Heisenberg rep.

1 single H.O. q, p, H

find classical soln. $q(t), p(t)$

$\tilde{q}(t), \tilde{p}(t)$

$$\langle \tilde{q}, q \rangle = \frac{i}{2}\left(\tilde{q}^* p - \tilde{p}^* q\right)$$

$$\langle q, q \rangle = \frac{i}{2}\left(q^* p - p^* q\right)$$

$$\frac{d}{dt}\langle \tilde{q}, q \rangle = 0 \quad \text{if } \tilde{q}(t), \tilde{p}(t) \text{ soln}$$

$q(t), p(t)$ soln

q, p $\quad \langle q, q \rangle > 0$

$\langle q, q \rangle = c$ $\quad q \neq \pm q^*$

$p \neq \pm p^*$

$\frac{q}{\sqrt{c}}, \frac{p}{\sqrt{c}}$ is soln.

$\langle q, q \rangle = 1$ Then

$\frac{dA}{dt} = 0$

$A = \langle q, Q \rangle$

$Q(t), P(t)$

$q(t), p(t)$

$Q = Aq + A^+ q^*$ $\quad P = Ap + A^+ p^*$

$\langle q, q^* \rangle = 0$

$A|\underline{0}\rangle = 0$ $\qquad$ $\underline{A^\dagger A}$

Hamiltonian X

q, p so that $H = \Omega A^\dagger A + c$

then each raised state will increase Energy-state by Ω

$A^\dagger|0\rangle \rightarrow$ estate with energy Ω "higher" than energy estate of $|0\rangle$

$$H = \frac{1}{2}\left(p^2 - \Omega^2 q\right)$$

$$q = q_0 \cosh \Omega t + \frac{p_0}{\Omega} \sinh \Omega t$$

$$p = p_0 \cosh \Omega t + \Omega q_0 \sinh \Omega t$$

$$p_0 = -i/\sqrt{2} \qquad q_0 = 1/\sqrt{2}$$

$$A = \langle q, P_0 \rangle \qquad A^+ A \,|0\rangle = 0$$

$$H = \frac{1}{2} P^2$$

$$P = P_0 \quad , \quad q = q_0 + P_0 t \, .$$

$$\partial_t P = 0 \qquad \partial_t q = P$$

$$P_0 = -i \quad , \quad q_0 = 1$$

$$P = -i \qquad q = 1 - i t$$

$$\frac{i}{2} \langle (1+it)(-i) - (i)(1-it) \rangle = 1$$

$$A \, , \quad |0\rangle \quad A|0\rangle = 0 \, , \quad A^{+n}|0\rangle$$

Multiple "oscillators"

$q_i, p_i \qquad i = 1, \dots, N$

$$p = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix} \qquad q = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix}$$

$$H = \frac{1}{2}\left(p^T \underset{\substack{\uparrow \\ \text{matrix}}}{B} p + q^T C q\right) \qquad \begin{aligned} B^T &= B \\ C^T &= C \end{aligned}$$

$$H^T = H \qquad p^T B^T p + q^T C^T q$$

$$\frac{dq}{dt} = \dot{q} = Bp \qquad \dot{p} = Cq$$

$$\langle \tilde{q}, q \rangle = \frac{i}{2}\left(\tilde{q}^{\dagger} p - \tilde{p}^{\dagger} q \right)$$

$$\frac{d}{dt}\langle \tilde{q}, q \rangle = \frac{i}{2}\left(\dot{\tilde{q}}^{\dagger} p + \tilde{q}^{\dagger} \dot{p} - \dot{\tilde{p}}^{\dagger} q - \tilde{p}^{\dagger} \dot{q} \right)$$

$$= \frac{i}{2}\left[\left(\tilde{p}^{\dagger} B p - \tilde{q}^{\dagger} C q \right) - \left(\tilde{p}^{\dagger} B p - \tilde{q}^{\dagger} C q \right) \right.$$

$$= 0$$

$$Q = \begin{pmatrix} Q_1 \\ \vdots \\ Q_N \end{pmatrix} \quad P = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$$

$$A = \langle q, Q \rangle \qquad \langle q, q \rangle = 1$$

Assume q_r, P_r is r^{th} soln.

$$\langle q_r, q_r \rangle = 1 \qquad \langle q_r, q_{r'} \rangle = 0$$

$$\langle q_r, q_r^* \rangle = 0 \qquad \langle q_r, q_{r'}^* \rangle = 0$$

$A_r = \langle q_r, Q \rangle$

$\sum_r A_r^\dagger A_r \rightarrow$ # "particles" in modes q_r

If q_r are such that

$\dot{q}_r = -i\omega q_r$ at $t=0$

$\dot{p}_r = -i\omega p_r$

$B p_r = -i\omega q_r$

$-C q_r = -i\omega p_r$

$\begin{pmatrix} 0 & B \\ -C & 0 \end{pmatrix}$

$$\langle q_r, q_r \rangle = \omega(q_r^\dagger q_r)$$

Annihilation ops $\Longrightarrow$ +ᵛ norm
not +ᵛ freq
(except sometimes

$$[A_r, A_r^\dagger] = 1$$

$$\tilde{g}, \; g_r$$

$$\langle g_r, \tilde{g} \rangle \neq 0$$

$$\langle g_r^*, \bar{g} \rangle = 0$$

$$\tilde{A} = \sum A_r$$

$$0 = \tilde{A}|0) \longrightarrow A_r|0\rangle = 0$$

$$\tilde{A} = A_r \cosh\theta + A_r^{+} \sinh\theta$$

$$|\tilde{0}\rangle = e^{-\tanh\theta \, A_r^{+2}/2} |0\rangle$$

$$\tilde{A} = \cosh\theta\, A_r + \sinh\theta\, A_s^\dagger$$
$$\hat{A} = \cosh\theta\, A_s + \sinh\theta\, A_r^\dagger$$
$$|\tilde{0}, \hat{0}\rangle \longrightarrow \tilde{A}|\tilde{0}, \hat{0}\rangle = \hat{A}|\tilde{0}, \hat{0}\rangle = 0$$
$$|\tilde{0}\hat{0}\rangle = f(A_r^\dagger, A_s^\dagger)|00\rangle$$
$$\left(\cosh\theta \frac{\partial}{\partial A_r^\dagger} + \sinh\theta\, A_s^\dagger\right) f = 0$$
$$\left(\cosh\theta \frac{\partial}{\partial A_s^\dagger} + \sinh\theta\, A_r^\dagger\right) f = 0$$

$$f(A_r^\dagger, A_s^\dagger) = e^{-\tanh\theta\, A_r^\dagger A_s^\dagger}$$

$$|\tilde{0},\hat{0}\rangle = e^{-\tanh\theta\, A_r^\dagger A_s^\dagger}|0,0\rangle$$

$$(A_r^\dagger A_s^\dagger)^n |0,0\rangle$$

2mode squeezed state.

$i \rightarrow \vec{x}$

Q.F.T.

0 fields in cosmology

(cosm. part creation)