$$H = \frac{1}{2}\left(p^2 + \Omega(t)^2 x^2\right)$$

$$S = \int \left(p\frac{dx}{dt} - H\right) dt$$

$$\frac{dx}{dt} = p \;; \; \frac{dp}{dt} = -\Omega^2(t)x$$

change variables , x, p

$$p\frac{dx}{dt} \Longrightarrow \bar{p}\frac{d\tilde{x}}{dt} \qquad H \to \tilde{H}(\tilde{p}, \tilde{x})$$

$$H = \frac{1}{2}(p^2 + \Omega^2 x^2) = \Omega\,\frac{1}{2}\left(\frac{p^2}{\Omega} + \Omega x^2\right)$$

$$\tilde{p} = \frac{p}{\sqrt{\Omega}} \qquad \tilde{x} = \sqrt{\Omega}\,x$$

$$P = \sqrt{\Omega}\,\tilde{p} \qquad X = \frac{\tilde{X}}{\sqrt{\Omega}}$$

$$S = \int \sqrt{\Omega}\,\tilde{p}\left(\frac{d}{dt}\frac{x}{\sqrt{\Omega}}\right) - \Omega\,\frac{1}{2}\left(\tilde{p}^2 + \tilde{x}^2\right) dt$$

$$= \int \tilde{p}\,\frac{d\tilde{x}}{dt} + \tilde{p}\tilde{x}\left(\sqrt{\Omega}\,\frac{d\frac{1}{\sqrt{\Omega}}}{dt}\right) + \Omega\,\frac{1}{2}\left(\tilde{p}^2 + \tilde{x}^2\right) dt$$

$$\tau = \int \Omega\, dt$$

$$\int (\cdot\cdot)\frac{d}{dt}(\ )\, dt = \int (\ )\frac{d}{d\tau}(\ )\, d\tau$$

$$d\tau = \frac{d\tau}{dt}\,dt$$

$$\frac{d}{d\tau} = \frac{dt}{d\tau}\frac{d}{dt}$$

$$S = \int \tilde{p} \frac{d}{d\tau} \tilde{x} + \underbrace{\tilde{p} \tilde{x} \sqrt{\Omega(\tau)} \frac{d \, 1/\sqrt{\Omega}}{d\tau}} - \frac{1}{2}\left(\tilde{p}^2 + \tilde{x}^2\right)$$

$$S = \int \tilde{p} \frac{d}{d\tau} \tilde{x} - \frac{1}{2}\left(\tilde{p}^2 + \tilde{x}^2 + 2 \underbrace{\frac{\frac{d}{d\tau}\sqrt{\Omega}}{\Omega}}_{\alpha} \tilde{p} \tilde{x}\right)$$

$$\hat{p} \quad \tilde{p} + \alpha \tilde{x} \qquad \hat{x} = \tilde{x}$$

$$\hat{p} \frac{d\hat{x}}{d\tau} - \underbrace{\alpha \hat{x} \frac{d\hat{x}}{d\tau}} - \frac{1}{2}\left(\hat{p}^2 + (1-\alpha^2)\hat{x}^2\right)$$

$$\cancel{\frac{1}{2}\frac{d}{d\tau}\left(\alpha \hat{x}^2\right)} - \frac{1}{2}\left(\frac{d\alpha}{d\tau}\right) \hat{x}^2$$

$$S = \int \left[\hat{p}\frac{d\hat{x}}{d\tau} - \frac{1}{2}\left(\hat{p}^2 + \hat{x}^2 \underbrace{\left(1 + \underbrace{\frac{\dot{\Omega}}{2\Omega} - \left(\frac{\dot{\Omega}}{\Omega}\right)^2}_{=0}\right)}_{\text{almost const}} \right) \right] d\tau$$

$$\underbrace{\qquad\qquad}_{\hat{\Omega}^2}$$

$$\cdot = \frac{d}{d\tau}$$

$$\hat{x} = \hat{x}_0 \cos\tau - \hat{p}_0 \sin\tau$$

$$\hat{p} = \cdots$$

$$\hat{x} = \tilde{x} \qquad \hat{p} = \tilde{p} + \frac{1}{2}\frac{\dot{\Omega}}{\Omega}\tilde{x}$$

$$\tilde{x} = \sqrt{\Omega}\, x \qquad \tilde{p} = \frac{1}{\sqrt{\Omega}} p$$

$$\tau = \int \Omega(t)\, dt$$

Keep going through procedure.

$\hat{\Omega}$ smaller time dep than Ω

- Approx procedure can only be taken a certain # time - gets worst.

Assymptotic expansion.

$\dot{q} = p \qquad \dot{p} = -\Omega^2 q$

$\ddot{q} = -\Omega^2 q$ (WKB is 1st expansion term)

$1 - \frac{1}{2}\frac{\ddot{\Omega}}{\Omega} + \frac{1}{4}\left(\frac{\dot{\Omega}}{\Omega}\right)^2)$ $\qquad \Omega$

$$S = \int -q\dot{p} - \frac{1}{2}\left(H(q,p) \right)$$

$$-p \sim q \qquad P = -q$$

$$q \sim p \qquad Q = p$$

$$\Rightarrow \int P\dot{Q} - \frac{1}{2} H(-P, Q)\, dt$$

Quantum cosmology

$a^2(t)(dx^2+dy^2+dz^2)$

$a\,dx \rightarrow$ distance in x direction

spacetime dist

$= a^2(dx^2+dy^2+dz^2) - dt^2$

$b(t)\,dt = d\tau$

$\tau = \int b(t)\,dt$

$$-\partial_t^2 \phi + \partial_x^2 \phi + \partial_y^2 \phi + \partial_z^2 \phi = 0$$

$$(\Box \phi = 0)$$

$$\Downarrow$$

$$\mathcal{L} = \frac{1}{2}\int \left((\partial_t \phi)^2 - \frac{1}{a^2}\left[(\partial_x \phi)^2 + (\partial_y \phi)^2 + (\partial_z \phi)^2 \right] \right) \text{dvol.}$$

$a^3\, dx\, dy\, dz$

$$= \frac{1}{2}\int a^3 \left((\partial_t \phi)^2 - \frac{1}{a^2} (\vec{\nabla}\phi)\cdot(\vec{\nabla}\phi) \right) d^3x$$

$\phi(t, x) \longrightarrow$ field $\longrightarrow$ config. var.

$\phi(t, x) \Longrightarrow q_i(t)$ $\qquad \phi \Longleftrightarrow q$

$\qquad\qquad\qquad\qquad\qquad x \Longleftrightarrow i$

$$\pi(t, x) = \frac{\delta I}{\delta\, \partial_t \phi(t,x)} \Longrightarrow \frac{\delta I}{\delta\, \partial_t q_i(t)} \qquad \int d^3x \Longrightarrow \sum_i$$

$$\Pi(t,x) = a^3 \partial_t \phi(t,x)$$

$$H = \sum p_i \dot{q}_i - \mathcal{L}\left(\partial_t \phi_i(q),\ \phi_i(t,x)\right)$$

$$H = \frac{1}{2}\int \left(\frac{\Pi^2}{a^3} + a \vec{\nabla}\phi \cdot \vec{\nabla}\phi\right) d^3x$$

$$\longrightarrow a^3 \left(\frac{1}{a^2}\right)$$

$$\left(\frac{\phi(x+\delta x) - \phi(x)}{\delta(x)}\right)^2$$

$$\frac{\Pi}{a^3} = \partial_t \phi$$

$$\nabla^2 \phi = -k^2 \phi$$

$$\partial_t \Pi = a \nabla^2 \phi$$

$$\phi = e^{i \vec{k} \cdot \vec{x}}$$

$$H = \int \frac{1}{2}\left(\frac{\Pi_k^2}{a^3} + a k^2 \phi_k^2\right) d^3k$$

$$\frac{1}{2}\int \left(\frac{\Pi_k^2}{a^2} + a\,\underline{k^2}\,\phi_k^2\right) d^3k$$

time dep.

$$= \cancel{\frac{1}{a}}\;\frac{1}{2}\int \frac{\Pi_{\vec{k}}^2}{k a^2} + k a^2 \phi_{\vec{k}}^2$$

$$k = \sqrt{\vec{k}\cdot\vec{k}}$$

$$A_k e^{-i(\omega t - \vec{k}\cdot\vec{x})} + A_k^+ e^{i(\omega t - kx)}$$

Not modes

$$\tau = \int \frac{dt}{a}$$

$A_k|0\rangle = 0 \Rightarrow$ Vacuum state

$A_k^+|0\rangle \rightarrow$ 1 particle in k mode.

Norm

$$\langle \tilde{\phi}, \phi \rangle = \frac{i}{2} \int \left[\tilde{\Phi}^*(t,x)\, \Pi(t,x) - \tilde{\Pi}^*(t,x)\, \phi(t,x) \right] d^3x$$

$$\frac{d}{dt}\langle \, , \, \rangle = 0 \qquad \phi(t,x) \text{ are solutions.}$$

quantum field Φ, Π

$$[\Phi(t,x), \Pi(t,x')] = i\,\delta(x-x')$$

$$\langle \phi_k, \phi_{k'} \rangle = \delta(k,k')$$

$$A_k = \langle \phi_k, \Phi \rangle$$