$$\frac{d}{dt}|\psi,t\rangle = -iH|\psi\rangle \qquad S.$$

$$\frac{d}{dt}A = -i[A,H] \qquad H.$$

$$\frac{d}{dt}$$

---

$$U \qquad U^{\dagger}U = \mathbb{1}$$

$$\frac{dU}{dt} = -iHU \Rightarrow U(t,t')$$

$$|\psi,t\rangle = U|\psi,t'\rangle$$

$$\frac{d}{dt}|\psi,t\rangle = \overbrace{\left(\frac{d}{dt}U\right)|\psi,t'\rangle}^{-iHU|\psi,t'\rangle} + \cancelto{0}{U\frac{d}{dt}|\psi,t'\rangle}$$

$$\frac{d}{dt} A \qquad A(t) = U^{\dagger} A(t') U$$

$$\frac{dA}{dt} = i\hbar [A, H]$$

$$\frac{d}{dt} U = -iH(t) U$$

$$U = e^{-i\int H(t)\, dt}$$

? X

$$e^{A} = \sum_{n=0}^{\infty} \frac{A^{n}}{n!}$$

$$U(t, t-\Delta) \approx e^{-iH(t)\Delta t}$$

Approx $H(t) = H(t-\Delta)$

$H(t) \quad t < t < t-\Delta$

commute.

Good approx $\displaystyle\lim_{\Delta t \to 0}$

$$U(t,t') = \lim_{\Delta t \to 0} U(t, t-\Delta t)\, U(t-\Delta t, t-2\Delta t) \cdots U(t'+\Delta t, t')$$

$$e^{-iH(t)\Delta t}\, e^{-iH(t-\Delta t)\Delta t} \cdots e^{-iH(t-\dots\Delta t)\Delta t}$$

$$[H(t), H(\bar{t})] \longrightarrow = e^{-i\int_{t'}^{t} H(t_1)\, dt_1}$$

$$\lim_{\Delta t \to 0} e^{-iH(t)\Delta t} \cdots e^{-iH(t+\Delta t)\Delta t}$$

$$T\, e^{-i\int_{t'}^{t} H(t_1)\, dt_1}$$

$$\vdots$$

Magnus

$$T\, e^{-i\int H\, dt} = e^{\left\{ -i\int_{t'}^{t} H(t')\, dt' + \frac{1}{2}\int_{t'}^{t}\int_{t'}^{t_1} (-i)^2 [H(t_1), H(t_2)]\, dt_2\, dt_1 + \frac{1}{6}\int_{t'}^{t}\int_{t'}^{t_1}\int_{t'}^{t_2} [H(t_1),[H(t_2),H(t_3)]] + [H(t_3),[H(t_2),H(t_1)]] + \cdots \right\}}$$

Truncated Magnus $\rightarrow$ unitary op.

Almost all other approx do not.

$$|\phi\rangle, |\psi\rangle \qquad \langle\phi|U^{\dagger}U|\psi\rangle = \langle\phi|\psi\rangle$$

Magnus preserves Prob

$$H = H_1 + H_2 \delta(t)$$

$$\frac{d\psi}{dt} = H\psi$$

$$e^{-i\int H\,dt}$$

# Entanglement.

$\mathcal{A}$ $|\phi\rangle$ $\qquad$ $\mathcal{B}$ $|\psi\rangle$

$A_i$ $\qquad$ $B_j$

$$[A_i, B_j] = 0$$

Hilbert space is "Direct Prod."
Joint $|\phi\rangle|\psi\rangle$ sums of products. $|\phi_1\rangle|\psi_1\rangle$, $|\phi_2\rangle|\psi_2\rangle$.
$|\phi_1\rangle|\psi_1\rangle + |\phi_2\rangle|\psi_2\rangle$ is also in Joint.

$$|\Psi\rangle = |\phi_1\rangle|\psi_1\rangle + |\phi_2\rangle|\psi_2\rangle$$

and $|\phi_1\rangle \neq |\phi_2\rangle$

$|\psi_1\rangle \neq |\psi_2\rangle$

Entangled.

If $|\Psi\rangle$ entangled Then there exist operators $A_e, B_e$

$$\langle\Psi|A_e B_e|\Psi\rangle \neq \langle\Psi|A_e|\Psi\rangle\langle\Psi|B_e|\Psi\rangle$$

Then $|\Psi\rangle$ is entangled.

$$\langle xy\rangle \neq \langle x\rangle\langle y\rangle$$ : Corr. funct

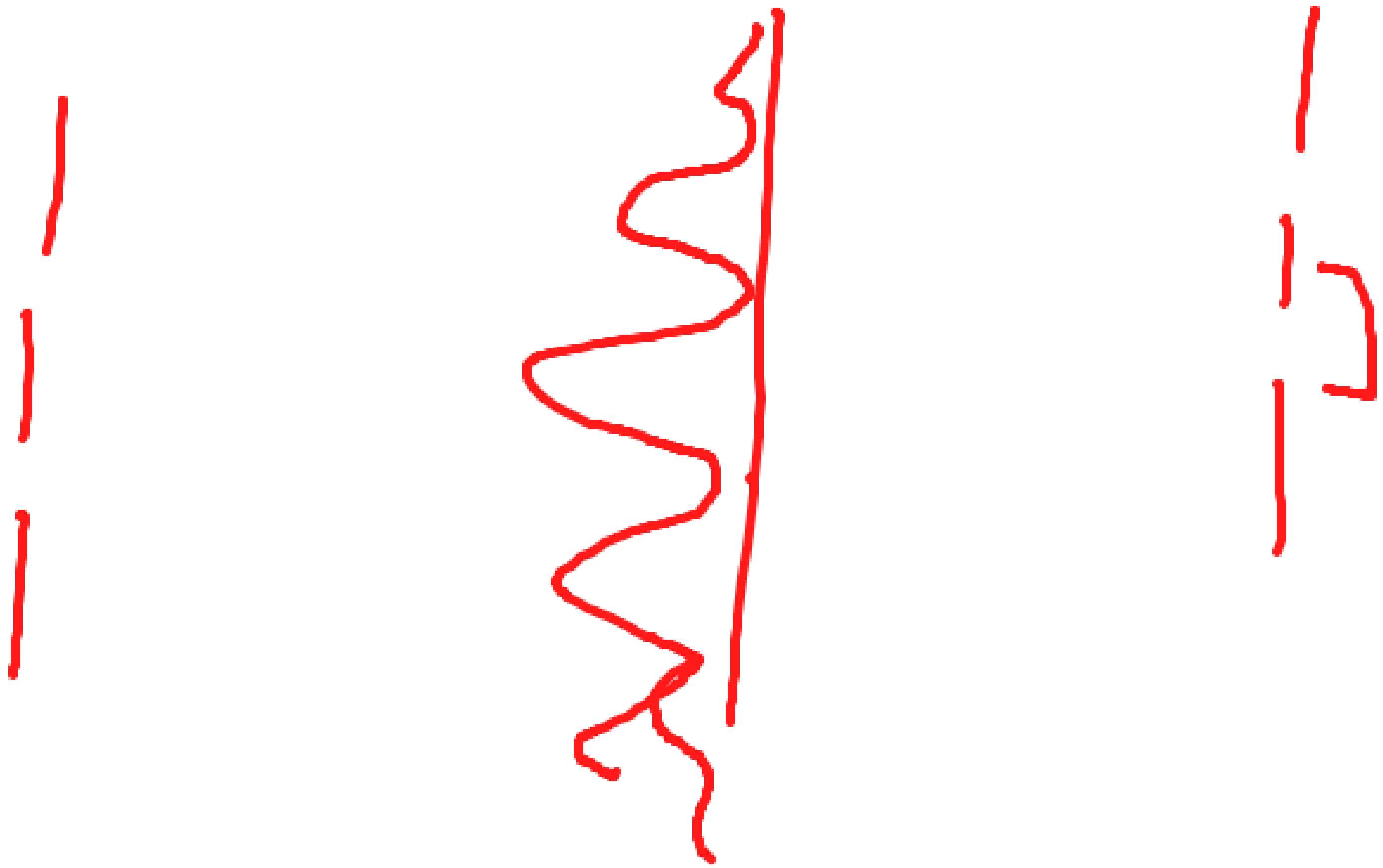
Entang in QM $\Longrightarrow$ Correlation

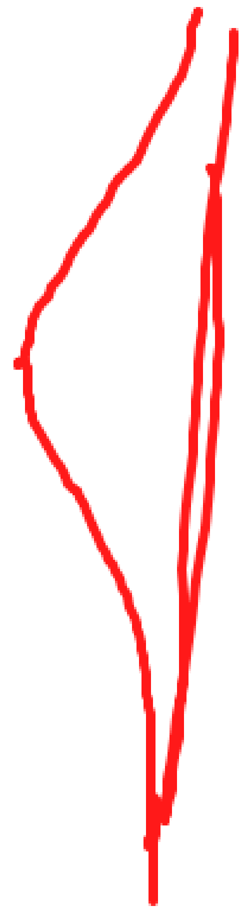
$$\langle (x-\langle x\rangle)^2 (p-\Delta p)^2 \rangle \geq |i\langle [x,p]\rangle|^2$$

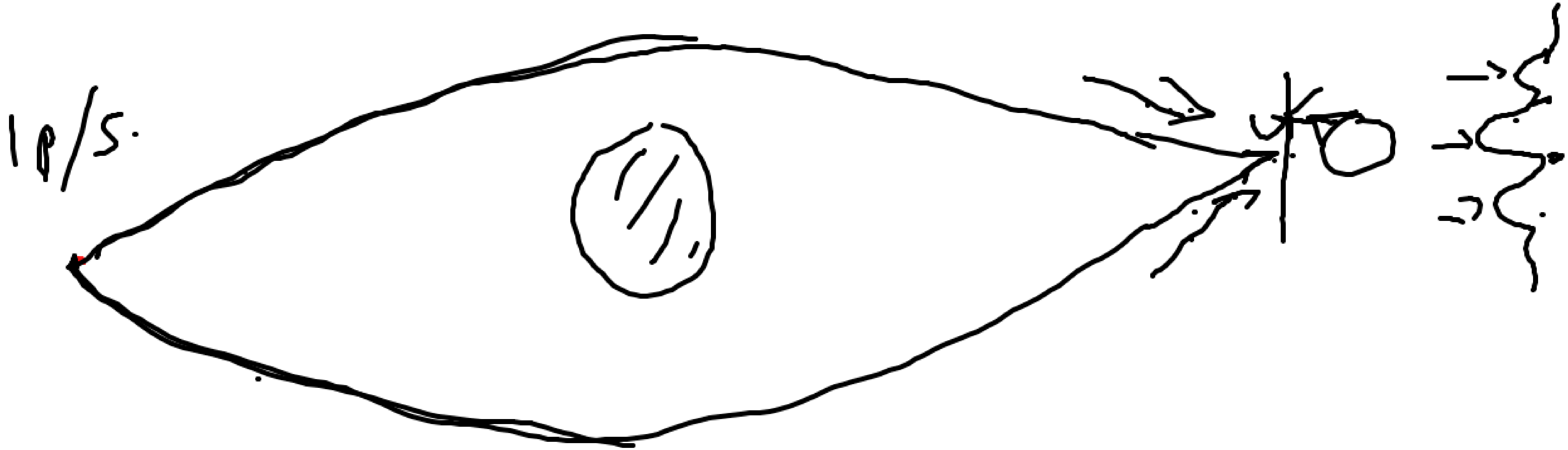
$$\frac{\hbar^2}{2^2}$$

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$|\psi\rangle$

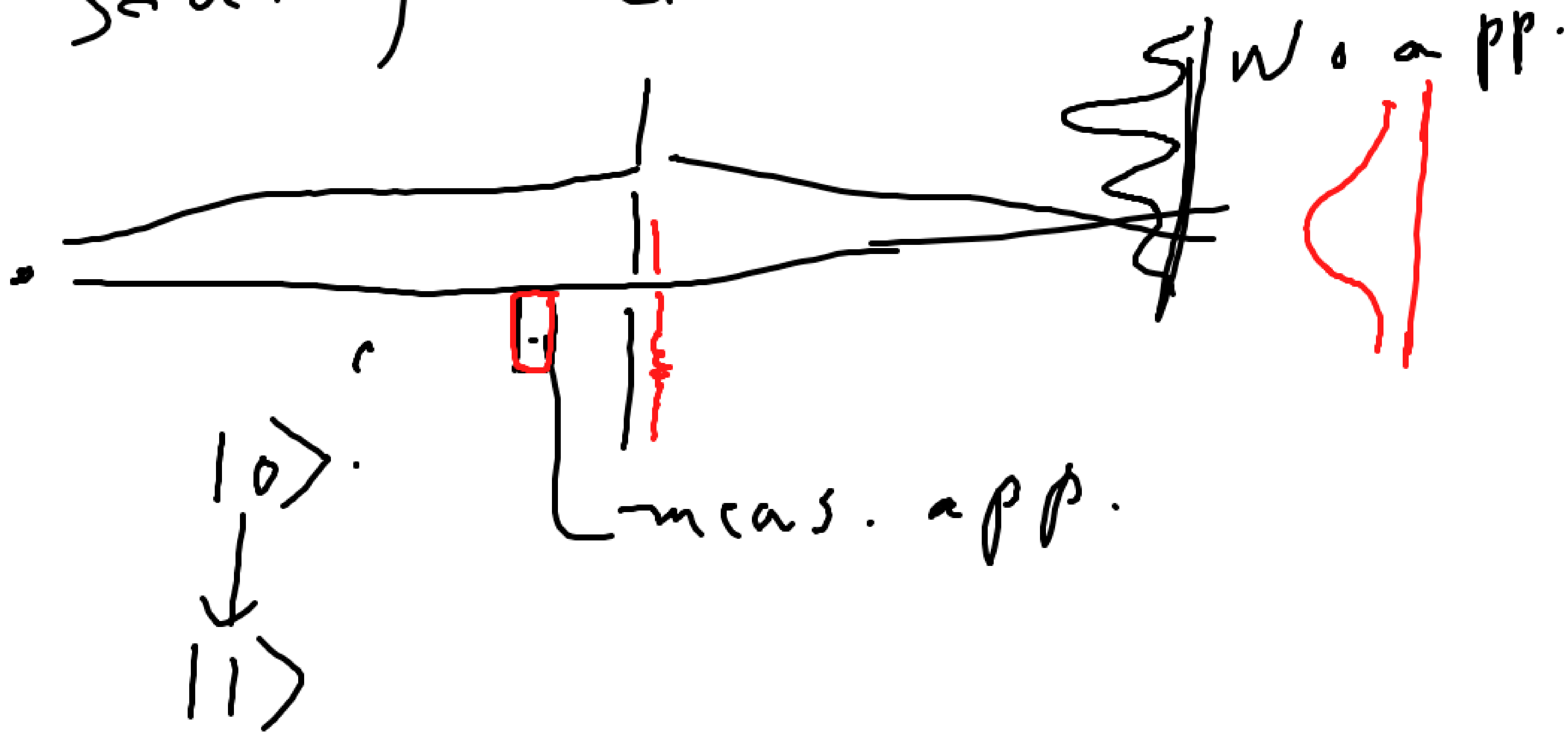


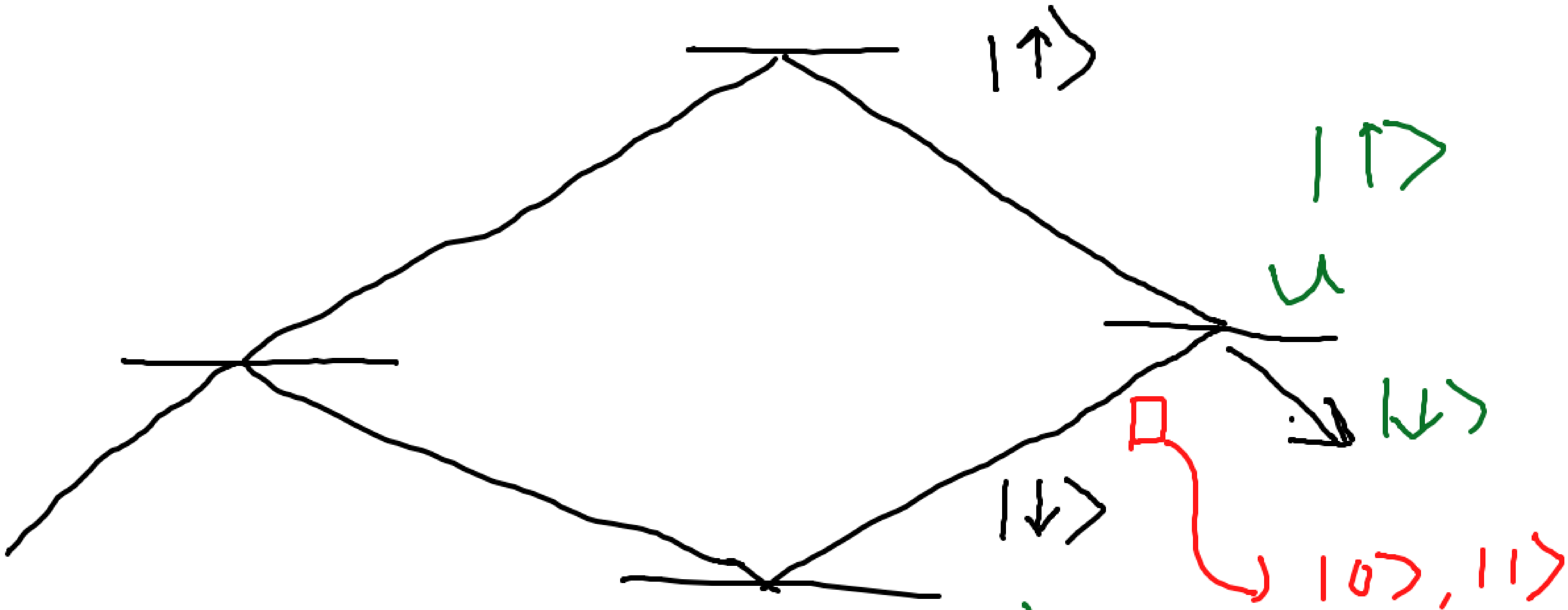





Delayed Choice.

Scully : Q. Eraser.






$$U|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

$$U|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)|0\rangle \Longrightarrow \underbrace{\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|0\rangle + |\downarrow\rangle|0\rangle\right)}_{|\psi\rangle}$$

$$U|\psi\rangle = \frac{1}{2}\Big(\left(|\uparrow\rangle - |\downarrow\rangle\right)|0\rangle + \left(|\uparrow\rangle + |\downarrow\rangle\right)|1\rangle\Big)$$

$$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \sigma_z$$

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \sigma_x$$


$$U|\psi\rangle = \frac{1}{2}\Big(|\uparrow\rangle\underbrace{\left(|0\rangle + |1\rangle\right)} + |\downarrow\rangle\left(-|0\rangle + |1\rangle\right)\Big)$$

$$\sigma \to |\sigma_z : 1\rangle + |\sigma_z : -1\rangle$$

$\boxed{\text{Bell's Thm}}$

Hardy's puzzle.