q , P $\Downarrow$ $-i(\epsilon T)p^A$ $\epsilon T = \underline{1}$

$\epsilon p^A \Rightarrow U = e^{-\frac{\tilde{q}}{2\Delta^2}}$

$\phi_i(q) = e^{-\frac{q^2}{2\Delta^2}} |\psi_i\rangle$

$U \tilde{\phi}_i(q) = \phi_i(q-A) |\psi_i\rangle$

$A \to a \quad |\psi_i\rangle = |a\rangle$

$U(\phi_i(q)|A\rangle) = \phi_i(q-a)|a\rangle$

$Max|a| \ll \Delta$

$\phi(q-a) \simeq 1 - \frac{A^2}{2\Delta^2}(q-a)^2$

final state $|\psi_f\rangle$

Amp for q:

$$\langle \psi_f | U | \psi_i \rangle \phi(q)$$

$$\approx \langle \psi_f | \Phi_i(q-A) | \psi_i \rangle$$

$$\simeq \langle \psi_f | \left(1 - \frac{(q-A)^2}{2\Delta^2}\right) | \psi_i \rangle$$

$$\simeq \langle \psi_f | \psi_i \rangle - \langle \psi_f | \psi_i \rangle \frac{q^2}{2\Delta^2} + \frac{q}{\Delta^2} \langle \psi_f | A | \psi_i \rangle - \frac{\langle A^2 \rangle}{2\Delta^2}$$

$$\partial_q (\quad) = 0$$

$\langle \phi_f | \phi(q-A) | \psi_i \rangle$ max where

$$\rightarrow q = \frac{\langle \psi_f | A | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

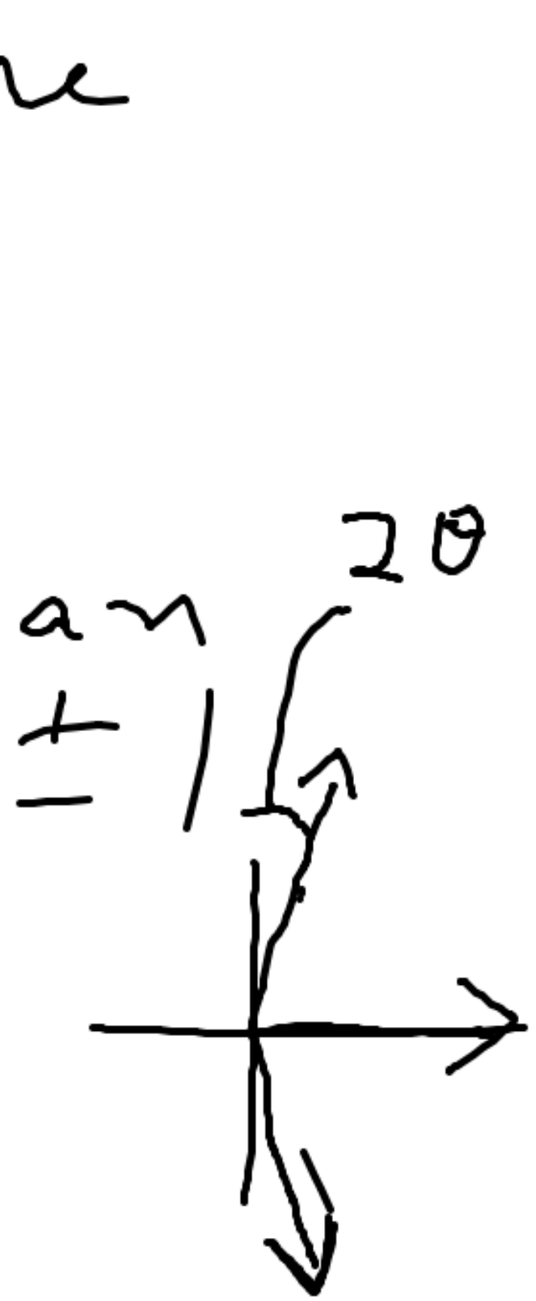
Aharonov, Albert, Vaidman

$$A \rightarrow \sigma_x \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|\psi_i\rangle = \cos\theta\,|\uparrow\rangle + \sin\theta\,|\downarrow\rangle$$

$$|\psi_f\rangle = \cos\theta\,|\downarrow\rangle + \sin\theta\,|\uparrow\rangle$$

$$\theta << \pi/2$$



Peak distr: $\frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi \rangle}$

$\langle \psi_f | \psi_i \rangle = (\cos\theta |\uparrow\rangle + \sin\theta |\downarrow\rangle)$
$(\cos\theta |\uparrow\rangle + \sin\theta |\downarrow\rangle)$

$\langle \psi_f | \sigma_x | \psi_i \rangle = (\cos\theta \langle\downarrow| + \sin\theta \langle\uparrow|)$
$(\cos\theta |\downarrow\rangle + \sin\theta |\uparrow\rangle)$

$= \sin 2\theta$

Peak of amp.: $\frac{1}{\sin 2\theta} \gg 1$ $\quad = 1$

$\sin 2\theta = 1/100$ then peak

occurs at $q = 100$

$q = 1 \Longrightarrow A \rightarrow \underline{1}$

$A \rightarrow$ value 100.

$S = 12.5$ $\quad S_z \to 12.5, 11.5, \ldots, -12.5$

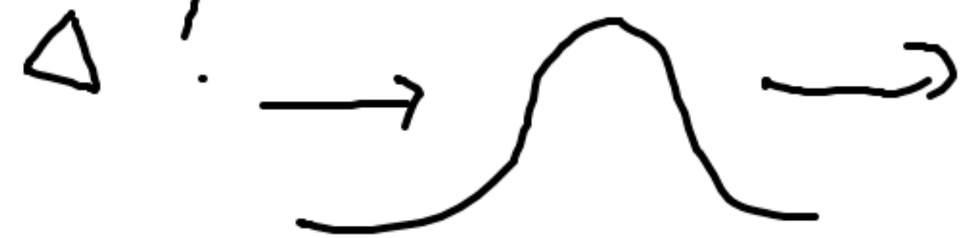
initial $|S_z = 12.5\rangle S_x \to$ $\quad$ "

final $|S_x = 12.5\rangle$ $\quad \underbrace{S_\theta}_{A} = \cos 45^\circ S_z + \sin 45^\circ S_x$

$$\langle \psi_f | A | \psi_i \rangle = \sqrt{2} \times 12.5 \simeq 17.8$$

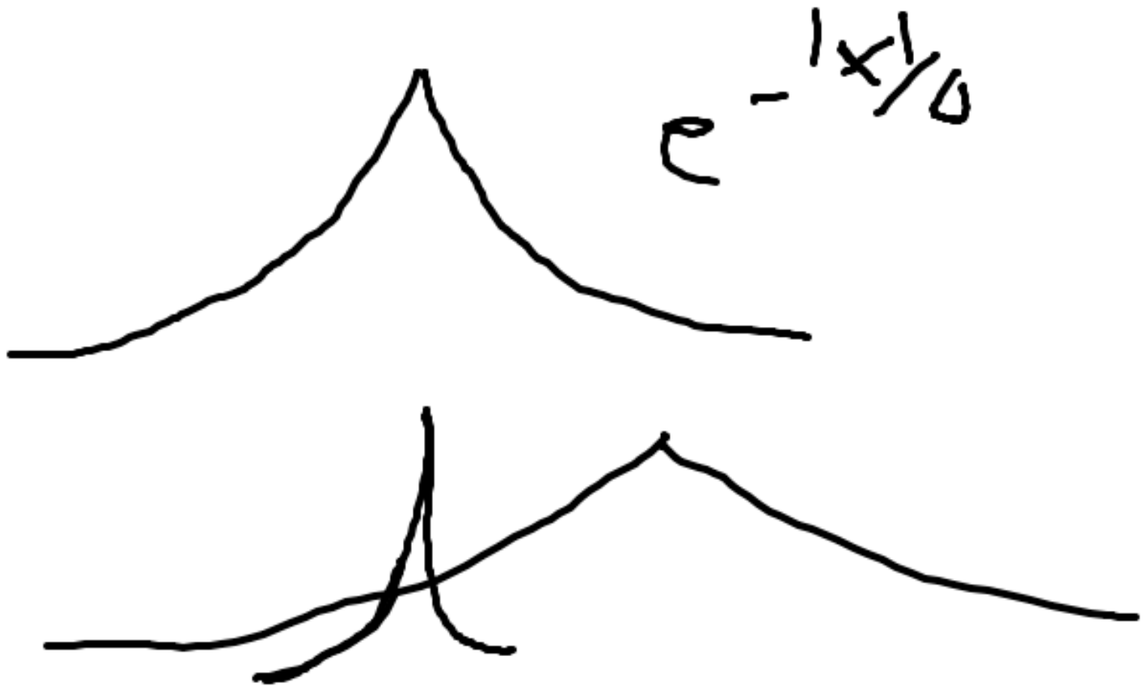
$$\overline{\langle \psi_f | \psi_i \rangle}$$

Δ ?



$e^{-i\lambda t}$

$e^{-i\lambda \sin t}$


$e^{-|x|/\delta}$

$\langle \psi_f | \psi_i \rangle$ small

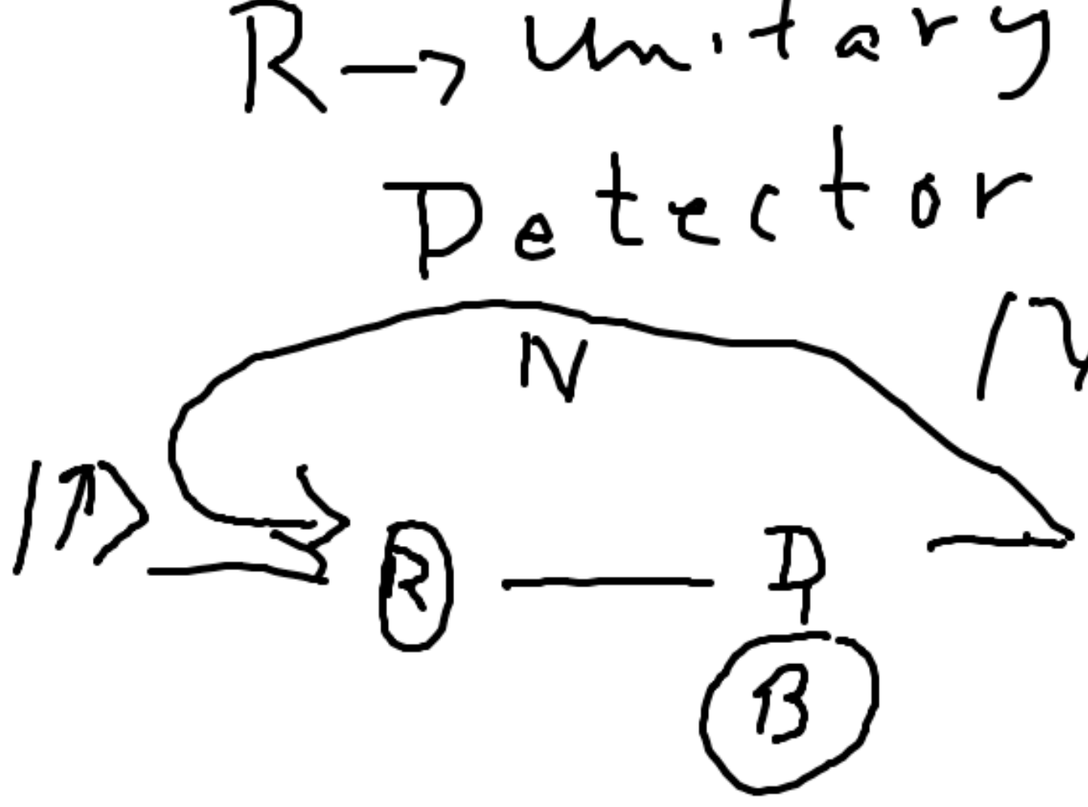
Vaidman Bomb Detector

2 level probe $|\uparrow\rangle, |\downarrow\rangle$

(R) $|\uparrow\rangle \rightarrow \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$

$|\downarrow\rangle \rightarrow \cos\theta|\downarrow\rangle - \sin\theta|\uparrow\rangle$

R → unitary.



Bomb

Dud

$|\psi\rangle \rightarrow |\psi\rangle$

$\theta = \frac{\pi}{2N}$

$\cos n\theta|\uparrow\rangle + \sin(n\theta)|\downarrow\rangle \to R$

$\cos((n+1)\theta|\uparrow\rangle + \sin((n+1)\theta)|\downarrow\rangle$

$\to$ Real bomb

$\underbrace{\cos\theta|\uparrow\rangle}_{\text{Bomb}} + \sin\theta|\downarrow\rangle$

$\cos^2\theta|\uparrow\rangle + \cancel{\cos\theta\sin\theta|\downarrow\rangle}$

$\vdots$

$\cos^N\theta|\uparrow\rangle$

$\approx \left(1 - \frac{1}{2}\left(\frac{\pi}{2N}\right)^2\right)^N$ Prob $\cos^{2N}\left(\frac{\pi}{2N}\right)$

$\sim 1 - \frac{\pi^2}{4N} \sim 1$

Dud.

$$\cos N\frac{\pi}{2N}\,|\uparrow\rangle + \sin N\frac{\pi}{2N}\,|\downarrow\rangle$$

$$= |\downarrow\rangle$$

after N, meas tells if Dud($|\downarrow\rangle$) or Dud($|\uparrow\rangle$)

prob. of det $\sim \frac{\pi^2}{4N}$ small / $N \gg 1$

Can tell with almost cert.
bomb is live or dead
without setting off.
(+ high prob).
Meas without disturbance