$t = \rho \sinh a\tau$ $\qquad \rho = \frac{1}{a}$ $\qquad \tau$ is proper time

$x = \rho \cosh a\tau$

$a =$ accn.

$u = t - x \qquad v = t + x$

$$\partial_t^2 \phi - \partial_x^2 \phi = 0$$

$$\Downarrow$$

$$\left((\partial_t v)\partial_v + (\partial_t u)\partial_u\right)\phi - \left(\partial_x v\, \partial_v + \partial_x u\, \partial_u\right)^2 \phi$$

$$= 4\partial_u \partial_v \phi = 0$$

$\phi(u)$ or $\phi(v)$ $\qquad \phi(t-x)$, $\phi(t+x)$

$$\phi_\omega^+ = \frac{e^{-i\omega u}}{\sqrt{2\pi|\omega|}} = \frac{e^{-i\omega(t-x)}}{\sqrt{2\pi\omega}} \qquad \phi_\omega^- = \frac{e^{-i\omega v}}{\sqrt{4\pi|\omega|}}$$

$$\phi^-_\Omega = (aV)^{-i\Omega/a}$$

$$V = \rho e^{a\tau} = \qquad aV = e^{a\tau + \ln a\rho}$$

$$(aV)^{-i\Omega/a} = e^{-i\frac{\Omega}{a}(a\tau + \ln a\rho)}$$

$$= e^{-i\Omega(\tau + a \ln a\rho)}$$

$$=$$

$$\langle \phi^-_{\Omega'}, \phi^-_{\Omega}(\Omega) \rangle = \frac{i}{2}\int \phi^*_{\Omega'} \frac{\partial}{\partial t} \phi_\Omega - \partial_t \phi^*_{\Omega'} \phi_\Omega \, dx$$

$$= \frac{i}{2}\int \phi^*_{\Omega'} \partial_V \phi_\Omega - \partial_V \phi^*_{\Omega'} \phi_\Omega \frac{dV}{\left(\frac{\partial V}{\partial x}\right)}$$

$$= \frac{i}{2}\int \phi^*_{\Omega'} \partial_V \phi_\Omega - \partial_V \phi^*_{\Omega'} \phi_\Omega \, dV$$

$$\frac{\cancel{2}\, i\, \Gamma(i\Omega)}{\cancel{2}\, a} \int \left((aV)^{-i\Omega'/a}\right)^* \frac{(aV)^{-i\Omega/a}}{V}\, dV$$

$$V > 0$$

$$= \frac{\Omega}{a} \int_{-\infty}^{\infty} e^{(\ln V + \ln a)\left(+ i\frac{\Omega'}{a} - i\Omega/a\right)}\, d\ln V$$

$$= \frac{\Omega}{a} \int_{-\infty}^{\infty} e^{i\frac{\Omega-\Omega'}{a}\ln a}\, \underbrace{e^{i\frac{\Omega'-\Omega}{a}\ln V}\, d\ln V}$$

$$= \frac{\Omega}{a} \qquad 2\pi\, \delta\left(\frac{\Omega'-\Omega}{a}\right) = \delta(\Omega-\Omega')\, 2\pi$$

$V < 0 \qquad \frac{dV}{V} = \frac{d|V|}{|V|}$

$$\int_{-\infty}^{0} \frac{dV}{V} = \int_{\infty}^{0} \frac{d|V|}{|V|}$$

$$= -\int_{0}^{\infty} \frac{d|V|}{|V|}$$

$$= -\int_{-\infty}^{\infty} d\ln|V|$$

$V < 0$

$$-\frac{i}{2}\int \left(\left(-|V|\right)^{-i\Omega/a}\right)^{*} \partial\left(-|V|\right)^{-i\Omega/a} \, d\ln|V|$$

$$(-1)^{-i\Omega/a} \qquad (-1)^{-i\Omega/a} \quad ?$$

$$(-1) = e^{\pm i\pi}$$

$$(-1)^{-i\frac{\Omega}{a}} = e^{\mp i\pi\Omega/a}$$

$v < 0$

$$-\frac{\Omega}{a} e^{-\pi\Omega/a} \, 2\pi \, \delta\left(\frac{\Omega - \Omega'}{a}\right)$$

$$= \boxed{-} \frac{\Omega}{\cancel{a}} e^{-\pi\Omega/a} \, 2\pi \cancel{a} \, \delta(\Omega - \Omega')$$

$$\langle \phi_{\Omega'}, \phi_\Omega \rangle = 2\pi\Omega \left(1 - e^{-\pi\Omega/a}\right) \delta(\Omega - \Omega')$$

– Norm is +ve for all $\Omega$

+ve and neg.

If I choose $(-1)^{-i\Omega/a} = e^{+i\pi(-i\Omega/a)} = e^{\pi\Omega/a}$

Not sign of $\Omega$ that determins whether $V^{-i\Omega/a}$ is +ve norm or -ve norm, it is whether I choose $(-1) = e^{-i\pi}$ or $e^{+i\pi}$

Not, I could have chosen my modes to be

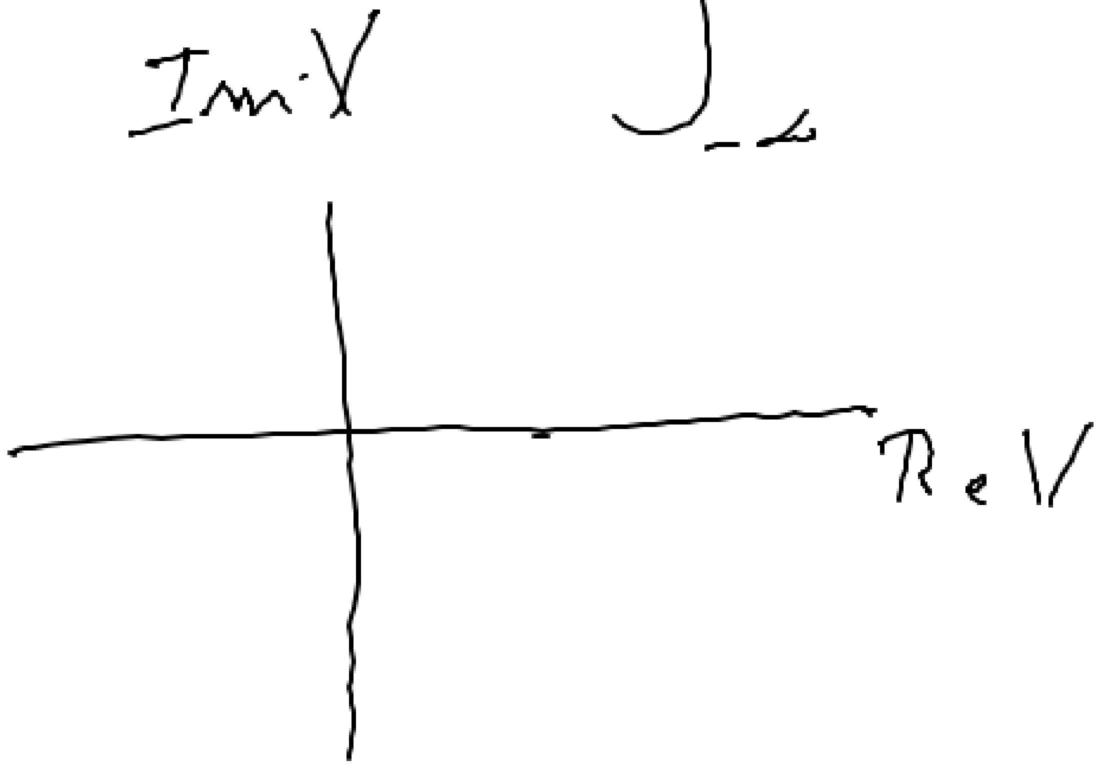
$$V^{-i\Omega/a} \quad V>0 \;, \quad 0 \quad V<0$$

$$(-V)^{-i\Omega/a} \quad V<0 \;, \quad 0 \quad V>0$$

$$\langle \phi^-_\omega , \phi^-_\Omega \rangle$$

$$= \frac{i}{2} \int e^{i\omega V} \partial_V V^{-i\Omega/a} - \partial_V e^{i\omega V} V^{-i\Omega/a} \, dV$$

$$= -i \int_{-\infty}^{\infty} (i\omega) \underline{e^{i\omega V}} V^{-i\Omega/a} \, dV$$




If $(-1) = e^{i\pi}$

Then $V^{-i\Omega/a}$ is made up of $e^{i\omega}$ modes

If $(-1) = e^{-i\pi}$

Then $V^{-i\Omega/a}$ for all $\Omega$ are $e^{-i\omega V}$ modes.

Normalization is

$$\sqrt{2\pi\Omega\left(1-e^{-\pi\Omega/a}\right)}$$

$$\Phi = \int_{-\infty}^{\infty} A_\Omega \, \phi_\Omega \, d\Omega + \text{H.C.}$$

$$\left(aV\right)^{-i\Omega/a} \quad V>0 \qquad 0 \quad V<0$$

$$\left(-aV\right)^{i\Omega/a} \quad V<0 \qquad 0 \quad V>0$$

$$\Phi = \int_0^{\infty} \frac{R_\Omega (aV)^{-i\Omega/a}\,\theta(V)}{\sqrt{2\pi|\Omega|}} + \frac{L_\Omega (-aV)^{i\Omega/a}\,\theta(-V)}{\sqrt{2\pi|\Omega|}}\, d\Omega$$

$$+ \text{ H.C.}$$

$$\theta(V) = \begin{cases} 1 & \text{if } V > 0 \\ 0 & \text{if } V < 0 \end{cases}$$

$R_\Omega$ and $L_\Omega$ are annih ops. for these Rindler modes.

$$e^{-i\Omega\tau} \qquad e^{i\Omega\tau}$$

$$R_\Omega = \frac{A_\Omega}{\sqrt{1-e^{-\pi\Omega/a}}} + \frac{A^\dagger_{-\Omega}}{\sqrt{e^{\pi\Omega/a}-1}}$$

$$L_\Omega = \frac{A_{-\Omega}}{\sqrt{1-e^{-\pi\Omega/a}}} + \frac{A^\dagger_\Omega}{\sqrt{e^{\pi\Omega}-1}}$$

$A_\Omega|0\rangle = 0$ is usual Mink vac.

$R_\Omega|0\rangle_R,\ L_\Omega|0\rangle_R = 0$ is "Rindler" Vacuum

$|0\rangle_R$ in terms of $|0\rangle_m$.

$$|0\rangle_M = e^{-\pi\Omega/a\, R^\dagger L^\dagger}|0\rangle_R .$$

$$e$$

Recall $$\frac{R^{\dagger n}}{\sqrt{n!}}|0\rangle_R = |n_R\rangle_R$$

$$|0\rangle_M = \sum_n e^{-\pi\Omega/a}|n_R\, n_R\rangle_R .$$

$$P_{n_R} = e^{-2\pi\Omega n/a} \left( = |\langle n_R|0\rangle_M|^2 \right.$$

$$= e^{-H_R/k_B T}$$

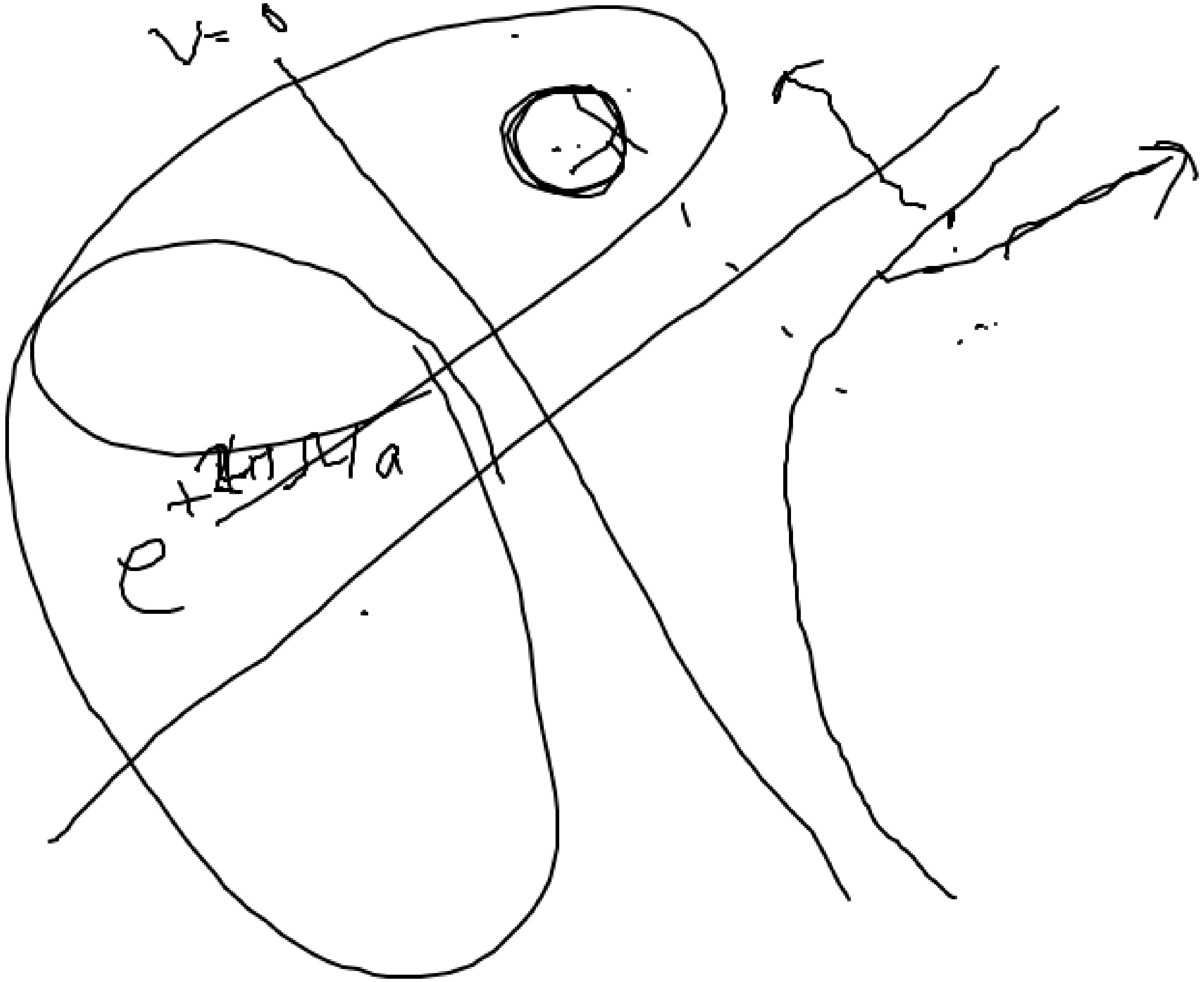
The modes from the Rindler point of view are populated as a thermal bath.

$$T = \frac{a}{2\pi}\left(\frac{\hbar}{c}\right)$$

Mink point of view.

$$\int e^{i\epsilon\tau}\,\Phi(\tau)\,|0\rangle \Longrightarrow A^{+}_{-\epsilon}\,|0\rangle$$

1 particle state.


V= 0