕ_1, ψ_1 dep. drive q.

+ q then drives ϕ, ψ.

Assume $e^{-i\omega t}$

$$\phi_0 = e^{-i\omega(t-x)} + e^{-i\omega(t+x)}$$

$$\psi_0 = e^{-i\omega(t-y)} + e^{-i\omega(t+y)}$$

$\phi_0 = e^{-i\omega t}$

$\psi_0 \to 0$

output

$e^{-i\omega(t+x)} \implies$ combination from ϕ, & χ channels.

<u>Next time</u>