$$\phi(t, x)$$

$$\mathcal{L} = \frac{1}{2}\int (\partial_t \phi)^2 - (\partial_x \phi)^2 \, dx$$

$$S = \int \mathcal{L} \, dt$$

$$\Pi = \frac{\delta \mathcal{L}}{\delta \partial_t \phi(t, x)} = \partial_t \phi$$

$$H = \int \pi \, \partial_t \phi - \mathcal{L} \qquad \partial_t \phi \to \Pi$$

$$= \frac{1}{2}\int \left(\Pi^2(t, x) + (\partial_x \phi)^2\right) dx$$

$$\partial_t \phi = \Pi \qquad \partial_t \Pi = \nabla^2 \phi = \underline{\partial_x^2 \phi}$$

$$\phi_k(x) = \frac{e^{i(kx)}}{\sqrt{2\pi}} \phi_k(t) \qquad \Pi_k(t, x) = \frac{e^{ikx}}{\sqrt{2\pi}} \Pi_k(t)$$

$$H_k = \frac{1}{2}\int \pi_k^2 + k^2 \phi_k^2$$

$$\langle \phi_k', \phi_k \rangle = \frac{i}{2}\left(\phi_k'^{*} \pi_k - \pi_k'^{*} \phi_k \right)$$

$$\phi_k = \frac{1}{\sqrt{|k|}} e^{-i|k|t} \qquad \pi_k = -i\sqrt{k}\, e^{i|k|t}$$

$$\langle \phi_k, \phi_k \rangle = 1$$

$$A_k, \; A_k^\dagger \qquad A_k|0\rangle = 0 \qquad \frac{A^{\dagger n}}{\sqrt{n!}}|0\rangle = |n\rangle$$

$\phi_k \to$ Positive freq mode.

$$[A_{k'}, A_k^\dagger] = \delta(k - k')$$

$$\mathcal{L} = \frac{1}{2}\int \underline{a^3(t)}(\partial_t \phi(t,\vec{x}))^2 - a\,(\partial_{\vec{x}}\phi)\cdot(\partial_{\vec{x}}\phi)\,d^3x$$

$$\pi = a^3 \partial_t \phi$$

$$H = \frac{1}{2}\int \frac{\pi^2}{a^3} + a\,(\nabla\phi)\cdot(\nabla\phi)\quad d^3x$$

$$\dot{\phi} = \frac{\pi}{a^3} \qquad \dot{\pi} = a\,\nabla^2\phi$$

$$\phi_k(t,x) = \phi_k(t)\,\frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{(2\pi)^3}}$$

$$\pi_k(t,x) = \pi_k(t)\,\frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{(2\pi)^3}}$$

$\dot{\phi}_k = a^3(t)\,\pi_k.$

$\dot{\pi}_k = -k^2 a(t)\,\phi_k.$

$k^2 = \vec{k}\cdot\vec{k}$

$$H = \frac{1}{2}\left(\frac{\pi_k^2}{a^3} + k^2 a\,\phi_k^2\right)$$

Particles are excitations of Energy.

$|0\rangle_t$: minimum of the energy.

Demand Min of energy.

$$\partial_t \phi_k = -i\omega \phi_k \qquad \partial_t \Pi_k = -i\omega \phi_k \qquad \omega > 0$$

$$-i\omega \phi_k = \frac{\Pi_k}{a^3(t)} \qquad -\omega^2 \phi_k = \frac{\Pi_k i\omega}{a^3} = -\frac{ak^2}{a^3}\phi_k$$

$$= -\frac{k^2}{a^2}\phi_k$$

$$-i\omega \Pi_k = -ak^2 \phi_k$$

$$\Pi_k = -i\frac{|k|}{a^2}\phi_k$$

$$\phi_k = +i\frac{a^2}{|k|}\Pi_k$$

$$\langle \phi, \Pi_{\cdot\cdot} \rangle = \frac{i}{2}\left(\phi_k^* \Pi_k - \Pi_k^* \phi_k\right)$$

$$= \frac{i}{2}\left(\phi_k^*\left(-i\frac{|k|}{a^2}\right)\phi_k\right.$$

$$\left. -\left(i\frac{k}{a^2}\right)\phi_k^* \phi_k\right) = \frac{k}{a^2}|\phi_k|^2$$

$$\phi_k = \frac{a}{\sqrt{|k|}}$$

$$\Pi = -i\frac{\sqrt{k}}{a}$$

But H diag is not a solution of the equations. Instant later in time.

$$\phi_k(t+\delta t) = \phi_k(t) + \frac{\Pi_k(t)}{a^2}\delta t$$

$$\pi_k(t+\delta t) = \pi_k(t) - ak^2\phi_k(t)\,\delta t$$

$$\phi_k(t+\delta t) = \frac{a(t+\delta t)}{\sqrt{|k|}}\ ; \quad \pi = \frac{\sqrt{k}}{a(t+\delta t)}(-i)$$

$$\phi_k(t+\delta t) \simeq \frac{a(t)}{\sqrt{|k|}} + \frac{\dot{a}}{\sqrt{k}}\delta t \qquad \pi = \Pi_k(t) - \frac{\sqrt{k}}{a^2}\dot{a}\,\delta t\,(-i)$$

Ham diag $A_{k\,\delta t} = \alpha A_{k(t)} + \underbrace{\frac{\dot a}{a}\delta t}_{\beta} A^{+}$

$\#$ exitations $\neq |\beta|^2 \approx \left(\frac{\dot a}{a}\delta t\right)^2$

Hubble const.

Total # particles

$$\int_0 \left(\frac{\dot a}{a}\delta t\right)^2 \left(k^2 dk\right) = \infty$$

Energy $\left(k\left(\frac{\dot a}{a}\delta t\right)^2 k^2 dk\right)$

$\propto K^4$

WRONG

Leonard Parker was in 1968

Assymp. expansion.

$$H = \frac{1}{2}\int \frac{\pi^2(x)}{a^3} + (\nabla\phi)^2 a \, d^3x$$

$$H_k = \frac{1}{2}\int \left(\frac{\pi_k^2}{a^3} + a k^2 \cdot \phi_k\right) d^3k$$

$$= \frac{1}{2}\frac{k}{a}\int\left(\frac{\pi_k^2}{ka^2} + a^2 k \, \phi_k\right) d^3x$$

$$T_k = \int \frac{k}{a} \, dt$$

$$\tilde{\pi}_k = \frac{\pi}{\sqrt{ka^2}} \qquad \tilde{\phi}_k = \sqrt{a^2 k}\,\phi_k$$

$$\tilde{H}_k = \frac{1}{2}\int \tilde{\pi}_k^2 + 2\frac{\dot{a}}{a}\Phi_k \pi_k - \tilde{\Phi}_k^2$$

$$\hat{H}_k = \frac{1}{2}\int \hat{\pi}_k^2 + \underbrace{\left(1 - \frac{\ddot{a}}{a}\right)}_{O\left(\frac{1}{k^2}\right)}\left(\hat{\phi}_k\right)^2 d^3k$$

$$' = \frac{d}{d\tau_k} = \frac{1}{k}\frac{d}{d\tau}$$

$$\tau = \int \frac{dt}{a}$$

particles created is finite

In GR. What is a particle.

FIELD Theory $\Rightarrow$

What is a particle?