Particle is what is detected by a particle Detetor

$\Phi(x)$, $\Pi(X)$

q_i $\quad$ P_i

$\phi_\alpha(x)$, $P_\alpha(x)$

$$\frac{1}{2}\int\left(\partial_t\phi(x)\right)^2 - (\nabla\phi)\cdot\nabla\phi - m^2\phi^2 \, d^3x$$

$$\pi = \partial_t\phi(x)$$

$$H = \frac{1}{2}\int\left(\pi(x)^2 + \nabla\phi\cdot\nabla\phi + m^2\phi^2\right)d^3x$$

$$\langle \phi_\alpha, \phi_\beta \rangle = \frac{i}{2}\int (\phi_\alpha^* \Pi_\beta - \Pi_\alpha^* \phi_\beta)\, d^3x$$

Modes $\{\phi_\alpha\}$ is +ve unit norm.

$$A_\alpha = \langle \phi_\alpha, \Phi \rangle$$

$$A_\alpha |0\rangle = 0$$

$$\Phi = \sum_\alpha \left(A_\alpha \phi_\alpha + A_\alpha^\dagger \phi_\alpha^* \right)$$

$$\left(\langle \phi_\alpha, \phi_\beta \rangle = \langle \phi_\alpha, \phi_\beta^* \rangle = 0, \alpha \neq \beta \right.$$

$$\langle \phi_\alpha, \phi_\alpha \rangle = 1$$

$$\phi_{\vec{k}} = \frac{e^{-i(\omega_k t - \vec{k}\cdot\vec{x})}}{\sqrt{(2\pi)^3 |\omega|}}$$

$$\omega_k^2 = \vec{k}\cdot\vec{k} + m^2$$

$$E^2 = p^2 + m^2$$

$$\Pi_k = \partial_t \phi$$

$$\Phi = \int \left(A_{\vec{k}}\,\phi_{\vec{k}} + A^{\dagger}_{\vec{k}}\,\phi_{\vec{k}}\right) d^3k$$

$$\Pi \quad \Pi = \partial_t \Phi = -i \int \frac{\omega_k A_k - A_k^{\dagger}\omega_k}{}\, d^3k$$

$$A_k|0\rangle = 0$$

$$A_k^{\dagger}|0\rangle \longrightarrow \text{"1 particle state."}$$

Detector → 2 level "atom"

2 states $\underset{g}{|\downarrow\rangle}$, $\underset{e}{|\uparrow\rangle}$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\downarrow\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\uparrow\rangle$$

$$H = \frac{E}{2}\sigma_z$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$[\sigma_x, \sigma_y] = 2i\sigma_z \qquad [\sigma_y, \sigma_z] = 2i\sigma_x$$

$$[\sigma_z, \sigma_x] = 2i\sigma_y$$

$$\sigma_- = \frac{1}{2}(\sigma_x - i\sigma_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_+ = \sigma_-^{\dagger} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$i\partial_t \sigma_- = [\sigma_-, H] = E\sigma_-$$

$$\sigma_-(t) = e^{-iEt}\sigma_-(0)$$

$$\sigma_+(t) = e^{iEt}\sigma_+(0)$$

$$\sigma_x = (\sigma_+ + \sigma_-) = \sigma_+(0)e^{iEt} + \sigma_-(0)e^{-iEt}$$

H. solns for free field
an free detector

$$[\sigma_z, H] = [\sigma_z, \sigma_z]\frac{E}{2} = 0$$

couple detector to
field.

$$|\psi\rangle$$

$$\underbrace{\epsilon\, \sigma_x(t)\, \Phi(t, \vec{0})}_{\text{Time Der.}}$$

$$H_I = \epsilon\, \sigma_x(t)\, \Phi(t, \vec{0}) + \cancel{H_K}$$

Heisenberg solns to free systems

$$i\partial_t|\psi,t\rangle = \underbrace{H_I(t)}_{\epsilon \ll 1}|\psi,t\rangle$$

State

$$\underbrace{|0,\downarrow\rangle}+\delta|\psi,t\rangle \quad 1^{st}\text{ order in }\epsilon$$

$$i\partial_t\,\delta|\psi,t\rangle = \epsilon H_I|0,\downarrow\rangle \quad 1^{st}\text{ order in }\epsilon$$

$$\delta|\psi,t\rangle = \int^t \epsilon\,\underline{\sigma_x(t)}\,\Phi(t,\vec{0})|0,\downarrow\rangle\,dt$$

$$\sigma_-|\downarrow\rangle = 0 \quad \begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\delta|\psi,t\rangle = \epsilon\int^{t} \sigma_+(0)e^{iEt'}\,\Phi(t',\vec{0})\,|0\rangle|\downarrow\rangle\, dt'$$

$$= \epsilon\,|\uparrow\rangle \int^{t} e^{iEt'}\,\Phi(t',\vec{0})\,|0\rangle\, dt'$$

$$P_{\uparrow} = |\langle\uparrow|\psi,t\rangle|^2$$

$$= \epsilon^2 \int^{t} \langle 0|e^{-iEt'}\,\Phi(t',0)\,e^{iEt''}\Phi(t'',0)|0\rangle\, dt'dt''$$

$$= \epsilon^2 \int e^{-i(t'-t'')E}\,\langle 0|\Phi(t',0)\,\Phi(t'',0)|0\rangle\, dt'dt''$$

$$\langle 0|\Phi(t',x')\,\Phi(t'',x'')|0\rangle$$

$$= W(t',\vec{x}';t'',x'')$$ is Wightman function

$$\int \Phi(t,\vec{0})\, e^{iE(t')}\, |0\rangle\, dt'$$

$$= \int^{t} e^{iEt'} \int \frac{\cancel{A_k e^{-i(\omega t' - k\cdot\vec{0})}} + A^{+} e^{i(\omega t' - k\cdot\vec{0})}}{\sqrt{|\omega|(2\pi)^3}}\, |0\rangle\, d^3k\, dt' \qquad = 0$$

$$= 2\pi \frac{\delta(\underline{E} + \underline{\omega_k})}{\sqrt{|\omega|(2\pi)^3}}\, d^3k = 0$$

If initial state is vacuum.
Detector does not click.
!!

Single particle

Mode $\phi_0 \rightarrow \langle \phi_0, \phi_0 \rangle = +1$

$$\langle \phi^*_{\vec{k}}, \phi_0 \rangle = 0$$

ϕ_0 is made up of only $\phi_{\vec{k}}$ pos. norm modes.

$A_\phi = \langle \phi_0, \Phi \rangle$ is made up of only $A_{\vec{k}}$.

$A_\phi |0\rangle = 0$. 1 particle in mode

$|1_\phi\rangle = A^\dagger_\phi |0\rangle$. $\rightarrow$ 1 particle ϕ_0 type.

$$|\psi\rangle = A^\dagger_\phi |0\rangle |\downarrow\rangle$$

$$\langle \uparrow | \delta | \psi \rangle = \epsilon \int^t e^{iEt'} \Phi(t, \vec{0}) A^\dagger_\phi |0\rangle \, dt'$$

$$= \epsilon \int e^{iEt} \overbrace{\left[\Phi(t', 0), A^\dagger_\phi \right]}^{\phi_0(t,0)} |0\rangle \, dt'$$

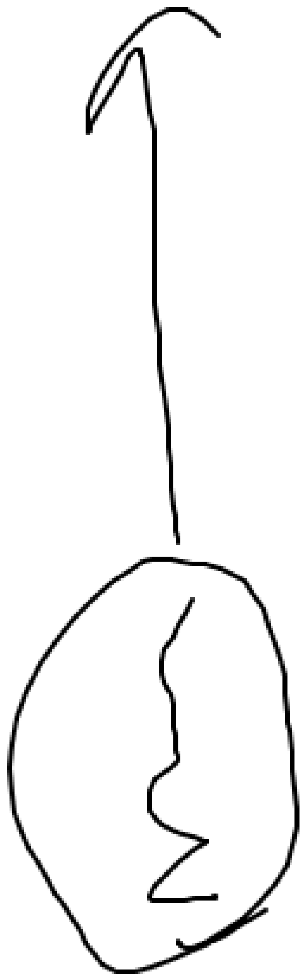
$$+ \epsilon \int^t e^{iEt'} A^\dagger_\phi \Phi(t', 0) |0\rangle \, dt'$$

$$= 0$$

$$\{ \phi_0 , \phi_i \} \qquad \langle \phi^*_k , \phi_i \rangle = 0$$

$$\langle \phi_i , \phi_j \rangle = \delta_{ij} , \quad \langle \phi^*_i , \phi_j \rangle = 0$$

$$\langle 1 | \delta | \psi, t \rangle = \int \underbrace{e^{iEt'} \phi_0(t', 0)}\, |0\rangle \, dt'$$



$\vec{0}$

If mode misses $\vec{0}$, then detector does not click

$$P_1 = \epsilon^2 \iint \underbrace{\phi^*(t', \vec{0})\, \phi(t'', 0)}_{\text{Schroed prob of particle at } \vec{0}} e^{-iE(t'-t'')} \, dt' \, dt''$$

If mod passes over posn of detector, detector clicks with Prob prop to Four. trans. of mode at posn of detector.

<u>Non local in time</u>

(if $t_r - t_i < 1/\epsilon$ then

$$\int_{t_i}^{t_r} e^{i\epsilon t}\, \underline{\Phi}(t,0)\,|0\rangle \neq 0$$

$$\int_{t_i}^{t_r} e^{i\epsilon t'} e^{i\omega t'}\, dt' \neq 0$$

Good model of particle detetor!

Glauber 1960's