$$\text{corr} = \langle A(X+Y) + B(X-Y) \rangle$$

A, B - Sys 1 $\quad X, Y$ Sys 2

class $\langle AX \rangle + \langle AY \rangle \rightarrow \langle AX + AY \rangle$

$$A = \sigma_z' \qquad B = \sigma_x'$$

$$X = (\sigma_z^2 + \sigma_x^2)/\sqrt{2} \qquad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}/\sqrt{2} \Rightarrow \pm 1$$

$$Y = (\sigma_z^2 - \sigma_x^2)/\sqrt{2}$$

$$X + Y = \sqrt{2}\,(\sigma_z^2) \qquad Y = \sqrt{2}\,(\sigma_x^2)$$

$$\text{Corr}: \langle\psi|(\underline{\sigma_z^1\sigma_z^2 + \sigma_x^1\sigma_x^2})\sqrt{2}|\psi\rangle$$

$$|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

$$\sigma_z|\uparrow\rangle = +1|\uparrow\rangle$$

$$\sigma_z|\downarrow\rangle = -|\downarrow\rangle$$

$$\sigma_x|\uparrow\rangle = |\downarrow\rangle$$

$$\sigma_x|\downarrow\rangle = |\uparrow\rangle$$

$$\sigma_z^1\sigma_z^2(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$\sigma_x^1\sigma_x^2(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) = (|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$$

$$\text{Corr} = 2\sqrt{2} > 2$$

Greenberger, Horne, Zeilinger

GHZ

3 particles $\sigma_z^1, \sigma_z^2, \sigma_z^3, \sigma_x^i, \sigma_y^i$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_y|\uparrow\rangle = i|\downarrow\rangle$$

$$\sigma_y|\downarrow\rangle = -i|\uparrow\rangle$$

$$|\psi\rangle = (|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)/\sqrt{2}$$

GHZ or Cat state

$$A = \sigma_x^1 \sigma_y^2 \sigma_y^3$$

$$B = \sigma_y^1 \sigma_x^2 \sigma_y^3$$

$$C = \sigma_y^1 \sigma_y^2 \sigma_x^3$$

$$D = \sigma_x^1 \sigma_x^2 \sigma_x^3$$

~~commute~~

$$D|\psi\rangle = +1|\psi\rangle$$

$$A|\psi\rangle = -1|\psi\rangle$$

$$B|\psi\rangle = -1|\psi\rangle$$

$$C|\psi\rangle = -1|\psi\rangle$$

$$ABC|\psi\rangle = -1|\psi\rangle$$

Class. ABC in statistical state

corr. $|\psi\rangle$

and Quantum $\supset$ direct contradiction

"local" realism or counterfactual realism

Things don't have values unless they are measured. (Bohr).

Hardy chain

2 system. 1, 2

$$|\psi\rangle = |\uparrow\uparrow\rangle \sin\theta + \cos\theta\, |\downarrow\rangle \left(\sin\phi|\uparrow\rangle + \cos\phi\, |\downarrow\rangle\right)$$

A, B – sys 1 X, Y – sys 2

a) I A meas a value +1 and X meas it has +1 always

then B ⇒ +1 always.

x) X ⇒ +1 ↳ measured val Y ⇒ +1

B) B ⇒ +1 then

" Y ⇒ ?

y) A ⇒ +1

$A \to \sigma_z{}^{1}$ $|\uparrow\rangle$ evalue

2 systems. 1, 2

$$|\Psi\rangle = \sum_a \alpha_a |a\rangle_1 |\phi_a\rangle_2$$

$\langle\Psi|\Psi\rangle = 1$

$\langle b|b\rangle = 1$

Meas $|\psi\rangle$ $B_1 \longrightarrow b, |b\rangle$

$$|\Phi\rangle = \sum \alpha_a \langle b|a\rangle |\phi_a\rangle_2$$

$$\frac{|\Phi\rangle}{\sqrt{\mathrm{Prob}_b}}$$

Prob b = $\langle\Phi|\Phi\rangle$

after meas $\dfrac{|b\rangle|\Phi\rangle}{\sqrt{\mathrm{Prob}_b}}$

$A = \sigma_z'$ $= \langle \uparrow_1 | \Psi \rangle = \sin\theta |\uparrow\rangle_2$

$A \Rightarrow 1 \Longrightarrow$ Prob $A \Rightarrow 1 = \sin^2\theta$

$|\uparrow\rangle$

$X = \sigma_z^2$

$|\uparrow\rangle_2 \rightarrow \sigma_z^2$

$= \sin\theta |\uparrow_1\rangle + \cos\theta \sin\phi |\downarrow\rangle$

$X \Rightarrow +1 \quad = \langle \uparrow_2 | \Psi \rangle = \sin\theta |\uparrow\rangle + \cos\theta \sin\phi |\downarrow\rangle$

Same for $|\phi_1\rangle$: $\dfrac{\sin\theta |\uparrow\rangle + \cos\theta \sin\phi |\downarrow\rangle}{\sqrt{\sin^2\theta + \cos^2\theta \sin^2\phi}}$

$|\phi_1\rangle$ is the +1 e state for operator B.

$B = (\sin\theta\,|\uparrow\rangle + \cos\theta\sin\phi\,|\downarrow\rangle)(\sin\theta\,|\uparrow\rangle + \cos\theta$
$+ \sin\phi\,|\downarrow\rangle)$

$y = N(\sin^2\theta\,|\uparrow_2\rangle + \sin^2\phi\cos^2\theta\,|\uparrow_2\rangle$
$+ \cos^2\theta\sin\phi\cos\phi\,|\downarrow\rangle)$

Assume θ, ϕ small. $\theta = \phi$

$y = N\left((\theta^2 + \phi^2)|\uparrow\rangle + \phi|\downarrow\rangle\right)$

$y = (2\phi)|\uparrow_2\rangle + |\downarrow_2\rangle$ $A \Rightarrow 1 \Longrightarrow |\uparrow_2\rangle$

If $A \Rightarrow 1$, y almost never $+1$

$\therefore$ Q.M. violates Class. logic $\because$

Prob $A \Rightarrow 1 \rightarrow \sin^2\theta = \phi^2$.