Physics 407-09
Midterm Exam
Nov 2 2009

This exam consists of four (4) questions. All problems are worth the same number of marks.

Note that after you receive back your marked exams, you will be allowed one week to redo the exam as an assignment. The mark you get for the midterm will be the average of the two marks (the midterm proper and the midterm done as an assignment) but in any case you cannot get less than the midterm mark.

1) Consider the metric

$$d\tau^2 = (1+r^2)dt^2 - \frac{1}{1+r^2}dr^2 \tag{1}$$

What are the geodesic equations for this metric?

i) Show that the time t it takes for a light-like particle to travel from $r=0$ to $r=\infty$ is finite.

ii) Show that the distance from $r=0$ to $r=\infty$ along a $t = const$ surface actually is infinite.

iii) Show that any timelike geodesic (straight line) never reaches $r=\infty$.

[Note that this is actually a spacetime called Anti-DeSitter spacetime which is important these days in string theory precisely because infinity is just a finite time away].

2)a) If $H^A{}_B$, T_{AB}, S^{ABC} are tensors, and U_A, W^B are vectors, which of the following expressions are also tensors and if not why not? For those that are tensors what kind of vectors and how many are they functions of?

i) $T_{AB} + H^A{}_B$

ii) $S^{ABC}T_{AB} + H^A{}_C U_A$

iii)$\nabla_A T_{AB} - S^{AXC} g_{AC} g_{XB}$

iv) $H^A{}_A - H^X{}_X$

v) T^{XX}

vi) Write the the tensor $S_A{}^{BC}$ in terms of the above tensors and the metric tensor.

b) Given the metric $r^2 d\phi^2 + dr^2$, what are its Christofel symbols.

3.)i)Describe the "Time dilation" in special relativity. Explain what is happening?

ii) Show that any curve $\gamma(\lambda)$ lying within a level surface of a function f has zero inner product with the cotangent vector associated with that function, $\frac{\partial}{\partial\gamma}^A df_A = 0$.

4.) Consider the surface $X^2 + Y^2 = Z^2$ in the space with metric

$$ds^2 = dX^2 + dY^2 + dZ^2 \tag{2}$$

Find the metric on that surface (with $Z > 0$) in terms of the coordinates r θ where

$$X = r\cos(\theta) \tag{3}$$

$$Y = r\sin(\theta) \tag{4}$$

What is Z as a function of r, θ? What can you say about the distance from the origin ($r = 0$) of the circle defined by constant r, as compared with the circumference of the circle ?