QFT

$\vec{E}, \vec{B}(t, \vec{x})$

Max eqns.

$\downarrow$

$A_\mu \qquad A_0, \vec{A}$

Simpler systems.

particle physics. $\longrightarrow$ All interaction are point int.

<u>Fields</u>

Scalar field

$$\longrightarrow \partial_t^2 \vec{E} - \nabla^2 \vec{E} = 0$$

$$\tilde{t} = \gamma(t - vx)$$
$$\tilde{x} = \gamma(x - vt)$$
$$\tilde{y} = y$$
$$\tilde{z} = z$$

$$\partial_{\tilde{t}}^2 \vec{\tilde{E}} - \tilde{\nabla}^2 \vec{\tilde{E}} = 0$$

$$\phi(t, x) \Longrightarrow \partial_t^2 \phi - \nabla^2 \phi + m^2 \phi = 0$$

$$e^{-i(\omega t - kx)} = 0$$

$$\omega^2 = k^2 + m^2$$

$$\hbar \omega = E \qquad E^2 = p^2 + m^2$$
$$\hbar \vec{k} = \vec{p}$$

$$\mathcal{L} = \frac{1}{2}\int\left((\partial_t\phi)^2 - (\vec{\nabla}\phi)^2 - m^2\phi^2\right)d^3x$$

$$\vec{\nabla}\phi\cdot\vec{\nabla}\phi$$

$$\frac{\delta I}{\delta\phi} \Longrightarrow \text{Eqn for } \phi.$$

$$H = \frac{1}{2}\int \pi^2 + \underbrace{\vec{\nabla}\phi\cdot\vec{\nabla}\phi + m^2\phi^2 \quad d^3x}$$

$$\left(\frac{\phi(x+\Delta x) - \phi(x)}{\Delta x}\right)^2$$

$$\phi(x) \quad \phi(x+\Delta x)$$

$$\partial_t\phi(t,x) = \frac{\delta H}{\delta\pi(t,x)} = \pi(t,x)$$

$$\partial_t\pi(t,x) = \frac{\delta H}{\delta\phi(t,x)} = \nabla^2\phi(t,x) - m^2\phi(t,x)$$

Norm

$$\langle \tilde{\phi}(t,x), \phi(t,x) \rangle = \frac{i}{2} \int \left(\tilde{\phi}^* \Pi - \tilde{\Pi}^* \phi \right) d^3x$$

$$\partial_t \langle \tilde{\phi}, \phi \rangle = 0$$

Modes.

$$\phi_{\vec{k}} = e^{-i(\omega t - \vec{k}\cdot\vec{x})} \Big/ \sqrt{(2\pi)^3 |\omega_k|}$$

$$\langle \phi_k, \phi_{k'} \rangle = \omega_k \underline{(2\pi)^3} \, \delta^3(\vec{k} - \vec{k}') \Big/ 2\pi |\omega_k|$$

$= 1$ if $\omega_k > 0$, +ve norm.

$\omega_k < 0$ −ve norm.

$$\Pi_k = \partial_t \Phi = \frac{-i\omega}{\sqrt{(2\pi)^3 |\omega|}} e^{-i(\omega t - \vec{k}\cdot x)}$$

$$\omega_k = \pm\sqrt{\vec{k}\cdot\vec{k} + m^2}$$

$$A_{\vec{k}} = \langle \phi_k , \Phi \rangle \qquad e^{-i(\omega t - \vec{k}\cdot x)}$$

$$A^\dagger_{\vec{k}} = \langle \phi^*_{\vec{k}} , \Phi \rangle$$

$$[\Phi(t,x), \Pi(t,x')] = i\hbar\, \delta^3(\vec{x} - \vec{x}')$$

$$[A_{\vec{k}}, A^\dagger_{\vec{k}'}] = \delta^3(\vec{k} - \vec{k}')$$

$$\underline{\Phi = \int A_{\vec{k}}\, \phi_{k} + A^{\dagger}_{\vec{k}}\, \phi^{*}_{\vec{k}}}$$

$$\Phi^{\dagger} = \Phi$$

$$\Pi_{k} = \int \left(A_{k} (-i\omega)\, \phi_{\vec{k}} - A^{\dagger}_{k} (i\omega)\, \phi^{*}_{\vec{k}} \right) d^{3}\vec{k}$$

$$H = \int \omega_{k} \left(\frac{1}{2} \left(A^{\dagger}_{k} A_{k} + A_{k} A^{\dagger}_{k} \right) \right) d^{3}\vec{k}$$

$$A^{\dagger}_{k} A |0\rangle = 0$$

Minimum Energy.

$\tilde{\Phi}_i$

$$\tilde{A}_i = \langle \phi_i , \Phi \rangle = \frac{i}{\hbar} \int \left(\tilde{\Phi}_i^* \Pi - \tilde{\Pi}_i^* \Phi \right) d^3x$$

$$\tilde{A}_i = \int \left(\ldots A_k + \ldots A_k^+ \right) d^3k$$

$$\tilde{A}_i = \int \left(\langle \tilde{\phi}_i , \phi_k \rangle \ldots \langle \tilde{\phi}_i , \phi_k^* \rangle \right)$$

If $\langle \phi_i , \phi_k^* \rangle = 0 \quad \forall k$

Then $\tilde{A}_i = \int \ldots \underline{A_k} \, d^3x$

$A_k \underline{|0\rangle} = 0 \qquad \tilde{A}^i |0\rangle = 0$

$\tilde{A}^i |\tilde{0}\rangle = 0$

If $\langle \phi , \phi_k^* \rangle \neq 0$

is squeezed state

for A_k

$j_l(kr) Y_{lm}(\theta, \phi)$

Vac. states under Lor. Trans. remains new vacuum.
(Not true for accln).

Particles - energy transport

$A^+ \longrightarrow$ create particles $e^{-i(\omega t - \vec{k}\cdot\vec{x})}$??

Cosm. context

$a(t) \Longrightarrow$ If find min energy state at time t, then it is ∞ bigger than min energy a time $t + \Delta t$.

$\Downarrow$

Redifine Φ, Π in time dep manner $\Longrightarrow$ $\hat{H}$

min $\hat{H}$ state. $\Longrightarrow$ finite large energy at insdant later.

Particle is what a particle detector detects.

Interactions, time dependent theory, ...