x, p $\quad [X], [P] \quad [X, P] = i\hbar$

$[X][P] - [P][X]$

$\underbrace{\partial_t}_{\frac{\partial}{\partial t}} [X] = i\hbar [X, H] \quad H = \frac{1}{2m} P^2$

$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$

1926 Schroed.

$i\partial_t \psi(x,t) = -\frac{1}{2m} \partial_x^2 \psi + V(x)\psi$

$P = i\hbar\partial_x$

$\Psi(x,t) \longrightarrow |\Psi\rangle$

$|\phi\rangle, |\Psi\rangle \Longrightarrow |\phi\rangle + |\Psi\rangle$

$a|\phi\rangle$ Vectors in Hilb. Sp.

Inner product. $\langle\Psi|$

$\langle\Psi|\phi\rangle$ $\quad \langle a\Psi|\phi\rangle = a^*\langle\Psi|\phi\rangle$

$\langle\Psi|\Psi\rangle \geq 0$ (= 0 only for $|\Psi\rangle \equiv 0$

$$\psi(x) \longrightarrow |\psi\rangle$$

$$\langle\psi| \longrightarrow \psi^*(x)$$

$$\langle\phi|\psi\rangle = \int \phi^*(x)\,\psi(x)\,dx$$

operators. linear

$$A\big(a|\phi\rangle + b|\psi\rangle\big) = a\,A|\phi\rangle + b\,A|\psi\rangle$$

$$|\psi(x)|^2$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2\,dx = 1$$

$$+\ |\psi(x)|^2$$

$\langle \psi | \psi \rangle = 1$ prob of something $= 1$.

$A|a\rangle = a|a\rangle$

Real

$A^\dagger = A$ $\quad \dagger = (A^T)^*$

$\{|a\rangle\} \Longrightarrow |\psi\rangle = \sum \langle a|\psi\rangle |a\rangle$

if $\langle a|a\rangle = 1$

$\langle a|a'\rangle = 0 \quad a \neq a'$

Meas. quantities must be Herm. Mat.

$$\partial_t |\psi\rangle = i\hbar H |\psi\rangle$$

$$\partial_t \langle\psi|\psi\rangle = \underbrace{\langle\psi| -i\hbar H^\dagger + i\hbar H |\psi\rangle}_{=0}$$

If state $|\phi\rangle$ is e-state of meas then

$|\langle\phi|\psi\rangle|^2 \rightarrow$ Prob of getting $\langle\phi|$ when meas $|\psi\rangle$

"Unitarity"

$|\psi\rangle$, A, $|a\rangle$ $a|$

$|\langle a|\psi\rangle|^2 = \text{Prob}_{\text{meas } A}^{a}$

$\psi(x) = \langle x|\psi\rangle$

$A \longrightarrow$ time indep

$|\psi\rangle \longrightarrow$ " dep.

$\partial_t |\psi\rangle = i\hbar |\psi\rangle$

$\partial_t A = i\hbar [A, H]$

$|\psi\rangle \to$ time indep.

Interp.

$|\psi\rangle$ $\quad A \quad P_a = |\langle a|\psi\rangle|^2$

$|\psi\rangle \Longrightarrow |a\rangle$

after meas

$|\psi\rangle \Longrightarrow |\psi, t\rangle \Longrightarrow$ Schroed.

$|\psi\rangle \Longrightarrow |a\rangle \Longrightarrow$ Meas.

$\psi(x,t)$

physical reality?

$$\partial_t A = i\hbar [A, H]$$

$|\psi\rangle \Longrightarrow |\psi\rangle$ prob.

$|a,t\rangle$

$A \Longrightarrow$ eigenstates.

$P_a \quad |\langle a,t|\psi\rangle|^2$

Dynamics $\longrightarrow$ operators.

state $\Longrightarrow$ Probabilities

$|\psi\rangle \longrightarrow$ like initial cond.

$A \longrightarrow$ dynamics.

Unitary transf.