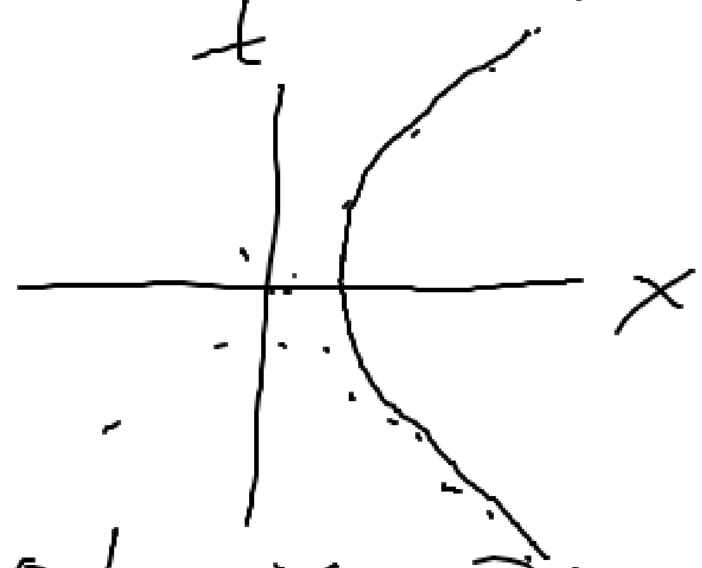
$$P_{\uparrow} \propto \int \langle \Phi(t(\tau)\, x(\tau))\, \Phi(t(\tau'), x(\tau')) \rangle \, \underline{e^{iE(\tau' - \tau)}} \, d\tau$$

$$t = \frac{1}{a} \sinh(a\tau)$$ const Rel. accn.

$$x = \frac{1}{a} \cosh(a\tau)$$

$$t^2 - x^2 = -\frac{1}{a^2}$$



Assymp. $t - x = 0, \; t + x = 0$

$\langle 0 | \Phi(t,x)\, \Phi(t',x') | 0 \rangle$

Wightman function.

Massles field

$$\partial_t^2 \phi - \nabla \cdot \nabla \phi = 0$$

$$\Phi = \int \left(A_\omega \frac{e^{-i(\omega t - \vec{k}\cdot\vec{x})}}{\sqrt{(2\pi)^3 |\omega|}} + \frac{A_\omega^+ e^{i(\omega t - \vec{k}\cdot\vec{x})}}{\sqrt{}} \right) d^3k$$

$\langle 0 | A^+ = (A|0\rangle)^+$

$\langle 0 | \Phi(t,x)\, \Phi(t',x') | 0 \rangle$

$$= \langle 0 | A_k \frac{e^{-i(\omega t - k\cdot x)}}{\sqrt{(2\pi)^3 |\omega|}} \frac{A_{k'}^+ e^{i(\omega' t' - k'\cdot x')}}{\sqrt{(2\pi)^3 |\omega'|}} d^3k | 0 \rangle$$

$$= \iint \delta^3(\vec{k}-\vec{k}') \frac{e^{i\omega(t'-t) - \vec{k}(x'-x)}}{(2\pi)^3 |\omega|} d^3k\, d^3k'$$

$$= \int$$

$\vec{k} \rightarrow$ polar. k, θ, ϕ.

$\theta \rightarrow$ angle between

$\vec{k}$, $(x'-x)$

$$\omega^2 = \vec{k}\cdot\vec{k} \rightarrow k^2$$

$$d^3k = k^2 dk \underbrace{\sin\theta\, d\theta}_{d\cos\theta} d\phi$$

$$\frac{1}{(2\pi)^3}\int e^{i(k(t-t') - k|x-x'|\cos\varphi)}\, k\, d\cos\theta\, d\phi$$

$$= \frac{1}{(2\pi)^2}\int_0^{\infty} \overbrace{\frac{-e^{i(k(t-t') - k|x-x'|)} + e^{ik(t-t')+k|x-x'|}}{-i\cancel{k}|x-x'|}\cancel{k}\,dk}^{e^{-\epsilon k}}$$

$\lim_{\epsilon\to 0}$

$$= \frac{1}{(2\pi)^2}\frac{1}{-i|x-x'|}\left(\frac{1}{i(t-t')-|\vec{x}-\vec{x}'|} - \frac{1}{i(t-t')-|x-x'|}\right)$$

$$= \frac{1}{2\pi^2}\,\cancel{\frac{1}{(-t')(i)}}\,1\left(\frac{1}{\underbrace{(t-t')^2 - |\vec{x}-\vec{x}'|^2}_{\frac{a^2}{2(1-\cosh a(\tau-\tau'))}}}\right)$$

Thermal spectrum.

$$P_\uparrow \quad (\;)\;\frac{e^{-2\pi E/a}}{1-e^{-2\pi E/a}}$$

$P_\downarrow$ – emission of particle.

$$e^{iE(T'-T)} \Longrightarrow e^{-iE(T'-T)}$$

$$P_\downarrow = (\;)\;\frac{1}{1-e^{2\pi E/a}} \qquad T = \frac{a}{2\pi}$$

$$1919 - \text{Einstein} \quad \frac{P_\uparrow}{P_\downarrow} = e^{-E/kT} = e^{-2\pi E/a}$$

Vacuum state.

Accel detector $\Longrightarrow T = \dfrac{a}{2\pi}$

$t, x, y, z \Longrightarrow$ New coord.

τ, ρ, y, z

$$t = \rho \sinh a\tau \qquad \rho = \frac{c^2}{a}$$

$$x = \rho \cosh a\tau$$

$$(\partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2)\phi = 0$$

$$\left(\frac{1}{\rho^2}\frac{\partial_\tau^2}{a^2} - \frac{1}{\rho}\partial_\rho \rho \partial_\rho - \partial_y^2 - \partial_z^2\right)\phi = 0$$

Drop to 2-D $\quad \tau, \rho$

$\quad t, x$

$$\partial_t^2 \phi - \partial_x^2 \phi = 0$$

$$\frac{1}{\rho^2} \partial_{a\tau}^2 \phi - \frac{1}{\rho} \partial_\rho (\rho \partial_\rho) \Phi = 0$$

$$\frac{1}{\rho^2} \left[\partial_{a\tau}^2 \phi - (\partial_{\ln \rho})^2 \phi \right] = 0$$

$$t = \rho \sinh a\tau$$

$$x = \rho \cosh a\tau$$

$$\phi_\Omega = \frac{e^{-i\omega(a\tau - x\ln\rho)}}{\sqrt{\underbrace{2\pi}\,|\omega|}} \qquad \omega^2 = k^2$$

not $(2\pi)^3$ $\longrightarrow$ no y, z

$\rho > 0$ $\ln\rho$ $-\infty$ to ∞.

$$\phi^*_\Omega = \frac{e^{+i(\omega a\tau - k\ln\rho)}}{\sqrt{2\pi|\omega|}}$$

$$\Pi_\Omega = \frac{1}{\rho} \partial_{a\tau} \phi_\Omega$$

$$\text{Norm} = i\int(\phi^*_\Omega \cdot \Pi_\Omega - \Pi^*_\Omega \cdot \phi_\Omega)\,dk$$

$$= \delta(k, k')$$

For $\rho < 0$

$$t = \rho \sinh a\tau$$

$\tau \uparrow \Rightarrow t \downarrow$ if $\rho < 0$.

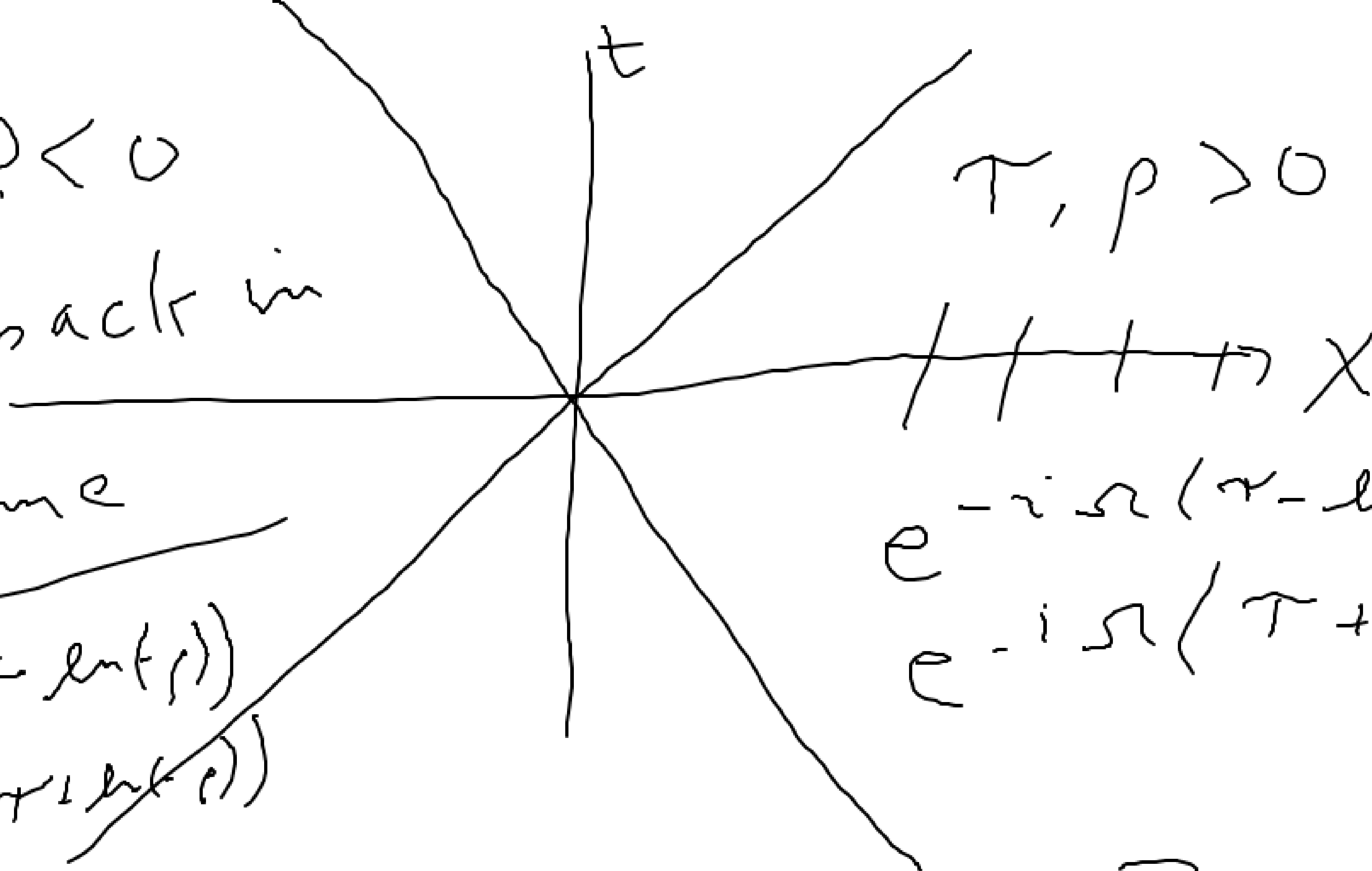
for $\rho < 0$. $\quad \Pi = \frac{1}{\rho} \frac{\partial \phi}{\partial \tau}$ $\quad d\rho$

N

$$= \frac{\partial \phi}{\partial \tau}$$

Norm for $-\rho$ switches sign.

$$-i \int \phi^* \partial_\tau \phi - \phi \partial_\tau \phi^* \, d\ln|\rho|$$



$T, \rho < 0$

T goes back in time

$e^{i\Omega(T - \ln(\rho))}$

$e^{i\Omega(T + \ln(\rho))}$

L_Ω

$T, \rho > 0$

$e^{-i\Omega(T - \ln \rho)}$ Right

$e^{-i\Omega(T + \ln \rho)}$ left moving

R_Ω

$R|0\rangle_{\text{Rindler}} = L|0\rangle_{\text{Rindler}} = 0$?

Falling $\rightarrow$ $|0\rangle_{\text{Rind}} \neq |0\rangle_{\text{mink.}}$

$$U = t - X \qquad u = a\tau - \ln\rho$$
$$V = t + X. \qquad v = a\tau + \ln\rho$$

Null coord.

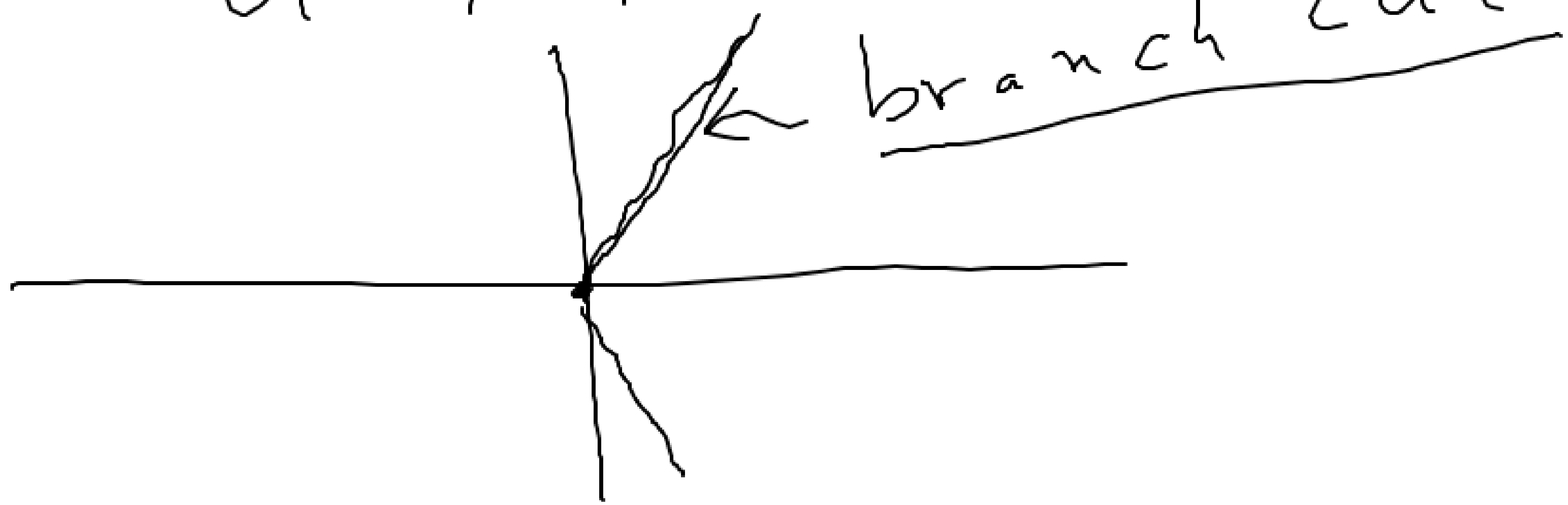
$$e^{-i\Omega(a\tau - \ln\rho)} = e^{-i\Omega(u)}$$
$$e^{-i\Omega(t-x)} = e^{-i\Omega(U)}$$
$$U = \rho(\sinh a\tau - \cosh a\tau) = -e^{-a\tau}\rho$$
$$= -e^{-(a\tau - \ln\rho)} = e^{-u}$$
$$u = -\ln(-U) \qquad \rho > 0$$
$$\underline{e^{-i\Omega u}} = \underbrace{(-U)^{i\Omega}} = e^{i\Omega \ln(-U)}$$

$\ln(-u)$ has branch cut at $u=0$.

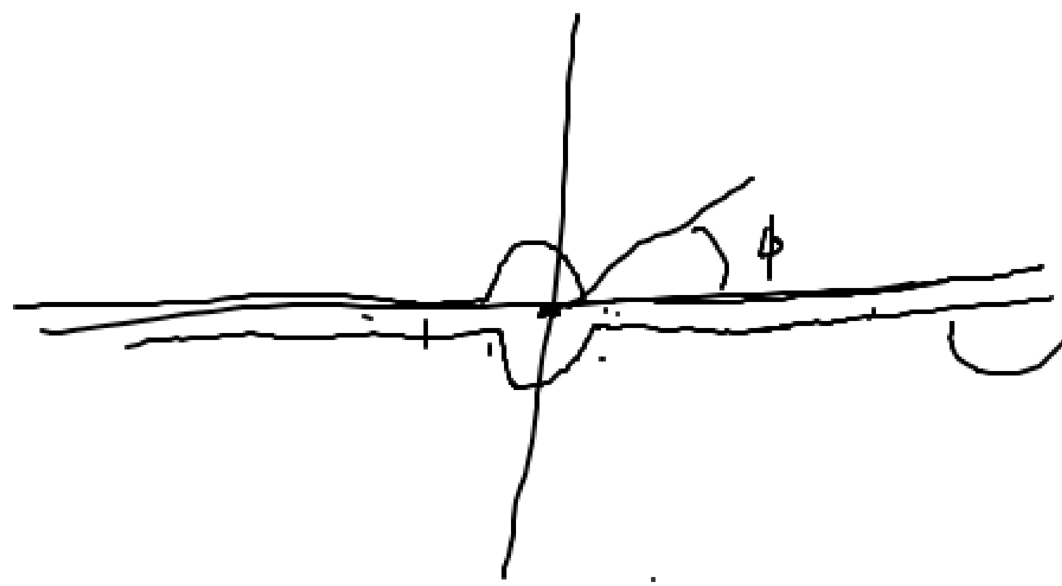
u complex

$$\ln(u) = \ln|u| + \underline{i\,\phi_u}$$

$$u = |u| e^{i\phi}$$



If I go from $u>0$ to $u<0$ → I can go around $u=0$ in compl plain either clockwise or counter clock.



$\ln u \to \ln(-u) - \pi$

$\ln u \to \ln(-u) + \pi$

$$\underline{\int e^{i\omega u}(-u)^{i\Omega}\,du}$$

$\omega > 0$, then as $\mathrm{Im}\, u \to \infty$

$e^{-\omega\infty} \to 0$

$\times 0$

$= 0$

$$\int e^{-i\omega u}(-u)^{i\Omega}\,du$$

$= 0$ with other branch cut

If I have branch cut to neg Im u.

$e^{-u^{i\Omega}}$ has zero components that go as $e^{+i\omega u}$. True for all values of Ω pos or neg.

Branch cut to lower u_{Im}.

$u^{i\Omega} = \int d\omega\, e^{i(\omega u)}\, d\omega$

$u^{i\Omega} = \int d\omega\, e^{-i\omega u}$ modes.

branch cut to $+$ u_I

Thus. modes $U^{i\Omega}$
branchcut down
are ~~on~~ Mink neg freq +
negnorm modes. for
all values of Ω, +ve
or neg.